Prep [3] - Second Term - Algebra - Unit [1] - Equations

Lesson [1] : Solving Two Equations Of First Degree In Two Variables

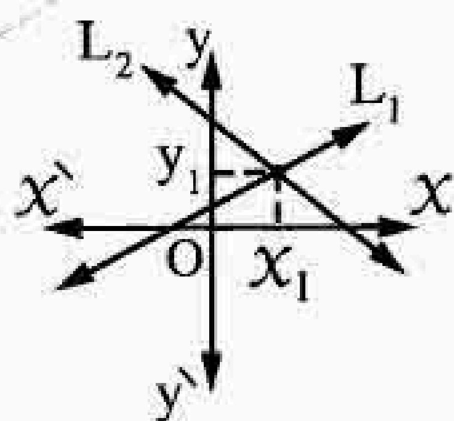
Graphically And Algebraically

First : Solving two equations of the first degree in two variables graphically

Then to solve the two equations graphically, we do as follows :

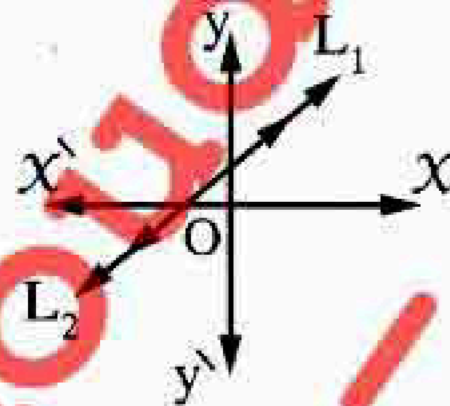
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 intersect at the point (x_1 , y_1)



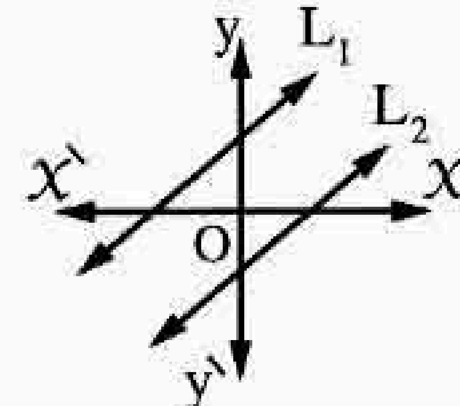
- There is a unique solution (x_1 , y_1)
- The S.S. = $\{(x_1 , y_1)\}$

2 L_1 and L_2 are coincident



- There is an infinite number of solutions

3 L_1 and L_2 are parallel



- There is no solution
- The S.S. = $\varnothing$

Second : Solving two equations of the first degree in two variables algebraically

For that purpose, we follow one of the two methods :

1 **Substituting method.** **2** **Omitting method.**

In the following, we will explain each of the two methods.

Solving life problems in this lesson

In this kind of problems, the solution takes the following steps :

1 Let one of the two unknown be x and the other be y

2 From the given data in the problem, form two equations of the first degree in x and y

3 Solve the two equations algebraically or graphically to get the values of x and y
It is preferable to solve them algebraically.

The following examples in the following table show each case of the previous cases.

Example (1)

$L_1 : 2x - y = 5$

$L_2 : x + 3y + 1 = 0$

$\because L_1 : y = 2x - 5$

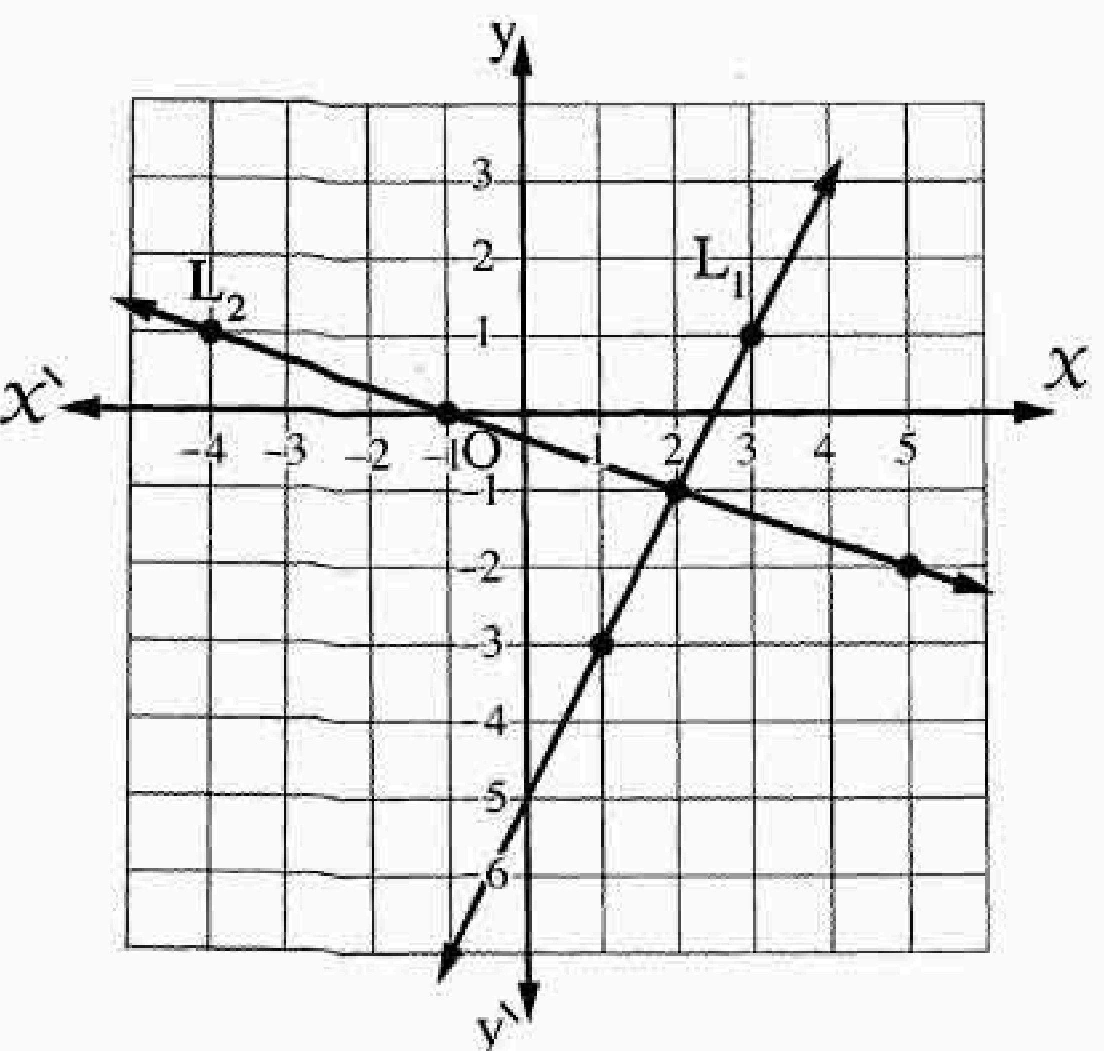
$\therefore$

x	1	2	3
y	−3	−1	1

$\because L_2 : x = -3y - 1$

$\therefore$

x	−1	−4	5
y	0	1	−2



The solution set in $\mathbb{R}^2 = \{(2, -1)\}$

Example (2)

$L_1 : y = 2x - 4$

$L_2 : 4x = 2y + 8$

$\because L_1 : y = 2x - 4$

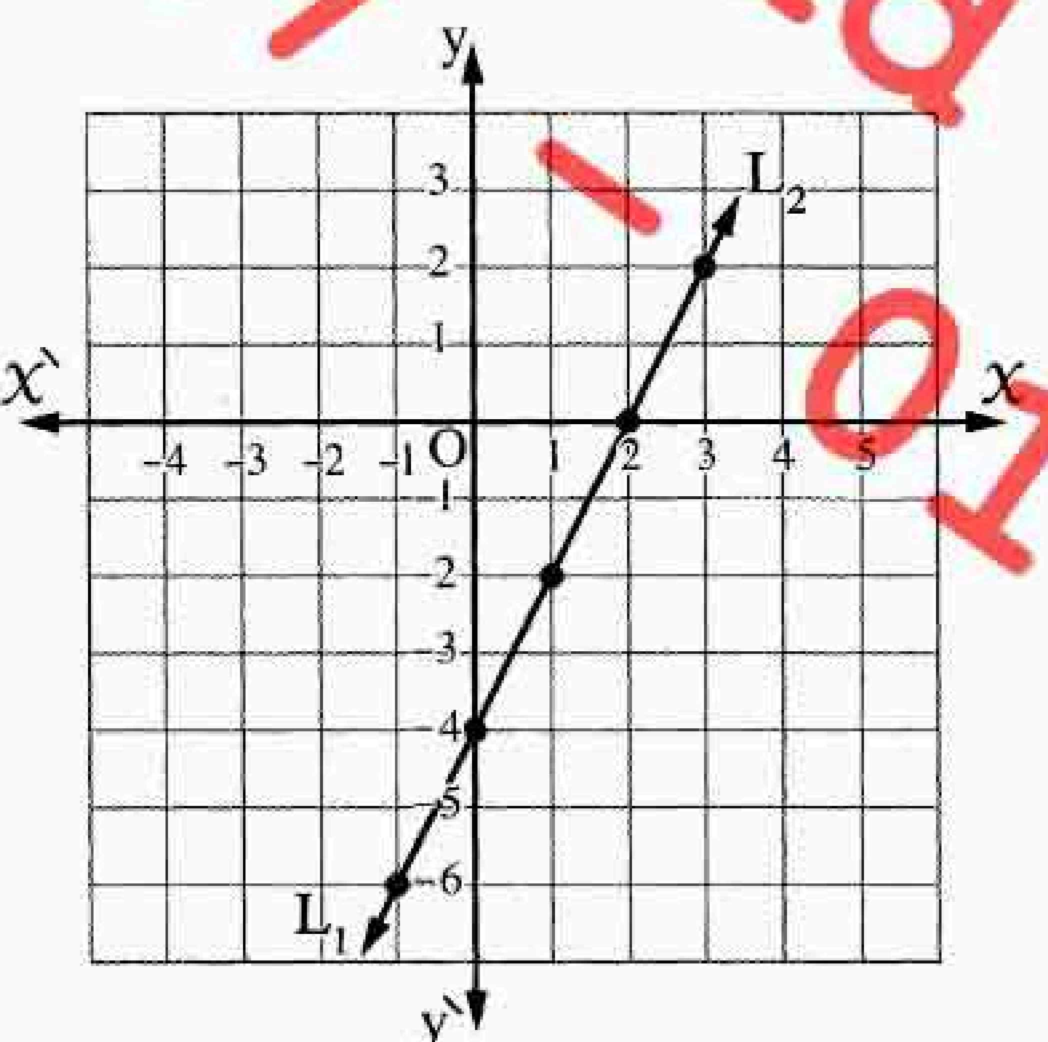
$\therefore$

x	0	1	−1
y	−4	−2	−6

$\because L_2 : x = \frac{2y+8}{4} = \frac{1}{2}y + 2$

$\therefore$

x	2	3	1
y	0	2	−2



The solution set in $\mathbb{R}^2$

$= \{(x, y) : y = 2x - 4, (x, y) \in \mathbb{R}^2\}$

Example (3)

$L_1 : y = 2x - 2$

$L_2 : 2y - 4x - 2 = 0$

$\because L_1 : y = 2x - 2$

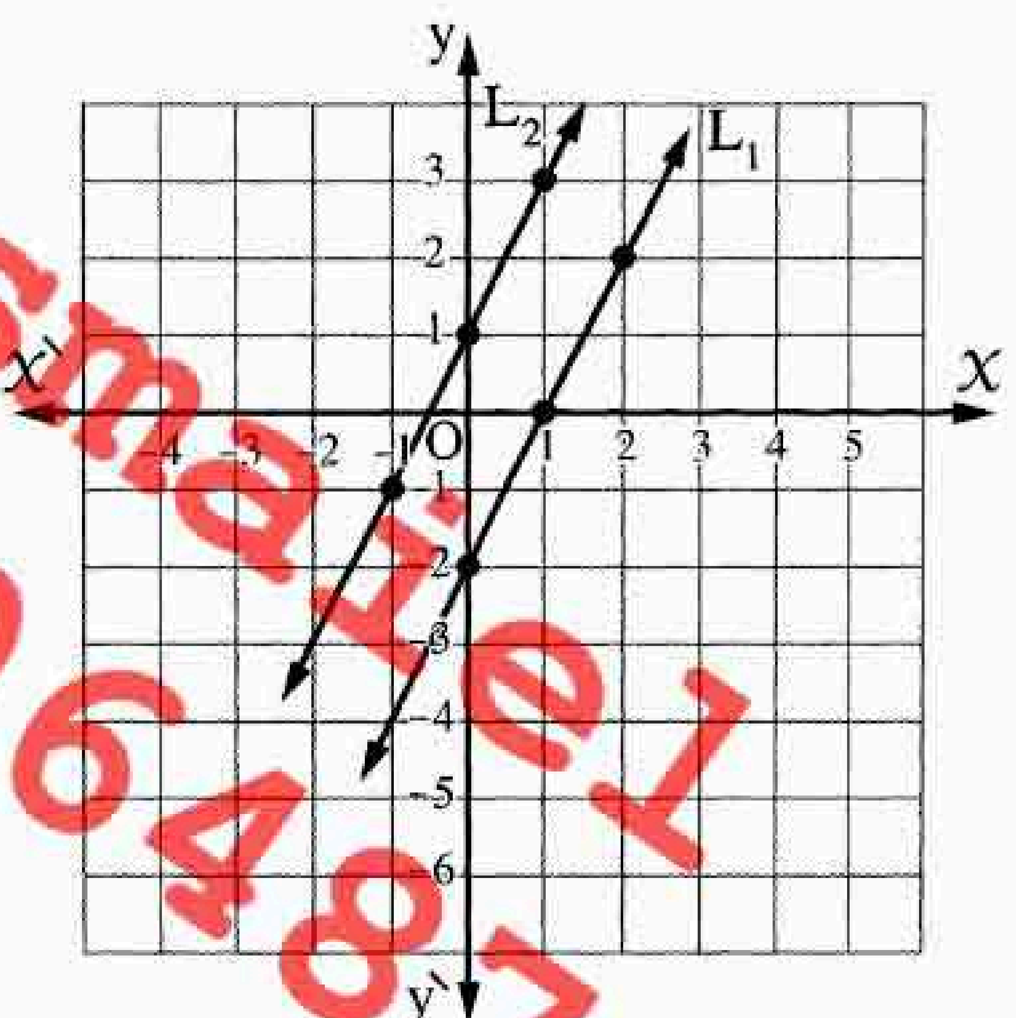
$\therefore$

x	2	1	0
y	2	0	−2

$\because L_2 : y = 2x + 1$

$\therefore$

x	0	1	−1
y	1	3	−1



The solution set in $\mathbb{R}^2 = \varnothing$

Examples :

1	🕮 **Find the solution set for each pair of the following two equations algebraically and graphically :** $y = x + 4$, $x + y = 4$ *(Souhag 2016 , Alexandria 2013)* « {(0 , 4)} »
2	**Find the value of a and b in each of the following :** 🕮 $a x + b y - 5 = 0$, $3 a x + b y = 17$ given that (3 , – 1) is a solution for the two equations *(El-Gharbia 2016 , El-Gharbia 2014)* « 2 , 1 »
3	If : $f(x) = a x^2 + b$, $f(1) = 5$, $f(2) = 11$, then find the value of a and b « 2 , 3 » *(El-Fayoum 2009)*
4	**In the opposite figure :** If the equation of straight line $L_1 : x + y = 6$ and equation of the straight line $L_2 : y - 2x = 0$ where $L_1 \cap L_2 = \{B\}$, O origin point , $A \in \overleftrightarrow{x x'}$ **Find :** The surface area of the triangle OAB *(El-Sharkia 2015)* « 12 square units »
5	The sum of two natural numbers is 63 and their difference is 11 **Find the two numbers.** *(El-Beheira 2016)* « 37 , 26 »
6	If three times a number is added to twice a second number the sum is 2 , and if the first number is added to three times the second number the sum is 10 , **find the two numbers.** *(El-Beheira 2015)* « – 2 , 4 »
7	🕮 A rectangle is with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. **Find the area of the rectangle.** *(Alex. 2012)* « 45 cm² »
8	🕮 Two acute angles in a right-angled triangle , the difference between their measures is 50 **Find the measure of each angle.** *(North Sinai 2015)* « 70° , 20° »
9	🕮 A two-digit number , the sum of its digits is 11 If the two digits are reversed , then the resulted number is 27 more than the original number , what is the original number ? *(Kafr El-Sheikh 2016)* « 47 »

Solutions

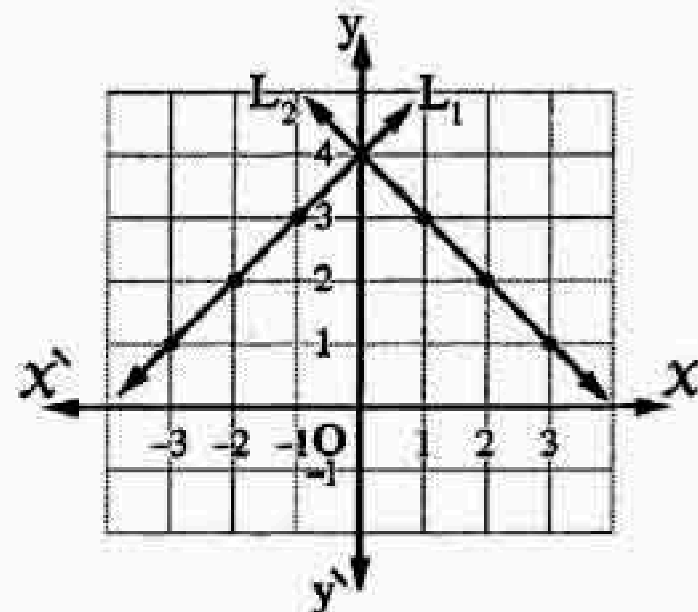
1

(1) **Graphically :**

$y = x + 4$, $y = 4 - x$

x	−1	−2	−3
y	3	2	1

x	1	2	3
y	3	2	1



from the graph , the S.S. = {(0 , 4)}

Algebraically :

Substituting by the value of y from the first equation in the second equation

$\therefore x + 4 + x = 4$ $\therefore 2x = 0$ $\therefore x = 0$

Substituting in the first equation : $\therefore y = 4$

$\therefore$ the S.S. = {(0 , 4)}

2

$\because$ (3 , − 1) is a solution for the equation

$a x + b y - 5 = 0$ $\therefore 3a - b = 5$ (1)

$\because$ (3 , − 1) is a solution for the equation

$3a x + b y = 17$ $\therefore 9a - b = 17$

$\therefore -9a + b = -17$ (2)

Adding (1) and (2) : $\therefore -6a = -12$ $\therefore a = 2$

Substituting in (1) : $\therefore b = 1$

3

$\because f(x) = a x^2 + b$, $f(1) = 5$

$\therefore a + b = 5$ (1)

, $\because f(2) = 11$

$\therefore 4a + b = 11$ (2)

Subtracting (1) from (2) :

$\therefore 3a = 6$ $\therefore a = 2$

Substituting in (1) : $\therefore b = 3$

4

$\because x + y = 6$ (1)

, $y - 2x = 0$ $\therefore y = 2x$ (2)

by substituting from (2) in (1)

$\therefore x + 2x = 6$ $\therefore 3x = 6$ $\therefore x = 2$

by substituting in (2) : $\therefore y = 4$ $\therefore$ B (2 , 4)

8

$\therefore$ the length of the altitude drawn from B to $\overleftrightarrow{AO}$ is 4 length units

, $\because$ A $\in$ straight line L_1 , A $\in \overleftrightarrow{xx'}$

at y = 0 in the equation $x + y = 6$

$\therefore x = 6$ $\therefore$ A (6 , 0)

$\therefore$ AO = 6 length units

$\therefore$ The area of Δ ABO = $\frac{1}{2} \times 6 \times 4 = 12$ square units

5

Let the two numbers be x and y

$\therefore x + y = 63$ (1) , $x - y = 11$ (2)

Adding (1) and (2) : $\therefore 2x = 74$ $\therefore x = 37$

Substituting in equ. (1) : $\therefore y = 26$

$\therefore$ The two numbers are 37 , 26

6

Let the two numbers be x and y

$\therefore x + y = 63$ (1) , $x - y = 11$ (2)

Adding (1) and (2) : $\therefore 2x = 74$ $\therefore x = 37$

Substituting in equ. (1) : $\therefore y = 26$

$\therefore$ The two numbers are 37 , 26

7

Let the length x cm. and the width be y cm.

$\therefore x - y = 4$ (1) , $2(x + y) = 28$ $\therefore x + y = 14$ (2)

Adding (1) and (2) : $\therefore 2x = 18$ $\therefore x = 9$

Substituting in (1) : $\therefore y = 5$

$\therefore$ The length = 9 cm. , the width = 5 cm.

$\therefore$ The area of the rectangle = $9 \times 5 = 45$ cm^2

8

Let the measure of the first angle be $x°$

and let the measure of the second angle be y°

$\therefore x + y = 90$ (1) , $x - y = 50$ (2)

Adding (1) and (2) : $\therefore 2x = 140$ $\therefore x = 70$

Substituting in (1) : $\therefore y = 20$

$\therefore$ The two measures are 70° , 20°

9

Let the units digit be x and the tens digit be y

$\therefore x + y = 11$ (1) ,

$(y + 10x) - (x + 10y) = 27$ $\therefore 9x - 9y = 27$

$\therefore x - y = 3$ (2)

Adding (1) and (2) : $\therefore 2x = 14$ $\therefore x = 7$

Substituting in (1) : $\therefore y = 4$

$\therefore$ The number is 47

Exercises

[A] : Choose The Correct Answer : -

1	The point (– 3 , 4) lies in .... quadrant. (a) fourth (b) third (c) second (d) first
2	If the point (5 , b – 7) lies on the X–axis , then b = ... (a) 2 (b) 3 (c) 5 (d) 7
3	The degree of the equation : $3X + 4y + Xy = 5$ is .... (a) zero. (b) first. (c) second. (d) third.
4	If $\frac{1}{3}X - \frac{5}{12} = \frac{1}{4}$, then X = ... (a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$
5	If $2X = 1$, then $\frac{1}{5}X$ = ... (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$
6	The solution set of the equation : $2X + 4 = 0$ in $\mathbb{N}$ is .... (a) {2} (b) {– 2} (c) {0} (d) $\varnothing$
7	If $3X = 1$, then $\frac{1}{5}X$ = ... (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
8	Twice the number X subtracted by 3 is .... (a) $X - 3$ (b) $2X + 3$ (c) $2X - 3$ (d) $3 - 2X$
9	If (5 , A – 4) = (B + 2 , 3) , then A + B = ... (a) 2 (b) 3 (c) 10 (d) 5
10	If (5 , X – 4) = (y , 3) , then X + y = ... (a) 25 (b) 12 (c) 8 (d) 6
11	If (7^{a-2} , 3) = (1 , b + 5) , then a + b = ... (a) – 1 (b) zero (c) 1 (d) 2
12	If $X + y = 5$, then $3X + 3y$ = ... (a) 5 (b) 3 (c) 8 (d) 15
13	If $X + 3y = 7$, then $X + 3(y + 5)$ = ... (a) 22 (b) 21 (c) 7 (d) 3
14	The solution set of the two equations : $X = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) {(– 1 , 1)} (b) {(1 , – 1)} (c) {(– 1 , – 1)} (d) {(1 , 1)}

15	The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point .... (a) $(-1, 2)$ (b) $(2, -1)$ (c) $(1, -2)$ (d) $(-1, -2)$
16	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are .... (a) parallel. (b) intersecting. (c) distant. (d) coincident.
17	The two straight lines : $3x = 7$, $2y = 9$ are .... (a) perpendicular. (b) coincide. (c) intersect and non perpendicular. (d) parallel.
18	The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is .... (a) $(-2, -3)$ (b) $(-2, 3)$ (c) $(2, -3)$ (d) $(2, 3)$
19	The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$
20	The point of intersection of the two straight lines : $x = 4$, $y - 3 = 0$ is .... (a) $(4, 3)$ (b) $(-4, 3)$ (c) $(-3, 4)$ (d) $(3, 4)$
21	The two straight lines : $x = 4$, $y = 3$ are intersecting in .... (a) $(4, 3)$ (b) $(0, 0)$ (c) $(3, 4)$ (d) $(-3, -4)$
22	The point of intersection of the two straight lines $x = 2$ and $x + y = 6$ is .... (a) $(2, 6)$ (b) $(2, 4)$ (c) $(4, 2)$ (d) $(6, 2)$
23	The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is .... (a) $(4, 2)$ (b) $(2, 4)$ (c) $(2, 2)$ (d) $(4, 4)$
24	The point of intersection of the two straight lines : $x + 2 = 0$ and $y = x$ is .... (a) $(2, 2)$ (b) $(2, 0)$ (c) $(-2, -2)$ (d) $(0, 0)$
25	If $x = 2$ and $y = 3$, then $(y - 2x)^{10} =$ .... (a) 10 (b) -1 (c) -10 (d) 1
26	The solution set of the two equations : $x + y = 0$, $y - 2 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(2, -2)\}$ (b) $\{(-2, 2)\}$ (c) $\{2, -2\}$ (d) $\{-2, 2\}$
27	The solution set of the two equations : $y - 5 = 0$, $y + x = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-5, 5)\}$ (b) $(5, -5)$ (c) $\{(0, 5)\}$ (d) $(-5, 5)$
28	Number of solutions of the two equations : $x + y = 2$, $y - 3 = 0$ together is .... (a) 3 (b) 2 (c) 1 (d) zero
29	The two straight lines : $x - 1 = 0$, $x + y = 5$ are .... (a) parallel. (b) coincide. (c) intersecting and not perpendicular. (d) perpendicular.

30 The number of solutions of the two equations : $x + y = 5$ and $y - 5 = 0$ is ....

(a) zero (b) 1 (c) 2 (d) 3

31 The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is ....

(a) (2 , 4) (b) (2 , 6) (c) (6 , 2) (d) (4 , 2)

32 The number of solutions of the two equations : $x + y = 2$ and $y + x = 3$ together in $\mathbb{R} \times \mathbb{R}$ is ....

(a) zero (b) 1 (c) 2 (d) 3

33 The number of solution of the two equations : $x + y = 2$, $x + y - 3 = 0$ is ....

(a) zero. (b) one. (c) two. (d) infinite numbers.

34 The two straight lines : $x + 2y = 1$ and $2x + 4y = 6$ are ....

(a) parallel (b) intersecting (c) perpendicular (d) coincide

35 The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(5 , 2)\}$ (b) $\{(2 , 4)\}$ (c) $\{(1 , 3)\}$ (d) $\{(3 , 1)\}$

36 If the point of intersection of the two straight lines $x - 1 = 0$ and $y = 2k$ lies on the fourth quadrant , then k may equal ....

(a) -5 (b) zero. (c) 1 (d) 5

37 The solution set of the two equation : $x + 3y = 5$ and $x - 3y = -1$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(2 , 1)\}$ (b) $\{(1 , 2)\}$ (c) $\{(2 , 3)\}$ (d) $\{(3 , 2)\}$

38 The two straight lines : $x + 5y = 1$, $x + 5y - 8 = 0$ are ....

(a) parallel. (b) coincide.
(c) perpendicular. (d) intersect and non perpendicular.

39 The number of solutions for the two equations : $x - \frac{1}{2}y = 4$, $2x - y = 2$ in $\mathbb{R}^2$ = ...

(a) 1 (b) 2 (c) infinit number. (d) zero.

40 The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersect in ....

(a) first quadrant. (b) second quadrant. (c) the origin point. (d) fourth quadrant.

41 If there is only one solution for the equation :

$x + 2y = 1$ and $2x + ky = 2$ in $\mathbb{R} \times \mathbb{R}$, then k cannot equal ....

(a) 2 (b) 4 (c) -2 (d) -4

42 If the two straight lines which represent the two equations : $x + 2y = 4$, $2x + ky = 11$ are parallel , then k = ...

(a) 7 (b) 6 (c) 4 (d) -4

43	If the two equations : $x + 3y = 4$, $2x + my = 8$ have infinite number of solutions , then m = ... (a) 2 (b) 6 (c) 3 (d) 1
44	If the two equations : $x + 3y = 4$, $x + ay = 7$ represent two parallel straight lines , then a = ... (a) $-\frac{1}{3}$ (b) -3 (c) 3 (d) 1
45	If the two equations : $x + 3y = 6$, $2x + ky = 12$ have an infinite number of solution in $\mathbb{R} \times \mathbb{R}$, then k = ... (a) 2 (b) 6 (c) 3 (d) 1
46	If the two equations : $x + 4y = 7$ and $3x + ky = 21$ have infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then k = ... (a) 4 (b) 7 (c) 12 (d) 21
47	If the sum of ages of a father and his sun now is 47 years , then the sum of their ages after 10 years = ... years. (a) 27 (b) 37 (c) 57 (d) 67
48	The solution set of the two equations : $x + 2y = 0$ and $2x - 3y = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-2, 0)\}$ (b) $\{(3, 2)\}$ (c) $\{(0, 0)\}$ (d) $\{(2, 3)\}$
49	If the age of a man now is x year , then his age after 5 years from now is .... years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
50	The sum of two consecutive integers is 17 , then the smaller number of them is .... (a) 8 (b) 9 (c) 17 (d) 72

[B] : Essay Problems : -

1	If $(2a + b , 3) = (18 , a - b)$: Find the value of a and b (Indicating the steps of the solution). 2018 Exam (5) Question (2) (a)
2	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two following equations simultaneously :** $x - y = 0$, $xy = 4$ 2017 Exam (8) Question (1) (b)
3	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $2x + y = 0$, $x + 2y = 3$ 2017 Exam (4) Question (2) (a)
4	**Find the solution set in $\mathbb{R} \times \mathbb{R}$:** $2x - y = 7$, $3x + y = 8$ (Explain your answer showing the steps solution)

	2018 Exam (9) Question (2) (a)
5	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations (graphically) :** $y = 3x - 1$, $x - y + 1 = \text{zero}$ 2018 Exam (8) Question (5) (b)
6	**Find the values of a , b knowing that (1 , – 1) is the solution of the two equations :** $ax + by = 7$ and $ax - by = 3$ 2017 Exam (5) Question (5) (b)
7	Two number , if three times a number is added to twice a second number the sum is 13 and if the first number is added to three times the second number the sum is 16 , find the two number. 2018 Exam (10) Question (5) (b)
8	A length of the rectangle is 5 cm. more than its width and its perimeter is 18 cm. , find the length and the width of the rectangle. 2017 Exam (8) Question (2) (b)
9	**Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :** $y + x = 7$, $y = 2x + 1$ 2017 Exam (2) Question (3) (a)
10	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 4$, $3x + 2y = 7$ 2018 Exam (4) Question (3) (a)
11	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $2x + y = 1$, $x + 2y = 5$ 2018 Exam (18) Question (4) (a)
12	**Find the solution set in $\mathbb{R} \times \mathbb{R}$:** $3x - y = 3$, $2x + y = 2$ (Explain your answer showing the steps of solution) 2017 Exam (9) Question (2) (a)
13	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S of the following two equations graphically :** $y = 2x - 3$, $x + 2y = 4$ 2018 Exam (23) Question (4) (a)
14	Find the value of a and b , knowing that : $\{(3 , -1)\}$ is the solution set of the two equations : $ax + by - 5 = 0$, $3ax + by = 17$ 2018 Exam (11) Question (2) (b)
15	The sum of two rational numbers is 12 , and three times the smallest number exceeds than twice the greatest number by one Find the two numbers. 2018 Exam (13) Question (4) (a)
16	A two-digit number the sum of its digits is 11 If the two digits are reversed , then the resulted number is 27 more than the original number. What is the original number ? 2017 Exam (11) Question (5) (b)

Homework

[A] : Choose The Correct Answer : -

1	The point (– 3 , 4) lies in .... quadrant. (a) fourth (b) third (c) second (d) first
2	If (5 , x – 4) = (y , 3) , then x + y = ... (a) 25 (b) 12 (c) 8 (d) 6
3	The point of intersection of the two straight lines : $x = 4$, $y - 3 = 0$ is .... (a) (4 , 3) (b) (– 4 , 3) (c) (– 3 , 4) (d) (3 , 4)
4	The number of solutions of the two equations : $x + y = 5$ and $y - 5 = 0$ is .... (a) zero (b) 1 (c) 2 (d) 3
5	The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersect in .... (a) first quadrant. (b) second quadrant. (c) the origin point. (d) fourth quadrant.
6	The sum of two consecutive integers is 17 , then the smaller number of them is .... (a) 8 (b) 9 (c) 17 (d) 72
7	If (5 , A – 4) = (B + 2 , 3) , then A + B = ... (a) 2 (b) 3 (c) 10 (d) 5
8	The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$
9	The two straight lines : $x - 1 = 0$, $x + y = 5$ are .... (a) parallel. (b) coincide. (c) intersecting and not perpendicular. (d) perpendicular.
10	The number of solutions for the two equations : $x - \frac{1}{2}y = 4$, $2x - y = 2$ in $\mathbb{R}^2$ = ... (a) 1 (b) 2 (c) infinit number. (d) zero.
11	If the age of a man now is x year , then his age after 5 years from now is .... years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
12	Twice the number x subtracted by 3 is .... (a) $x - 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $3 - 2x$
13	The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is .... (a) (– 2 , – 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (2 , 3)
14	Number of solutions of the two equations : $x + y = 2$, $y - 3 = 0$ together is .... (a) 3 (b) 2 (c) 1 (d) zero

15	The two straight lines : $x + 5y = 1$, $x + 5y - 8 = 0$ are …………… (a) parallel. (b) coincide. (c) perpendicular. (d) intersect and non perpendicular.
16	The solution set of the two equations : $x + 2y = 0$ and $2x - 3y = 0$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) $\{(-2, 0)\}$ (b) $\{(3, 2)\}$ (c) $\{(0, 0)\}$ (d) $\{(2, 3)\}$
17	If $3x = 1$, then $\frac{1}{5}x =$ …………… (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
18	The two straight lines : $3x = 7$, $2y = 9$ are …………… (a) perpendicular. (b) coincide. (c) intersect and non perpendicular. (d) parallel.
19	The solution set of the two equations : $y - 5 = 0$, $y + x = 0$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) $\{(-5, 5)\}$ (b) $(5, -5)$ (c) $\{(0, 5)\}$ (d) $(-5, 5)$
20	The solution set of the two equation : $x + 3y = 5$ and $x - 3y = -1$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) $\{(2, 1)\}$ (b) $\{(1, 2)\}$ (c) $\{(2, 3)\}$ (d) $\{(3, 2)\}$
21	If the sum of ages of a father and his sun now is 47 years , then the sum of their ages after 10 years = …………… years. (a) 27 (b) 37 (c) 57 (d) 67
22	The solution set of the equation : $2x + 4 = 0$ in $\mathbb{N}$ is …………… (a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $\varnothing$
23	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are …………… (a) parallel. (b) intersecting. (c) distant. (d) coincident.
24	The solution set of the two equations : $x + y = 0$, $y - 2 = 0$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) $\{(2, -2)\}$ (b) $\{(-2, 2)\}$ (c) $\{2, -2\}$ (d) $\{-2, 2\}$
25	If the point of intersection of the two straight lines $x - 1 = 0$ and $y = 2k$ lies on the fourth quadrant , then k may equal …………… (a) -5 (b) zero. (c) 1 (d) 5
26	If the two equations : $x + 4y = 7$ and $3x + ky = 21$ have infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then $k =$ …………… (a) 4 (b) 7 (c) 12 (d) 21
27	The solution set of the two equations : $x = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) $\{(-1, 1)\}$ (b) $\{(1, -1)\}$ (c) $\{(-1, -1)\}$ (d) $\{(1, 1)\}$

28	The point of intersection of the two straight lines : $x + 2 = 0$ and $y = x$ is …………… (a) (2 , 2) (b) (2 , 0) (c) (– 2, – 2) (d) (0 , 0)	
29	If $2x = 1$, then $\frac{1}{5}x =$ …………… (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$	
30	The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point …………… (a) (– 1 , 2) (b) (2 , – 1) (c) (1 , – 2) (d) (– 1 , – 2)	
31	If $x = 2$ and $y = 3$, then $(y - 2x)^{10} =$ …………… (a) 10 (b) – 1 (c) – 10 (d) 1	
32	The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is …………… (a) {(5 , 2)} (b) {(2 , 4)} (c) {(1 , 3)} (d) {(3 , 1)}	
33	If the two equations : $x + 3y = 6$, $2x + ky = 12$ have an infinite number of solution in $\mathbb{R} \times \mathbb{R}$, then k = …………… (a) 2 (b) 6 (c) 3 (d) 1	
34	If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then $x =$ …………… (a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$	
35	The two straight lines : $x + 2y = 1$ and $2x + 4y = 6$ are …………… (a) parallel (b) intersecting (c) perpendicular (d) coincide	
36	If the two equations : $x + 3y = 4$, $x + ay = 7$ represent two parallel straight lines , then a = …………… (a) $-\frac{1}{3}$ (b) – 3 (c) 3 (d) 1	
37	The degree of the equation : $3x + 4y + xy = 5$ is …………… (a) zero. (b) first. (c) second. (d) third.	
38	If $x + 3y = 7$, then $x + 3(y + 5) =$ …………… (a) 22 (b) 21 (c) 7 (d) 3	
39	The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is …………… (a) (4 , 2) (b) (2 , 4) (c) (2 , 2) (d) (4 , 4)	
40	The number of solution of the two equations : $x + y = 2$, $x + y - 3 = 0$ is …………… (a) zero. (b) one. (c) two. (d) infinite numbers.	
41	If the two equations : $x + 3y = 4$, $2x + my = 8$ have infinite number of solutions , then m = …………… (a) 2 (b) 6 (c) 3 (d) 1	

42 If the point $(5 , b - 7)$ lies on the X–axis , then $b =$ ....

(a) 2 (b) 3 (c) 5 (d) 7

43 If $X + y = 5$, then $3X + 3y =$ ....

(a) 5 (b) 3 (c) 8 (d) 15

44 The point of intersection of the two straight lines $X = 2$ and $X + y = 6$ is ....

(a) (2 , 6) (b) (2 , 4) (c) (4 , 2) (d) (6 , 2)

45 The number of solutions of the two equations : $X + y = 2$ and $y + X = 3$ together in $\mathbb{R} \times \mathbb{R}$ is ....

(a) zero (b) 1 (c) 2 (d) 3

46 If the two straight lines which represent the two equations : $X + 2y = 4$, $2X + ky = 11$ are parallel , then $k =$ ....

(a) 7 (b) 6 (c) 4 (d) – 4

47 If $(7^{a-2} , 3) = (1 , b + 5)$, then $a + b =$ ....

(a) – 1 (b) zero (c) 1 (d) 2

48 The two straight lines : $X = 4$, $y = 3$ are intersecting in ....

(a) (4 , 3) (b) (0 , 0) (c) (3 , 4) (d) (– 3 , – 4)

49 The point of intersection of the two straight lines : $y = 2$, $X + y = 6$ is ....

(a) (2 , 4) (b) (2 , 6) (c) (6 , 2) (d) (4 , 2)

50 If there is only one solution for the equation :

$X + 2y = 1$ and $2X + ky = 2$ in $\mathbb{R} \times \mathbb{R}$, then k cannot equal ....

(a) 2 (b) 4 (c) – 2 (d) – 4

[B] : Essay Problems : -

1 **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations graphically :**

$X + y = 4$, $2X - y = 2$

2018 Exam (2) Question (5) (a)

2 **Find in $\mathbb{R} \times \mathbb{R}$ algebraically the solution set of the two equations :**

$X - 2y = 0$, $2X + 3y = 7$

2017 Exam (14) Question (2) (a)

3 **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of :** $2X + y = 5$, $X = 7 - 2y$ graphically.

2017 Exam (10) Question (2) (a)

4 **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$2X = 1 - y$, $X + 2y = 5$ in $\mathbb{R} \times \mathbb{R}$

	2018 Exam (3) Question (2) (b)
5	**Find in ℝ × ℝ the solution set of the two equations graphically :** $y = x + 1$, $2x + y = 7$ 2018 Exam (14) Question (4) (a)
6	If (a , 2 b) is a solution for the equations $3x - y = 5$ and $x + y = -1$, find the value of a and b 2018 Exam (7) Question (2) (b)
7	Two acute angles in a right-angled triangle the difference between their measures is 50° Find the measure of each angle. 2018 Exam (11) Question (4) (a)
8	**Find in ℝ × ℝ the solution set of the following two equations simultaneously :** $x - y = 5$ and $3x + y = 11$ 2017 Exam (1) Question (3) (a)
9	**Find in ℝ × ℝ the solution set of the two following equations algebraically :** $x + 3y = 7$, $5x - y = 3$ 2018 Exam (1) Question (2) (a)
10	**Find algabrically the solution set of the two equations :** $2x - y = 3$, $x + 2y = 4$ 2018 Exam (17) Question (2) (a)
11	**Find in ℝ × ℝ the solution set of the two equations graphically :** $y = x - 1$, $2x + y = 5$ 2017 Exam (13) Question (3) (a)
12	**Find in ℝ × ℝ the solution set of the two equations :** $y = x + 4$, $x + y = 4$ algebraically and graphically 2017 Exam (17) Question (3) (b)
13	The sum of two rational numbers is 63 , and the difference between them is 11 , find the two numbers. 2017 Exam (12) Question (4) (a)
14	A rectangle with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. , find the area of the rectangle. 2018 Model Exam (1) Question (4) (a)

Lesson [2] : Solving An Equation Of Second Degree In One Unknown

Graphically And Algebraically

First : Solving an equation of the second degree in one unknown graphically :

To solve an equation of the second degree in one unknown graphically , we do the following steps :

1. Put the equation in the form : $a x^2 + b x + c = 0$
2. Assume that : $f(x) = a x^2 + b x + c$
3. Draw the curve of the function f by the method that you studied previously.
4. Determine the points of intersection of the function curve and x-axis , then the x-coordinates of these points of intersection are the solutions of the equation : $f(x) = 0$

   i.e. $a x^2 + b x + c = 0$

Second : Solving an equation of the second degree in one unknown using the general rule (general formula) :

The general rule (general formula) for solving an equation of the second degree in one unknown :

- If $a x^2 + b x + c = 0$ where a , b and c are real numbers , $a \neq 0$

, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e. The solution set of the equation $= \left\{ \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} , \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}$

Example [1]

Find in $\mathbb{R}$ the S.S. of the equation : $x^2 + 2x - 6 = 0$ to the nearest 3-decimal digits.

$\because x^2 + 2x - 6 = 0 \qquad \therefore a = 1 \ , \ b = 2 \ , \ c = -6$

$\because x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore x = \dfrac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-6)}}{2 \times 1} = \dfrac{-2 \pm \sqrt{4 + 24}}{2} = \dfrac{-2 \pm \sqrt{28}}{2}$

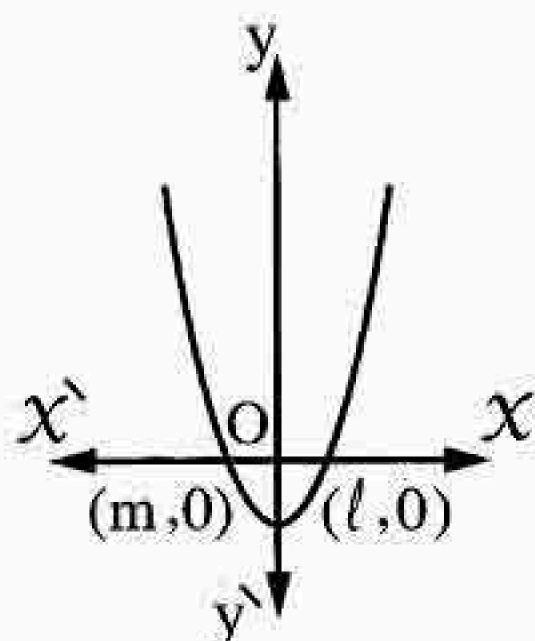
$\therefore x = \dfrac{-2 + \sqrt{28}}{2} \simeq 1.646$ or $x = \dfrac{-2 - \sqrt{28}}{2} \simeq -3.646$

$\therefore$ The S.S. $= \{1.646 , -3.646\}$

According to that , We find three cases :

Case (1)

The curve intersects x-axis at **two points**

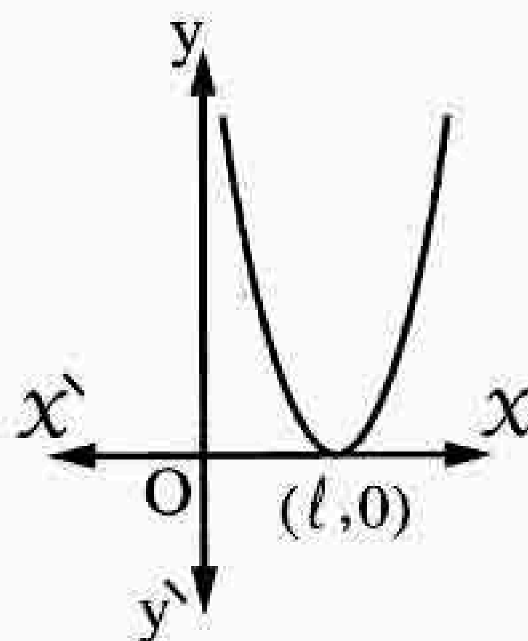


There are **two solutions** in $\mathbb{R}$

The S.S. = $\{\ell , m\}$

Case (2)

The curve touches x-axis at **one point**

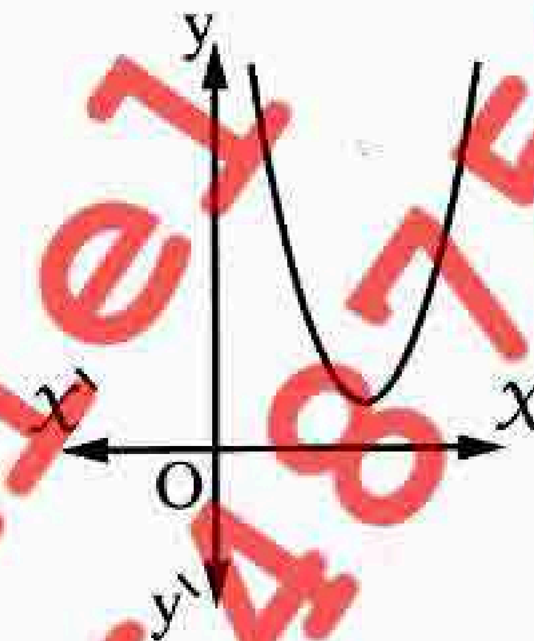


There is **a unique solution** in $\mathbb{R}$

The S.S. = $\{\ell\}$

Case (3)

The curve **does not intersect** x-axis



There is **no solution** in $\mathbb{R}$

The S.S. = $\varnothing$

The following examples show the previous cases :

Example (1)

Find graphically in $\mathbb{R}$ the S.S. of the equation :

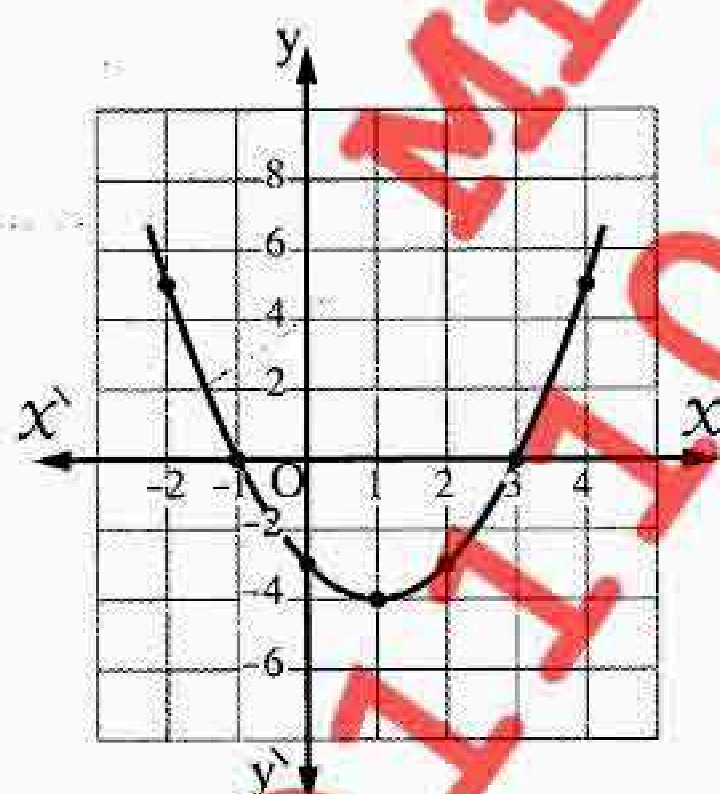
$x^2 - 2x - 3 = 0$

on the interval $[-2 , 4]$

Solution

Let $f(x) = x^2 - 2x - 3$

x	−2	−1	0	1	2	3	4
y	5	0	−3	−4	−3	0	5



From the graph ,

the S.S. = $\{3 , -1\}$

Example (2)

Find graphically in $\mathbb{R}$ the S.S. of the equation :

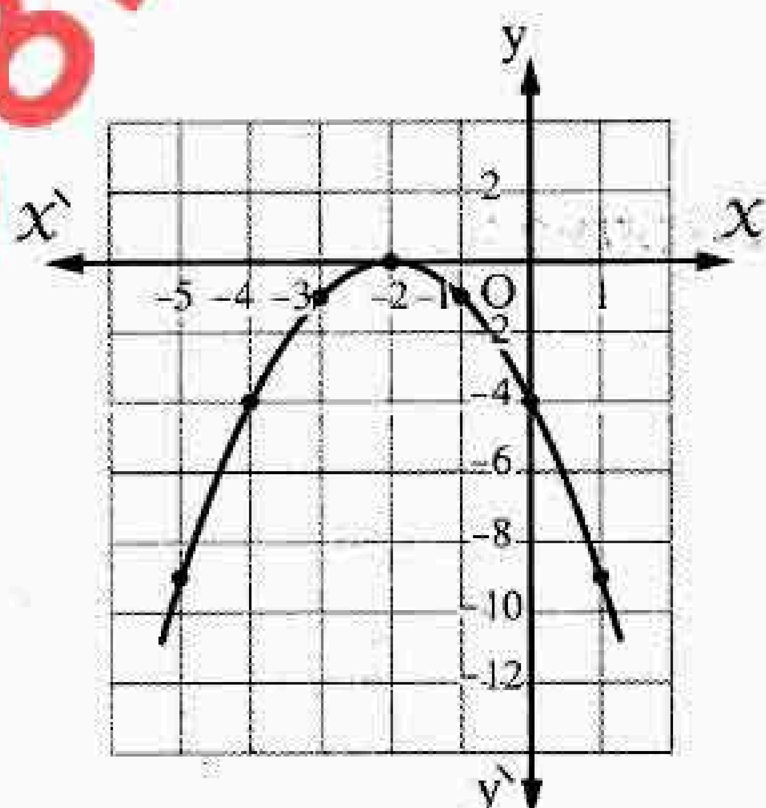
$-x^2 - 4x - 4 = 0$

on the interval $[-5 , 1]$

Solution

Let $f(x) = -x^2 - 4x - 4$

x	−5	−4	−3	−2	−1	0	1
y	−9	−4	−1	0	−1	−4	−9



From the graph ,

the S.S. = $\{-2\}$

Example (3)

Find graphically in $\mathbb{R}$ the S.S. of the equation :

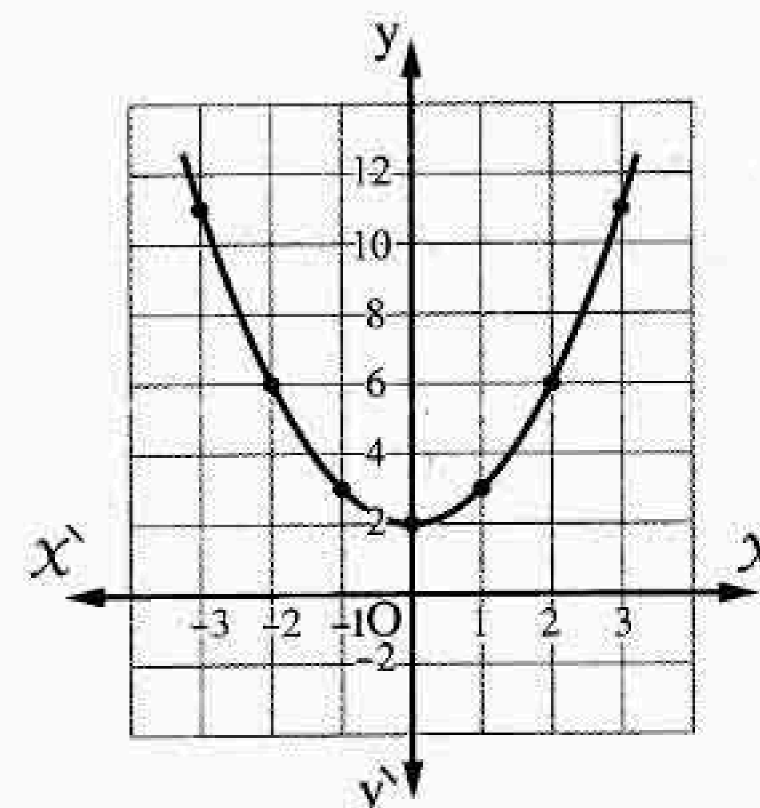
$x^2 + 2 = 0$

on the interval $[-3 , 3]$

Solution

Let $f(x) = x^2 + 2$

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11



From the graph ,

the S.S. = $\varnothing$

Examples :

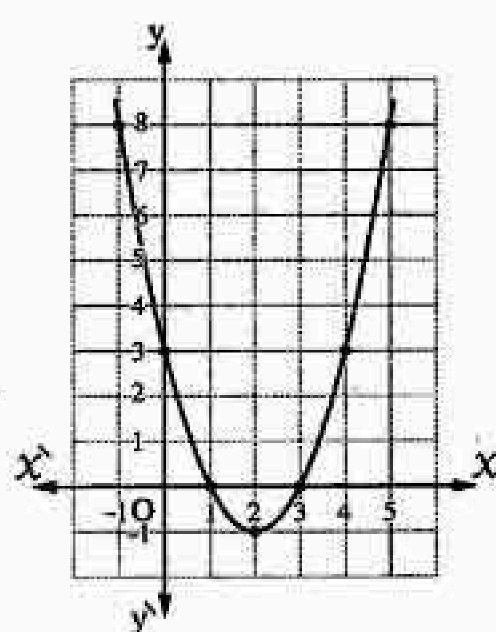
1	**Graph the function $f : f(x) = x^2 - 4x + 3$ on the interval $[-1 , 5]$ and from the graph , find :** (1) The minimum value of the function. (2) The equation of the axis of symmetry. (3) The S.S. of the equation $f(x) = 0$ *(Monofia 2012)*
2	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2 + 7x + 2 = 0$ approximating the result to the nearest tenth. *(El-Kalyoubia 2016)*
3	**Find in ℝ the S.S. of each of the following equations using the general formula :** 🕮 $2x^2 - 4x + 1 = 0$ rounding the result to three decimal digits. *(Qena 2012)*
4	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2 + 8x + 9 = 0$, where $\sqrt{7} \approx 2.65$ *(Ismailia 2009)*
5	**Find in ℝ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :** 🕮 $x(x-1) = 4$ *(Kafr El-Sheikh 2016)*
6	**Find in ℝ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :** $x - 3 = \frac{-1}{x}$ *(El-Monofia 2011)*
7	**Find in ℝ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :** 🕮 $\frac{8}{x^2} + \frac{1}{x} = 1$ *(El-Fayoum 2012)*

Solutions

1

$f(x) = x^2 - 4x + 3$

x	−1	0	1	2	3	4	5
y	8	3	0	−1	0	3	8



From the graph :

(1) The minimum value = − 1

(2) The equation of the axis of symmetry is $x = 2$

(3) The S.S. = {1 , 3}

2

$\because a = 1 , b = 7 , c = 2$

$\therefore x = \frac{-7 \pm \sqrt{49-8}}{2} = \frac{-7 \pm \sqrt{41}}{2}$

$\therefore x \approx -0.3$ or $x \approx -6.7$

$\therefore$ The S.S. = $\{-0.3 , -6.7\}$

3

$\because a = 2 , b = -4 , c = 1$

$\therefore x = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4 \pm \sqrt{8}}{4}$

$\therefore x \approx 0.293$ or $x \approx 1.707$

$\therefore$ The S.S. = $\{0.293 , 1.707\}$

4

$\because a = 1 , b = 8 , c = 9$

$\therefore x = \frac{-8 \pm \sqrt{64-36}}{2} = \frac{-8 \pm 2\sqrt{7}}{2}$

$= -4 \pm \sqrt{7} = -4 \pm 2.65$

$\therefore$ The S.S. = $\{-1.35 , -6.65\}$

5

$\because x^2 - x - 4 = 0$

$\therefore a = 1 , b = -1 , c = -4$

$\therefore x = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$

$\therefore x \approx 2.562$ or $x \approx -1.562$

$\therefore$ The S.S. = $\{2.562 , -1.562\}$

6

$\because x^2 - 3x + 1 = 0$

$\therefore a = 1 , b = -3 , c = 1$

$\therefore x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$\therefore x \approx 0.382$ or $x \approx 2.618$

$\therefore$ The S.S. = $\{0.382 , 2.618\}$

7

$\because -x^2 + x + 8 = 0$

$\therefore a = -1 , b = 1 , c = 8$

$\therefore x = \frac{-1 \pm \sqrt{1+32}}{-2} = \frac{-1 \pm \sqrt{33}}{-2}$

$\therefore x \approx -2.372$ or $x \approx 3.372$

$\therefore$ The S.S. = $\{-2.372 , 3.372\}$

Exercises

[A] : Choose The Correct Answer : -

1	If $2x^2 = 5$, then $6x^2 =$ .... (a) 5 (b) 10 (c) 15 (d) 20
2	If $f(x) = 6x^2 + 3x(1 - 2x)$ is a polynomial function , then its degree is .... (a) first. (b) second. (c) third. (d) fourth.
3	The solution set of the equation : $a x^2 + b x + c = 0$, $a \neq 0$ graphically is the set of x coordinates of the points of intersection of the curve of the function $f : f(x) = a x^2 + b x + c$ with the .... (a) y-axis (b) x-axis (c) symmetric line (d) straight line $y = 2$
4	If the curve of the quadratic function does not intersect the x-axis at any point , then the number of solutions of the equation $f(x) = 0$ in $\mathbb{R}$ is .... (a) zero (b) one solution. (c) two solutions. (d) an infinite number.
5	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1 , -1\}$ (c) $\{-1\}$ (d) $\varnothing$
6	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4 , 1\}$ (d) $\varnothing$
7	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit
8	If the solution set of the equation $x^2 - a x + 4 = 0$ is $\{-2\}$, then $a =$ .... (a) -2 (b) -4 (c) 2 (d) 4
9	If the solution set of the equation : $x^2 + m x + 9 = 0$ is $\{-3\}$, then $m =$ .... (a) 5 (b) 6 (c) ± 6 (d) zero
10	If $x = 3$ belongs to the solution set of the equation : $x^2 - a x - 6 = 0$, then $a =$ .... (a) 3 (b) 2 (c) 1 (d) -1
11	If $x = 1$ is the solution of the equation : $x^2 + m x + 4 = 0$, then $m =$ .... (a) 1 (b) -1 (c) zero (d) -5
12	If the curve of the function $f(x) = x^2 - a$ passing through the point (2 , 0) , then $a =$ .... (a) 4 (b) 7 (c) 9 (d) 16
13	If $(5 , x - 4) = (y , 3)$, then $x + y =$ .... (a) 25 (b) 12 (c) 8 (d) 6

14 The point of intersection of the two straight lines : $x = 4$, $y - 3 = 0$ is ....

(a) (4 , 3) (b) (– 4 , 3) (c) (– 3 , 4) (d) (3 , 4)

15 The number of solutions of the two equations : $x + y = 5$ and $y - 5 = 0$ is ....

(a) zero (b) 1 (c) 2 (d) 3

16 If the curve of the function f where $f(x) = x^2 - a$ passes through the point (1 , 0) , then a = ...

(a) – 2 (b) – 1 (c) zero (d) 1

17 If the curve of the function $f : f(x) = x^2 - x + c$ passing through the point (2 , 1) , then c = ...

(a) 2 (b) 1 (c) – 2 (d) – 1

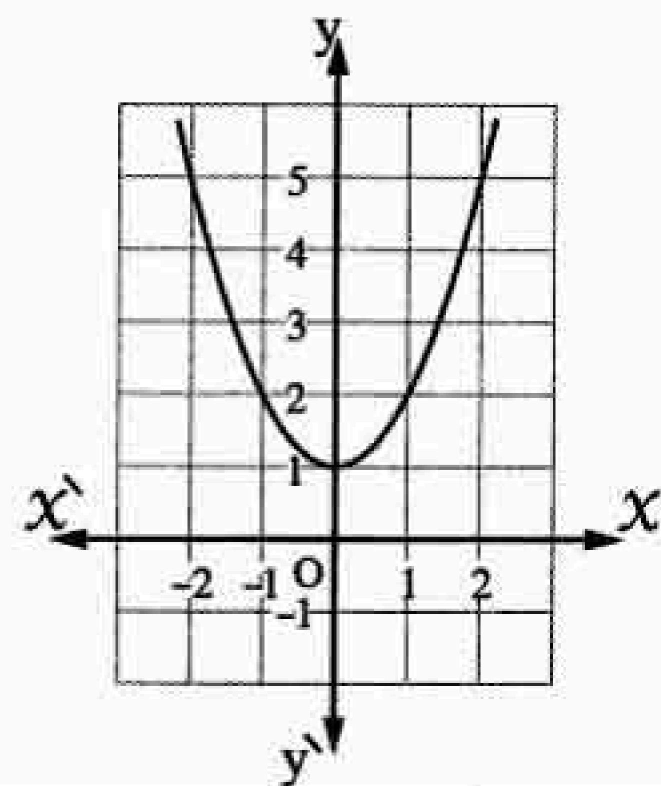
18 If the curve of the quadratic function f passes through the points (2 , 0) , (– 3 , 0) and (0 , – 6) , then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is ....

(a) $\{-2 , 3\}$ (b) $\{3 , 2\}$ (c) $\{2 , -3\}$ (d) $\{-3 , -6\}$

19 The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is ....

(a) $x = -4$ (b) $x = 0$ (c) $y = 0$ (d) $y = -4$

20 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(x) = 0$ is ....

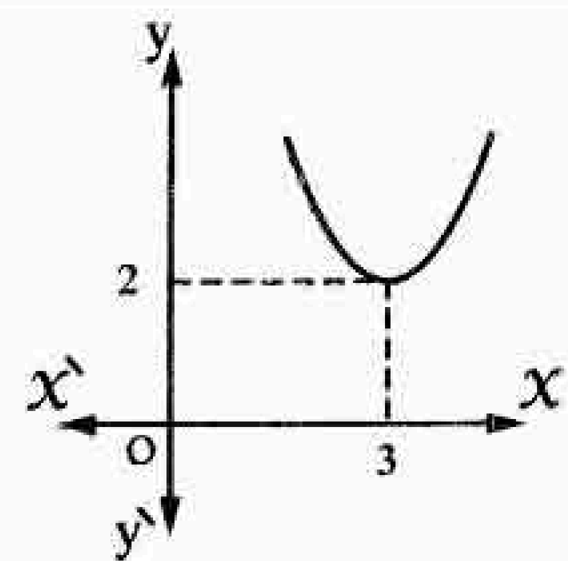


(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0 , 1)\}$

21 **In the opposite figure :**

The solution set of $f : f(x) = 0$ is ....



(a) $\varnothing$ (b) $\{3\}$

(c) $\{2 , 3\}$ (d) $\{2\}$

22 The point (– 3 , 4) lies in .... quadrant.

(a) fourth (b) third (c) second (d) first

23 If (5 , A – 4) = (B + 2 , 3) , then A + B = ...

(a) 2 (b) 3 (c) 10 (d) 5

24 The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(3 , 4)\}$ (b) $\{(4 , 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$

25	The two straight lines : $x - 1 = 0$, $x + y = 5$ are .... (a) parallel. (b) coincide. (c) intersecting and not perpendicular. (d) perpendicular.
26	Twice the number x subtracted by 3 is .... (a) $x - 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $3 - 2x$
27	The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is .... (a) $(-2, -3)$ (b) $(-2, 3)$ (c) $(2, -3)$ (d) $(2, 3)$
28	Number of solutions of the two equations : $x + y = 2$, $y - 3 = 0$ together is .... (a) 3 (b) 2 (c) 1 (d) zero
29	If $3x = 1$, then $\frac{1}{5}x =$ .... (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
30	The two straight lines : $3x = 7$, $2y = 9$ are .... (a) perpendicular. (b) coincide. (c) intersect and non perpendicular. (d) parallel.
31	The solution set of the two equations : $y - 5 = 0$, $y + x = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-5, 5)\}$ (b) $(5, -5)$ (c) $\{(0, 5)\}$ (d) $(-5, 5)$
32	The solution set of the equation : $2x + 4 = 0$ in $\mathbb{N}$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $\varnothing$
33	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are .... (a) parallel. (b) intersecting. (c) distant. (d) coincident.
34	The solution set of the two equations : $x + y = 0$, $y - 2 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(2, -2)\}$ (b) $\{(-2, 2)\}$ (c) $\{2, -2\}$ (d) $\{-2, 2\}$
35	If $2x = 1$, then $\frac{1}{5}x =$ .... (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$
36	The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point .... (a) $(-1, 2)$ (b) $(2, -1)$ (c) $(1, -2)$ (d) $(-1, -2)$
37	If $x = 2$ and $y = 3$, then $(y - 2x)^{10} =$ .... (a) 10 (b) -1 (c) -10 (d) 1
38	If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then $x =$ .... (a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$

39	The solution set of the two equations : $x = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-1, 1)\}$ (b) $\{(1, -1)\}$ (c) $\{(-1, -1)\}$ (d) $\{(1, 1)\}$
40	The point of intersection of the two straight lines : $x + 2 = 0$ and $y = x$ is .... (a) (2 , 2) (b) (2 , 0) (c) (– 2 , – 2) (d) (0 , 0)
41	The two straight lines : $x + 2y = 1$ and $2x + 4y = 6$ are .... (a) parallel (b) intersecting (c) perpendicular (d) coincide
42	The degree of the equation : $3x + 4y + xy = 5$ is .... (a) zero. (b) first. (c) second. (d) third.
43	If $x + 3y = 7$, then $x + 3(y + 5) =$ .... (a) 22 (b) 21 (c) 7 (d) 3
44	The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is .... (a) (4 , 2) (b) (2 , 4) (c) (2 , 2) (d) (4 , 4)
45	The number of solution of the two equations : $x + y = 2$, $x + y - 3 = 0$ is .... (a) zero. (b) one. (c) two. (d) infinite numbers.
46	If the point (5 , b – 7) lies on the x–axis , then b = ... (a) 2 (b) 3 (c) 5 (d) 7
47	If $x + y = 5$, then $3x + 3y =$ .... (a) 5 (b) 3 (c) 8 (d) 15
48	The point of intersection of the two straight lines $x = 2$ and $x + y = 6$ is .... (a) (2 , 6) (b) (2 , 4) (c) (4 , 2) (d) (6 , 2)
49	The number of solutions of the two equations : $x + y = 2$ and $y + x = 3$ together in $\mathbb{R} \times \mathbb{R}$ is .... (a) zero (b) 1 (c) 2 (d) 3
50	If $(7^{a-2}, 3) = (1, b + 5)$, then a + b = ... (a) – 1 (b) zero (c) 1 (d) 2
51	The two straight lines : $x = 4$, $y = 3$ are intersecting in .... (a) (4 , 3) (b) (0 , 0) (c) (3 , 4) (d) (– 3 , – 4)
52	The point of intersection of the two straight lines : $y = 2$, $x + y = 6$ is .... (a) (2 , 4) (b) (2 , 6) (c) (6 , 2) (d) (4 , 2)

[B] : Essay Problems : -

1	Graph the function f where $f(x) = x^2 - 1$, $x \in [-3 , 3]$, from the graph find the solution set of the equation : $x^2 - 1 = 0$ 2018 Model Exam (2) Question (5) (b)
2	Using the general formula to find the solution set of the equation : $x^2 - 2x - 4 = 0$ approximating the result to the nearest one decimal place. 2017 Exam (1) Question (2) (a)
3	**Find in $\mathbb{R}$ the solution set for the following equations by using the formula in :** $x^2 - 3x + 1 = 0$, knowing that $\sqrt{5} = 2.24$ 2018 Exam (9) Question (4) (a)
4	**By using the general rule , find in $\mathbb{R}$ the solution of the equation :** $x^2 + 7x + 2 =$ zero , approximating the result to the nearest tenth. 2017 Exam (4) Question (4) (a)
5	**By using the formula find in $\mathbb{R}$ the solution set of the equation** $3x^2 - 5x + 1 = 0$ rounding the result to two decimal places. 2018 Exam (21) Question (5) (a)
6	**Using the general rule find in $\mathbb{R}$ the S.S. of the equation :** $3x^2 = 5x - 1$ (given that $\sqrt{13} \approx 3.61$) 2018 Exam (20) Question (3) (b)
7	Find in $\mathbb{R}$ the solution set of the equation $x(x - 3) = -1$, using the general formula (approximating the results to the nearest tenth) 2018 Exam (7) Question (1) (b)
8	**Find in $\mathbb{R}$ the solution set of the equation :** $\frac{x}{3} = \frac{1}{5 - x}$, using the general rule and rounding the results to two decimal places. 2017 Exam (7) Question (2) (a)
9	Draw the function curve f where $f(x) = x^2 - 2x + 1$ in the interval $[-1 , 3]$ **From the drawing find :** the solution set of the equation $x^2 - 2x + 1 = 0$ 2017 Exam (15) Question (2) (a)
10	**Using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $x^2 - 2x + 4 =$ zero 2017 Exam (17) Question (5) (a)
11	**Find in $\mathbb{R}$ the solution set of the following equation by using the general rule :** $x^2 - 4x + 1 = 0$ rounding the results to two decimal places. 2018 Exam (1) Question (3) (a)
12	**By using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $2x^2 - 5x + 1 = 0$ "approximate the result to the nearest one decimal". 2018 Model Exam (1) Question (2) (a)

Homework

[A] : Choose The Correct Answer : -

1	The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(5, 2)\}$ (b) $\{(2, 4)\}$ (c) $\{(1, 3)\}$ (d) $\{(3, 1)\}$
2	If the two equations : $x + 3y = 4$, $x + ay = 7$ represent two parallel straight lines , then a = ... (a) $-\frac{1}{3}$ (b) -3 (c) 3 (d) 1
3	If the solution set of the equation $x^2 - ax + 4 = 0$ is $\{-2\}$, then a = ... (a) -2 (b) -4 (c) 2 (d) 4
4	The solution set of the equation : $ax^2 + bx + c = 0$, $a \neq 0$ graphically is the set of x coordinates of the points of intersection of the curve of the function $f : f(x) = ax^2 + bx + c$ with the .... (a) y-axis (b) x-axis (c) symmetric line (d) straight line y = 2
5	If the two equations : $x + 3y = 4$, $2x + my = 8$ have infinite number of solutions , then m = ... (a) 2 (b) 6 (c) 3 (d) 1
6	If the solution set of the equation : $x^2 + mx + 9 = 0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero
7	If the curve of the function $f : f(x) = x^2 - x + c$ passing through the point (2 , 1) , then c = ... (a) 2 (b) 1 (c) -2 (d) -1
8	If the two straight lines which represent the two equations : $x + 2y = 4$, $2x + ky = 11$ are parallel , then k = ... (a) 7 (b) 6 (c) 4 (d) -4
9	If $x = 3$ belongs to the solution set of the equation : $x^2 - ax - 6 = 0$, then a = ... (a) 3 (b) 2 (c) 1 (d) -1
10	If the curve of the quadratic function does not intersect the x-axis at any point , then the number of solutions of the equation $f(x) = 0$ in $\mathbb{R}$ is .... (a) zero (b) one solution. (c) two solutions. (d) an infinite number.
11	If there is only one solution for the equation : $x + 2y = 1$ and $2x + ky = 2$ in $\mathbb{R} \times \mathbb{R}$, then k cannot equal .... (a) 2 (b) 4 (c) -2 (d) -4

12	If $2x^2 = 5$, then $6x^2 =$ .... (a) 5 (b) 10 (c) 15 (d) 20
13	If the curve of the quadratic function f passes through the points $(2, 0)$, $(-3, 0)$ and $(0, -6)$, then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is .... (a) $\{-2, 3\}$ (b) $\{3, 2\}$ (c) $\{2, -3\}$ (d) $\{-3, -6\}$
14	The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersect in .... (a) first quadrant. (b) second quadrant. (c) the origin point. (d) fourth quadrant.
15	The sum of two consecutive integers is 17 , then the smaller number of them is .... (a) 8 (b) 9 (c) 17 (d) 72
16	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\varnothing$
17	The number of solutions for the two equations : $x - \frac{1}{2}y = 4$, $2x - y = 2$ in $\mathbb{R}^2 =$ .... (a) 1 (b) 2 (c) infinit number. (d) zero.
18	If the age of a man now is x year , then his age after 5 years from now is .... years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
19	The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is .... (a) $x = -4$ (b) $x = 0$ (c) $y = 0$ (d) $y = -4$
20	The two straight lines : $x + 5y = 1$, $x + 5y - 8 = 0$ are .... (a) parallel. (b) coincide. (c) perpendicular. (d) intersect and non perpendicular.
21	The solution set of the two equations : $x + 2y = 0$ and $2x - 3y = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-2, 0)\}$ (b) $\{(3, 2)\}$ (c) $\{(0, 0)\}$ (d) $\{(2, 3)\}$
22	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4, 1\}$ (d) $\varnothing$
23	If $x = 1$ is the solution of the equation : $x^2 + mx + 4 = 0$, then $m =$ .... (a) 1 (b) -1 (c) zero (d) -5
24	If the curve of the function $f(x) = x^2 - a$ passing through the point $(2, 0)$, then $a =$ .... (a) 4 (b) 7 (c) 9 (d) 16

25 If the sum of ages of a father and his sun now is 47 years , then the sum of their ages after 10 years = ... years.

(a) 27 (b) 37 (c) 57 (d) 67

26 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(X) = 0$ is ....

(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0, 1)\}$

27 If the point of intersection of the two straight lines $X - 1 = 0$ and $y = 2k$ lies on the fourth quadrant , then k may equal ....

(a) -5 (b) zero. (c) 1 (d) 5

28 If the two equations : $X + 4y = 7$ and $3X + ky = 21$ have infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) 4 (b) 7 (c) 12 (d) 21

29 In equation : $aX^2 + bX + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots.

(a) 1 (b) 2 (c) zero. (d) infinit

30 If $f(X) = 6X^2 + 3X(1 - 2X)$ is a polynomial function , then its degree is ....

(a) first. (b) second. (c) third. (d) fourth.

31 If the two equations : $X + 3y = 6$, $2X + ky = 12$ have an infinite number of solution in $\mathbb{R} \times \mathbb{R}$, then k = ...

(a) 2 (b) 6 (c) 3 (d) 1

32 **In the opposite figure :**

The solution set of $f : f(X) = 0$ is ....

(a) $\varnothing$ (b) $\{3\}$

(c) $\{2, 3\}$ (d) $\{2\}$

33 If the curve of the function f where $f(X) = X^2 - a$ passes through the point $(1, 0)$, then a = ...

(a) -2 (b) -1 (c) zero (d) 1

[B] : Essay Problems : -

1	Draw the graphical representation of the function $f(x) = x^2 - 2x - 3$ in the interval $[-2, 4]$ and from the drawing , find the solution set of the equation $x^2 - 2x - 3 = 0$ 2018 Exam (6) Question (4) (a)
2	By using the formula , find in $\mathbb{R}$ the solution set of the equation : $x^2 - 2x - 6 = 0$ (Approximate to the nearest one decimal) 2018 Exam (3) Question (4) (b)
3	**By using the general rule find in $\mathbb{R}$ the solution set of the equation :** $x^2 - 5x + 3 = 0$, approximating the result to the nearest one decimal digit. 2018 Exam (14) Question (3) (a)
4	**By using the formula find in $\mathbb{R}$ the solution set of the equation :** $2x^2 - 5x - 1 = 0$ approximating the result to the nearest one decimal. 2018 Exam (22) Question (3) (b)
5	**Find in $\mathbb{R}$ the solution set of the following equation by using the general formula :** $2x^2 = 5x + 1$ 2017 Exam (14) Question (3) (a)
6	**Find the solution set of the equation :** $x^2 = 2(x + 6)$ where $x \in \mathbb{R}$, given that : $\sqrt{52} = 7.2$ 2017 Exam (6) Question (2) (a)
7	**By using the general formula find in $\mathbb{R}$ the S.S. of :** $x^2 - x - 4 = 0$ where $\sqrt{17} \approx 4.12$ 2018 Exam (16) Question (3) (a)
8	**Find in $\mathbb{R}$ the solution set of the equation :** $x^2 - 2x - 9 = 0$ where $\sqrt{10} \simeq 3.16$ 2017 Exam (18) Question (3) (a)
9	**Find in $\mathbb{R}$ the solution set for the following equation by using the formula :** $x^2 - 6x + 7 = 0$ (Rounding the results to two decimal places) 2017 Exam (9) Question (3) (a)
10	**Find in $\mathbb{R}$ the solution set of the equation :** $3x^2 - 5x - 1 = 0$ approximating the result to the nearest two decimals. 2018 Exam (17) Question (4) (a)
11	**Solve in $\mathbb{R}$ using the (general rule) the equation :** $3x^2 = 5x + 4$ approximating the result to the nearest two decimals. 2018 Exam (12) Question (3) (a)
12	**By using the general rule solve in $\mathbb{R}$ the equation :** $x(x - 1) = 4$ taking $\sqrt{17} \approx 4.12$ 2018 Exam (4) Question (2) (a)
13	**By using the general formula , find in $\mathbb{R}$ the solution set for the following equation :** $(x - 4)(x - 2) = 1$ (knowing that : $\sqrt{2} \approx 1.41$) 2018 Exam (5) Question (2) (b)

Lesson [3] : Solving Two Equations In Two Variables, One Is Of The First Degree And The Other Is Of The Second Degree

Remember that :

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$

Remember that :

- Perimeter of the rectangle = (Length + width) × 2
- Area of the rectangle = Length × width
- Perimeter of the square = side length × 4
- Area of the square = side length × itself

Examples :

1	**Find the S.S. algebraically :** $x - 2 = 0$, $x^2 + xy + y^2 = 7$ in $\mathbb{R} \times \mathbb{R}$ 2015 Exam (10) Question (4) (a)
2	**Find the solution set of the two equations :** $x + y =$ zero and $2x^2 - y^2 = 4$ in $\mathbb{R} \times \mathbb{R}$ 2016 Exam (4) Question (4) (a)
3	**Find the solution set of the two following equations in** $\mathbb{R} \times \mathbb{R}$ **:** $x - 2y =$ zero , $x^2 - y^2 = 3$ 2016 Exam (10) Question (4) (a)
4	**Find in** $\mathbb{R} \times \mathbb{R}$ **the solution set of the two following equations :** $x - 3y = 0$, $x^2 - y^2 = 32$ 2016 Exam (2) Question (5) (a)
5	**Find in** $\mathbb{R} \times \mathbb{R}$ **the S.S. of the two equations :** $x = y + 2$, $x^2 - y^2 = 8$ 2016 Exam (8) Question (2) (b)
6	**Find the solution set of the two equations in** $\mathbb{R} \times \mathbb{R}$ **:** $x - y = 3$, $xy = 4$ 2016 Exam (5) Question (3) (b)
7	**Find in** $\mathbb{R} \times \mathbb{R}$ **the solution set of the two equations :** $y = x + 3$, $2x^2 - xy = 10$

	2016 Exam (7) Question (4) (b)
8	**Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x + y = 9$, $x^2 - y^2 = 27$ simultaneously. 2016 Exam (1) Question (5) (a)
9	The sum of two real positive numbers is 17 and their product is 72. **Find the two numbers.** *(Alex. 2009)* « 8 , 9 »
10	The sum of two real numbers is 9 and the difference between their squares equals 45 **Find the two numbers.** *(Kafr El-Sheikh 2013)* « 7 , 2 »
11	Two positive numbers , one of them exceeds three times the other by 1 and the sum of their squares is 17 **What are the two numbers ?** *(Sharkia 2004)* « 1 , 4 »
12	The perimeter of a rectangle is 18 and its area is 18 cm^2. **Find its two dimensions.** *(New Valley 2016)* « 6 cm. , 3 cm. »
13	🕮 A length of a rectangle is 3 cm. more than its width and its area is 28 cm^2. **Find its perimeter.** *(El-Fayoum 2012)* « 22 cm. »
14	🕮 A right angled-triangle of hypotenuse length 13 cm. and its perimeter is 30 cm. **Find the lengths of the other two sides.** *(El-Monofia 2015)* « 5 cm. , 12 cm. »
15	The length of a rectangle is x cm. and its width is y cm. and its area = 77 cm^2. If its length decreases by 2 cm. and its width increases 2 cm. , then it will become a square. **Find the area of the square.** *(North Sinai 2005)* « 81 cm^2 »

Solutions

1

$\because x - 2 = 0 \qquad \therefore x = 2 \qquad (1)$

$, x^2 + xy + y^2 = 7 \qquad (2)$

Substituting from (1) in (2) :

$\therefore (2)^2 + 2y + y^2 = 7$

$\therefore 4 + 2y + y^2 - 7 = 0$

$\therefore y^2 + 2y - 3 = 0 \qquad \therefore (y-1)(y+3) = 0$

$\therefore y = 1$ or $y = -3$

The S.S. = $\{(2, 1), (2, -3)\}$

2

$\because x + y = 0 \qquad \therefore x = -y \qquad (1)$

$, 2x^2 - y^2 = 4 \qquad (2)$

substituting from (1) in (2) :

$\therefore 2(-y)^2 - y^2 = 4 \qquad \therefore 2y^2 - y^2 = 4$

$\therefore y^2 = 4 \qquad \therefore y = 2$ hance $x = -2$

$\therefore$ or $y = -2$ hance $x = 2$

$\therefore$ The S.S. = $\{(-2, 2), (2, -2)\}$

3

$\because x - 2y = 0 \qquad \therefore x = 2y \qquad (1)$

$, x^2 - y^2 = 3 \qquad (2)$

substituting from (1) in (2) :

$\therefore (2y)^2 - y^2 = 3 \qquad \therefore 4y^2 - y^2 = 3$

$\therefore 3y^2 = 3 \qquad \therefore y^2 = 1$

$\therefore y = 1$ hance $x = 2$ or $y = -1$ hance $x = -2$

$\therefore$ The S.S. = $\{(2, 1), (-2, -1)\}$

4

$\because x - 3y = 0 \qquad \therefore x = 3y \qquad (1)$

$, x^2 - y^2 = 32 \qquad (2)$

substituting from (1) in (2) :

$\therefore (3y)^2 - y^2 = 32 \qquad \therefore 9y^2 - y^2 = 32$

$\therefore 8y^2 = 32 \qquad \therefore y^2 = 4$

$\therefore y = 2$ hance $x = 6$ or $y = -2$ hance $x = -6$

$\therefore$ The S.S. = $\{(6, 2), (-6, -2)\}$

5

$\because x = y + 2 \qquad (1)$

$, x^2 - y^2 = 8 \qquad (2)$

substituting from (1) in (2) :

$\therefore (y+2)^2 - y^2 = 8 \qquad \therefore y^2 + 4y + 4 - y^2 = 8$

$\therefore 4y = 4$

$\therefore y = 1$ substituting in (1) : $\therefore x = 3$

$\therefore$ The S.S. = $\{(3, 1)\}$

6

$\because x - y = 3 \qquad \therefore x = y + 3 \qquad (1)$

$, xy = 4 \qquad (2)$

substituting from (1) in (2) : $(y + 3)y = 4$

$\therefore y^2 + 3y - 4 = 0 \qquad \therefore (y+4)(y-1) = 0$

$\therefore y = -4$ hance $x = -1$ or $y = 1$ hance $x = 4$

$\therefore$ The S.S. = $\{(-1, -4), (4, 1)\}$

7

$\because y = x + 3 \quad (1) \quad , \quad 2x^2 - xy = 10 \quad (2)$

substituiting from (1) in (2) : $\therefore 2x^2 - x(x + 3) = 10$

$\therefore 2x^2 - x^2 - 3x = 10 \qquad \therefore x^2 - 3x - 10 = 0$

$\therefore (x - 5)(x + 2) = 0$

$\therefore x = 5$ hance $y = 8$ or $x = -2$ hance $y = 1$

$\therefore$ The S.S. = $\{(5, 8), (-2, 1)\}$

8

$\because x + y = 9 \qquad \therefore x = 9 - y \qquad (1)$

$, x^2 - y^2 = 27 \qquad (2)$

substituting from (1) in (2) :

$\therefore (9 - y)^2 - y^2 = 27$

$\therefore 81 - 18y + y^2 - y^2 - 27 = 0$

$\therefore 54 - 18y = 0 \qquad \therefore 18y = 54$

$\therefore y = 3$ hance $x = 6 \qquad \therefore$ The S.S. = $\{(6, 3)\}$

9

Let the two numbers be x and y :

$\therefore x + y = 17 \qquad (1)$

$, xy = 72 \qquad (2)$

From (1) : $\therefore x = 17 - y \qquad (3)$

Substituting from (3) in (2) :

$\therefore (17 - y)y = 72 \qquad \therefore 17y - y^2 - 72 = 0$

$\therefore y^2 - 17y + 72 = 0 \qquad \therefore (y - 9)(y - 8) = 0$

$\therefore y = 9$ or $y = 8$

Substituting in (3) : $\therefore x = 8$ or $x = 9$

$\therefore$ The two numbers are 8 and 9

10

Let the two numbers be x and y :

$\therefore x + y = 9 \qquad (1)$

$, x^2 - y^2 = 45 \qquad (2)$

From (1) : $\therefore x = 9 - y \qquad (3)$

Substituting from (3) in (2) : $\therefore (9 - y)^2 - y^2 = 45$

$\therefore 81 - 18y + y^2 - y^2 = 45 \qquad \therefore 81 - 18y = 45$

$\therefore 18y = 36 \qquad \therefore y = 2$

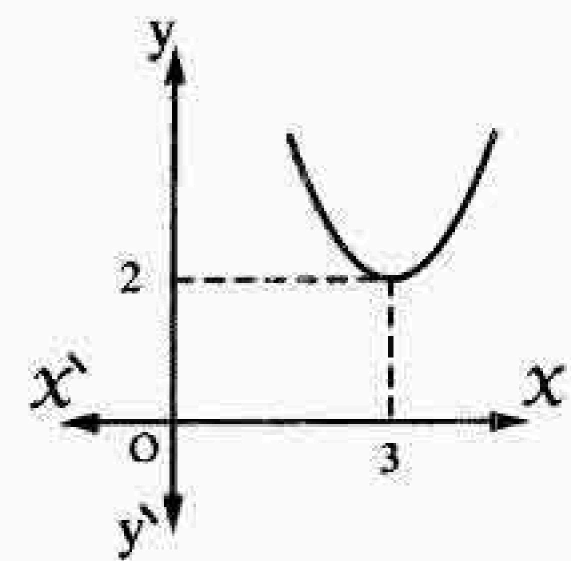
Substituting in (3) : $\therefore x = 9 - 2 = 7$

$\therefore$ The two numbers are 7 and 2

11	Let the two numbers be x and y : $\therefore x - 3y = 1$ (1) $, x^2 + y^2 = 17$ (2) From (1) : $\therefore x = 1 + 3y$ (3) Substituting in (2) : $\therefore (1 + 3y)^2 + y^2 = 17$ $\therefore 1 + 6y + 9y^2 + y^2 - 17 = 0$ $\therefore 10y^2 + 6y - 16 = 0$ $\therefore 5y^2 + 3y - 8 = 0$ $\therefore (5y + 8)(y - 1) = 0$ $\therefore y = \frac{-8}{5}$ (refused) or $y = 1$ And from (3) : $\therefore x = 4$ $\therefore$ The two numbers are 1 and 4
12	Let the length of the rectangle = x cm. and the width = y cm. $\therefore (x + y) \times 2 = 18$ $\therefore x + y = 9$ (1) $, xy = 18$ (2) From (1) : $\therefore y = 9 - x$ (3) Substituting in (2) : $\therefore x(9 - x) = 18$ $\therefore 9x - x^2 = 18$ $\therefore x^2 - 9x + 18 = 0$ $\therefore (x - 3)(x - 6) = 0$ $\therefore x = 3$ or $x = 6$ Substituting in (3) : $\therefore y = 6$ or $y = 3$ $\therefore$ The two dimensions are 6 cm. and 3 cm.
13	Let the length of the rectangle be x cm. and its width be y cm. $\therefore x - y = 3$ (1) $, xy = 28$ (2) From (1) : $\therefore x = y + 3$ (3) Substituting from (3) in (2) : $\therefore y(y + 3) = 28$ $\therefore y^2 + 3y - 28 = 0$ $\therefore (y + 7)(y - 4) = 0$ $\therefore y = -7$ (refused) or $y = 4$ Substituting in (3) : $\therefore x = 7$ $\therefore$ The two dimensions of the rectangle are 4 cm. and 7 cm. $\therefore$ The perimeter of the rectangle = $(7 + 4) \times 2 = 22$ cm.
14	Let the lengths of the two sides of the right angle be x cm. and y cm. $\therefore x + y + 13 = 30$ $\therefore x + y = 17$ (1) $, \because x^2 + y^2 = 169$ (2) From (1) : $\therefore x = 17 - y$ (3) Substituting in (2) : $\therefore (17 - y)^2 + y^2 = 169$ $\therefore y^2 - 34y + 289 + y^2 - 169 = 0$ $\therefore 2y^2 - 34y + 120 = 0$ $\therefore y^2 - 17y + 60 = 0$ $\therefore (y - 12)(y - 5) = 0$ $\therefore y = 12$ or $y = 5$ Substituting in (3) : $\therefore x = 5$ or $x = 12$ $\therefore$ The side lengths of the right angle are 5 cm. and 12 cm.
15	$\because xy = 77$ (1) $, x - 2 = y + 2$ $\therefore x = y + 4$ (2) Substituting in (1) : $\therefore (y + 4) \times y = 77$ $\therefore y^2 + 4y - 77 = 0$ $\therefore (y + 11)(y - 7) = 0$ $\therefore y = -11$ (refused) or $y = 7$ Substituting in (2) : $\therefore x = 11$ $\therefore$ The side length of the square = $x - 2 = 9$ cm. $\therefore$ The area of the square = 81 cm^2

Exercises

[A] : Choose The Correct Answer : -

1	The solution set of the two equations : $x = 2$ and $x y = 6$ is .... (a) $\{(2 , 3)\}$ (b) $\{2 , 3\}$ (c) $\{(3 , 2)\}$ (d) $\{3\}$
2	If the two equations : $x + 3 y = 4$, $x + a y = 7$ represent two parallel straight lines , then a = ... (a) $-\frac{1}{3}$ (b) -3 (c) 3 (d) 1
3	If $2 x^2 = 5$, then $6 x^2 =$ .... (a) 5 (b) 10 (c) 15 (d) 20
4	The solution set of the equation : $x^2 - 4 x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4 , 1\}$ (d) $\varnothing$
5	**In the opposite figure :** The solution set of $f : f(x) = 0$ is .... (a) $\varnothing$ (b) $\{3\}$ (c) $\{2 , 3\}$ (d) $\{2\}$ 
6	The S.S. of the two equations : $x - 2 y = 1$, $3 x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(5 , 2)\}$ (b) $\{(2 , 4)\}$ (c) $\{(1 , 3)\}$ (d) $\{(3 , 1)\}$
7	If there is only one solution for the equation : $x + 2 y = 1$ and $2 x + k y = 2$ in $\mathbb{R} \times \mathbb{R}$, then k cannot equal .... (a) 2 (b) 4 (c) -2 (d) -4
8	The solution set of the two equations : $x + 2 y = 0$ and $2 x - 3 y = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-2 , 0)\}$ (b) $\{(3 , 2)\}$ (c) $\{(0 , 0)\}$ (d) $\{(2 , 3)\}$
9	If the two equations : $x + 3 y = 6$, $2 x + k y = 12$ have an infinite number of solution in $\mathbb{R} \times \mathbb{R}$, then k = ... (a) 2 (b) 6 (c) 3 (d) 1
10	If the sum of two numbers is 8 , and their product is 15 , then the two numbers are .... (a) 2 , 6 (b) 3 , 5 (c) 4 , 4 (d) 1 , 15
11	If the curve of the quadratic function does not intersect the x-axis at any point , then the number of solutions of the equation $f(x) = 0$ in $\mathbb{R}$ is .... (a) zero (b) one solution. (c) two solutions. (d) an infinite number.

12	The two straight lines : $x + 5y = 1$, $x + 5y - 8 = 0$ are .... (a) parallel. (b) coincide. (c) perpendicular. (d) intersect and non perpendicular.
13	If $f(x) = 6x^2 + 3x(1 - 2x)$ is a polynomial function , then its degree is .... (a) first. (b) second. (c) third. (d) fourth.
14	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit
15	If $2xy = 6$, $x^2 y + x y^2 = 6$, then $x + y =$ .... (a) 1 (b) 2 (c) 6 (d) $\frac{1}{2}$
16	If $x = 3$ belongs to the solution set of the equation : $x^2 - a x - 6 = 0$, then a = ... (a) 3 (b) 2 (c) 1 (d) -1
17	The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is .... (a) $x = -4$ (b) $x = 0$ (c) $y = 0$ (d) $y = -4$
18	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit
19	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4 , 1\}$ (d) $\varnothing$
20	If $x = y + 1$, $(y - x)^2 + y = 3$, then $x =$ .... (a) 5 (b) 4 (c) 3 (d) 2
21	If the two straight lines which represent the two equations : $x + 2y = 4$, $2x + ky = 11$ are parallel , then k = ... (a) 7 (b) 6 (c) 4 (d) -4
22	If the age of a man now is x year , then his age after 5 years from now is .... years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
23	If the two equations : $x + 4y = 7$ and $3x + ky = 21$ have infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then k = ... (a) 4 (b) 7 (c) 12 (d) 21
24	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1 , -1\}$ (c) $\{-1\}$ (d) $\varnothing$
25	One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is .... (a) $(-4 , 2)$ (b) $(2 , -4)$ (c) $(3 , 1)$ (d) $(4 , 2)$
26	If the solution set of the equation : $x^2 + m x + 9 = 0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero

27 The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

(a) $\{1\}$ (b) $\{1 , -1\}$ (c) $\{-1\}$ (d) $\varnothing$

28 If the curve of the function $f : f(x) = x^2 - x + c$ passing through the point (2 , 1) , then c = ...

(a) 2 (b) 1 (c) – 2 (d) – 1

29 The number of solutions for the two equations : $x - \frac{1}{2}\,y = 4$, $2x - y = 2$ in $\mathbb{R}^2$ = ...

(a) 1 (b) 2 (c) infinit number. (d) zero.

30 If the point of intersection of the two straight lines $x - 1 = 0$ and $y = 2k$ lies on the fourth quadrant , then k may equal ....

(a) – 5 (b) zero. (c) 1 (d) 5

31 If the curve of the quadratic function does not intersect the x-axis at any point , then the number of solutions of the equation $f(x) = 0$ in $\mathbb{R}$ is ....

(a) zero (b) one solution. (c) two solutions. (d) an infinite number.

32 The ordered pair which satisfies the two equations : $xy = 2$ and $x - y = 1$ is ....

(a) (1 , 1) (b) (2 , 1) (c) (1 , 2) (d) $(\frac{1}{2} , 1)$

33 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(x) = 0$ is ....

(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0 , 1)\}$

34 The solution set of the equation : $ax^2 + bx + c = 0$, $a \neq 0$ graphically is the set of x coordinates of the points of intersection of the curve of the function $f : f(x) = ax^2 + bx + c$ with the ....

(a) y-axis (b) x-axis (c) symmetric line (d) straight line y = 2

35 If y = 2 and $x^2 - y^2 = 5$, then x = ...

(a) – 3 (b) 3 (c) ± 3 (d) 9

36 If the two equations : $x + 3y = 4$, $2x + my = 8$ have infinite number of solutions , then m = ...

(a) 2 (b) 6 (c) 3 (d) 1

37	The sum of two consecutive integers is 17 , then the smaller number of them is …………… (a) 8 (b) 9 (c) 17 (d) 72
38	If the sum of ages of a father and his sun now is 47 years , then the sum of their ages after 10 years = …………… years. (a) 27 (b) 37 (c) 57 (d) 67
39	If $f(x) = 6x^2 + 3x(1 - 2x)$ is a polynomial function , then its degree is …………… (a) first. (b) second. (c) third. (d) fourth.
40	If $x - 3 = 0$, $y^2 = x + 6$, then y = …………… (a) -3 (b) 3 (c) ± 3 (d) 9
41	The solution set of the equation : $a x^2 + b x + c = 0$, $a \neq 0$ graphically is the set of x coordinates of the points of intersection of the curve of the function $f : f(x) = a x^2 + b x + c$ with the …………… (a) y-axis (b) x-axis (c) symmetric line (d) straight line y = 2
42	The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersect in …………… (a) first quadrant. (b) second quadrant. (c) the origin point. (d) fourth quadrant.
43	If the curve of the function $f(x) = x^2 - a$ passing through the point (2 , 0) , then a = …………… (a) 4 (b) 7 (c) 9 (d) 16
44	If $2x^2 = 5$, then $6x^2 =$ …………… (a) 5 (b) 10 (c) 15 (d) 20
45	If the solution set of the equation $x^2 - a x + 4 = 0$ is $\{-2\}$, then a = …………… (a) -2 (b) -4 (c) 2 (d) 4
46	If the curve of the quadratic function f passes through the points (2 , 0) , (– 3 , 0) and (0 , – 6) , then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is …………… (a) $\{-2 , 3\}$ (b) $\{3 , 2\}$ (c) $\{2 , -3\}$ (d) $\{-3 , -6\}$
47	If $x = 1$ is the solution of the equation : $x^2 + m x + 4 = 0$, then m = …………… (a) 1 (b) -1 (c) zero (d) -5
48	If the curve of the function f where $f(x) = x^2 - a$ passes through the point (1 , 0) , then a = …………… (a) -2 (b) -1 (c) zero (d) 1

[B] : Essay Problems : -

1	**Find in $\mathbb{R} \times \mathbb{R}$ algebraically the solution set of the two equations :** $x - 1 = 0$, $x^2 + y^2 = 10$ 2018 Exam (9) Question (3) (a)
2	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of pair of equations :** $x + y = 3$ and $x^2 + xy = 6$ 2018 Exam (7) Question (3) (b)
3	**Find in $\mathbb{R}^2$ the solution set of the two equations :** $x + y = 9$, $x^2 - y^2 = 45$ 2017 Exam (4) Question (3) (a)
4	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 1$, $x^2 + y^2 = 25$ 2018 Model Exam (2) Question (3) (a)
5	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $y = x + 1$, $x^2 + y^2 = 13$ 2018 Exam (14) Question (2) (b)
6	**Solve in $\mathbb{R} \times \mathbb{R}$ the two equations :** $x - 2y = 1$, $x^2 - xy = 0$ 2018 Exam (18) Question (3) (a)
7	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - 2 = 0$, $x^2 + xy + y^2 = 7$ 2018 Exam (15) Question (4) (b)
8	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations (algebraically) :** $x + y = 5$, $x^2 + xy = 15$ 2018 Exam (8) Question (3) (a)
9	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 0$ and $x^2 + xy + y^2 = 27$ 2018 Model Exam (1) Question (3) (a)
10	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x = y + 1$, $(x - y)^2 + y = 3$ 2017 Exam (16) Question (3) (b)
11	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :** $x - y = 1$, $x^2 - xy = 0$ 2018 Exam (22) Question (4) (a)
12	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $y = x - 3$, $x^2 + y^2 = 17$ 2018 Exam (24) Question (4) (b)
13	A right angled triangle in which the length of one of the sides of right angled is 5 cm. and its perimeter is 30 cm. Find the area of the triangle. (Indicating the steps of the solution). 2018 Exam (5) Question (4) (b)

Homework

[A] : Choose The Correct Answer : -

1 If the solution set of the equation $x^2 - a x + 4 = 0$ is $\{-2\}$, then a = ...

(a) – 2 (b) – 4 (c) 2 (d) 4

2 If the curve of the function $f : f(x) = x^2 - x + c$ passing through the point (2 , 1) , then c = ...

(a) 2 (b) 1 (c) – 2 (d) – 1

3 The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is ....

(a) (– 2 , – 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (2 , 3)

4 If $x = 2$ and $y = 3$, then $(y - 2x)^{10}$ = ...

(a) 10 (b) – 1 (c) – 10 (d) 1

5 If $x + y = 5$, then $3x + 3y$ = ...

(a) 5 (b) 3 (c) 8 (d) 15

6 If $x = y + 1$, $(y - x)^2 + y = 3$, then x = ...

(a) 5 (b) 4 (c) 3 (d) 2

7 If the curve of the function f where $f(x) = x^2 - a$ passes through the point (1 , 0) , then a = ...

(a) – 2 (b) – 1 (c) zero (d) 1

8 Twice the number x subtracted by 3 is ....

(a) $x - 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $3 - 2x$

9 The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point ....

(a) (– 1 , 2) (b) (2 , – 1) (c) (1 , – 2) (d) (– 1 , – 2)

10 If the point (5 , b – 7) lies on the x–axis , then b = ...

(a) 2 (b) 3 (c) 5 (d) 7

11 One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is ....

(a) (– 4 , 2) (b) (2 , – 4) (c) (3 , 1) (d) (4 , 2)

12 The number of solutions of the two equations : $x + y = 5$ and $y - 5 = 0$ is ....

(a) zero (b) 1 (c) 2 (d) 3

13 The two straight lines : $x - 1 = 0$, $x + y = 5$ are ....

(a) parallel. (b) coincide.

(c) intersecting and not perpendicular. (d) perpendicular.

14	If $2X = 1$, then $\frac{1}{5}X =$ .... (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$
15	The number of solution of the two equations : $X + y = 2$, $X + y - 3 = 0$ is .... (a) zero. (b) one. (c) two. (d) infinite numbers.
16	The ordered pair which satisfies the two equations : $Xy = 2$ and $X - y = 1$ is .... (a) (1 , 1) (b) (2 , 1) (c) (1 , 2) (d) $(\frac{1}{2}, 1)$
17	The point of intersection of the two straight lines : $X = 4$, $y - 3 = 0$ is .... (a) (4 , 3) (b) (– 4 , 3) (c) (– 3 , 4) (d) (3 , 4)
18	The solution set of the two equations : $X = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) {(3 , 4)} (b) {(4 , 3)} (c) $\mathbb{R}$ (d) $\varnothing$
19	The solution set of the two equations : $X + y = 0$, $y - 2 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) {(2 , – 2)} (b) {(– 2 , 2)} (c) {2 , – 2} (d) {– 2 , 2}
20	The point of intersection of the two straight lines : $y = 2$, $X + y = 6$ is .... (a) (4 , 2) (b) (2 , 4) (c) (2 , 2) (d) (4 , 4)
21	If $y = 2$ and $X^2 - y^2 = 5$, then $X =$ .... (a) – 3 (b) 3 (c) ± 3 (d) 9
22	If $(5 , X - 4) = (y , 3)$, then $X + y =$ .... (a) 25 (b) 12 (c) 8 (d) 6
23	If $(5 , A - 4) = (B + 2 , 3)$, then $A + B =$ .... (a) 2 (b) 3 (c) 10 (d) 5
24	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are .... (a) parallel. (b) intersecting. (c) distant. (d) coincident.
25	If $X + 3y = 7$, then $X + 3(y + 5) =$ .... (a) 22 (b) 21 (c) 7 (d) 3
26	If $X - 3 = 0$, $y^2 = X + 6$, then $y =$ .... (a) – 3 (b) 3 (c) ± 3 (d) 9
27	If the curve of the function $f(X) = X^2 - a$ passing through the point (2 , 0) , then a = ... (a) 4 (b) 7 (c) 9 (d) 16
28	The point (– 3 , 4) lies in .... quadrant. (a) fourth (b) third (c) second (d) first

29 The solution set of the equation : $2x + 4 = 0$ in $\mathbb{N}$ is ....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $\varnothing$

30 The degree of the equation : $3x + 4y + xy = 5$ is ....

(a) zero. (b) first. (c) second. (d) third.

31 The solution set of the two equations : $x = 2$ and $xy = 6$ is ....

(a) $\{(2, 3)\}$ (b) $\{2, 3\}$ (c) $\{(3, 2)\}$ (d) $\{3\}$

32 If $x = 1$ is the solution of the equation : $x^2 + mx + 4 = 0$, then m = ...

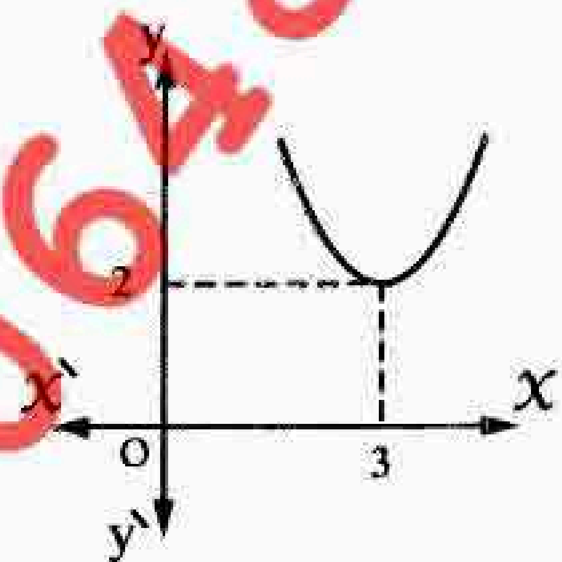
(a) 1 (b) -1 (c) zero (d) -5

33 **In the opposite figure :**

The solution set of f : $f(x) = 0$ is ....

(a) $\varnothing$ (b) $\{3\}$

(c) $\{2, 3\}$ (d) $\{2\}$



34 The solution set of the two equations : $y - 5 = 0$, $y + x = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(-5, 5)\}$ (b) $(5, -5)$ (c) $\{(0, 5)\}$ (d) $(-5, 5)$

35 The two straight lines : $x + 2y = 1$ and $2x + 4y = 6$ are ....

(a) parallel (b) intersecting (c) perpendicular (d) coincide

36 The two straight lines : $x = 4$, $y = 3$ are intersecting in ....

(a) $(4, 3)$ (b) $(0, 0)$ (c) $(3, 4)$ (d) $(-3, -4)$

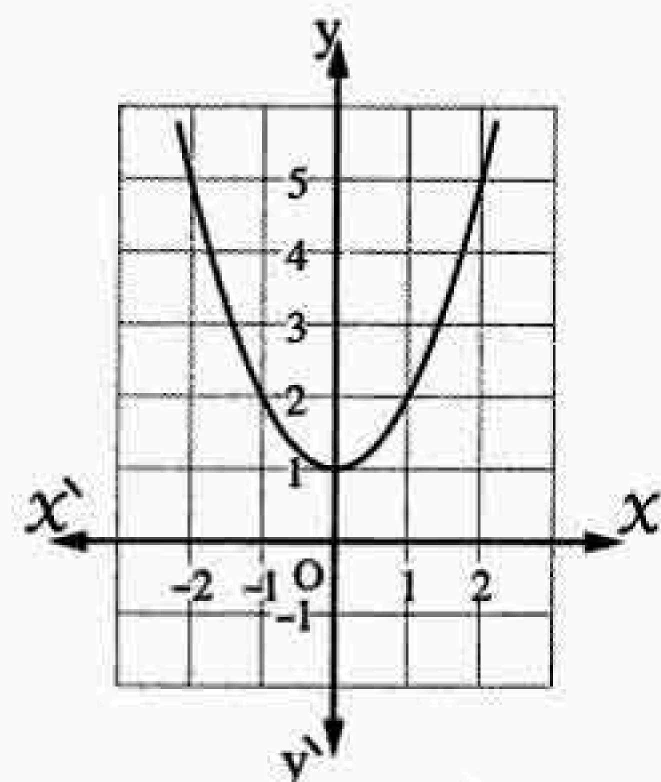
37 If $x = 3$ belongs to the solution set of the equation : $x^2 - ax - 6 = 0$, then a = ...

(a) 3 (b) 2 (c) 1 (d) -1

38 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(x) = 0$ is ....

(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0, 1)\}$



39 The two straight lines : $3x = 7$, $2y = 9$ are ....

(a) perpendicular. (b) coincide.

(c) intersect and non perpendicular. (d) parallel.

40 The point of intersection of the two straight lines : $x + 2 = 0$ and $y = x$ is ....

(a) $(2, 2)$ (b) $(2, 0)$ (c) $(-2, -2)$ (d) $(0, 0)$

41	If $(7^{a-2}, 3) = (1, b+5)$, then $a + b =$ .... (a) -1 (b) zero (c) 1 (d) 2
42	The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(5, 2)\}$ (b) $\{(2, 4)\}$ (c) $\{(1, 3)\}$ (d) $\{(3, 1)\}$
43	If the solution set of the equation : $x^2 + mx + 9 = 0$ is $\{-3\}$, then $m =$ .... (a) 5 (b) 6 (c) ± 6 (d) zero
44	The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is .... (a) $x = -4$ (b) $x = 0$ (c) $y = 0$ (d) $y = -4$
45	If $3x = 1$, then $\frac{1}{5}x =$ .... (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
46	The solution set of the two equations : $x = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(-1, 1)\}$ (b) $\{(1, -1)\}$ (c) $\{(-1, -1)\}$ (d) $\{(1, 1)\}$
47	The number of solutions of the two equations : $x + y = 2$ and $y + x = 3$ together in $\mathbb{R} \times \mathbb{R}$ is .... (a) zero (b) 1 (c) 2 (d) 3
48	If the sum of two numbers is 8, and their product is 15, then the two numbers are .... (a) 2 , 6 (b) 3 , 5 (c) 4 , 4 (d) 1 , 15
49	If the curve of the quadratic function f passes through the points $(2, 0)$, $(-3, 0)$ and $(0, -6)$, then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is .... (a) $\{-2, 3\}$ (b) $\{3, 2\}$ (c) $\{2, -3\}$ (d) $\{-3, -6\}$
50	Number of solutions of the two equations : $x + y = 2$, $y - 3 = 0$ together is .... (a) 3 (b) 2 (c) 1 (d) zero
51	If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then $x =$ .... (a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$
52	The point of intersection of the two straight lines $x = 2$ and $x + y = 6$ is .... (a) $(2, 6)$ (b) $(2, 4)$ (c) $(4, 2)$ (d) $(6, 2)$
53	If $2xy = 6$, $x^2y + xy^2 = 6$, then $x + y =$ .... (a) 1 (b) 2 (c) 6 (d) $\frac{1}{2}$

[B] : Essay Problems : -

1	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of :** $x + y = \text{zero}$, $5y^2 - 4x^2 = 36$ 2018 Exam (16) Question (2) (a)
2	**Find in $\mathbb{R} \times \mathbb{R}$ algebraically the solution set of the two equations :** $x + y = 7$, $xy = 12$ 2018 Exam (19) Question (4) (a)
3	**Find in $\mathbb{R} \times \mathbb{R}$ algebraically the solution set of the two equations :** $x - y = 0$, $2x^2 - y^2 = 4$ 2017 Exam (9) Question (4) (a)
4	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two following equations :** $x - y = 1$, $x^2 - y^2 = 25$ 2018 Exam (1) Question (5) (a)
5	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $y - x = 2$ and $x^2 + xy - 4 = 0$ 2018 Exam (6) Question (3) (a)
6	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - 2y = 0$, $x^2 - y^2 = 27$ 2017 Exam (14) Question (4) (b)
7	Find the number which is formed from two digits , if the units digit is twice the tens digit , and if the product of the two digits equals $\frac{1}{3}$ the original number. 2017 Exam (6) Question (4) (a)
8	**Solve the following two equations together in $\mathbb{R} \times \mathbb{R}$:** $x + y = 3$, $x^2 - y = 3$ 2017 Exam (2) Question (5) (a)
9	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x + y = 7$, $x^2 + y^2 = 25$ 2018 Exam (2) Question (3) (b)
10	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 0$, $x^2 + y^2 = 32$ 2017 Exam (15) Question (3) (a)
11	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 1$, $x^2 - y^2 = 5$ 2018 Exam (4) Question (4) (a)
12	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of the following two equations :** $x - y = 2$, $x^2 - 5y = 4$ 2018 Exam (23) Question (3) (a)
13	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - 2y = 0$, $x^2 - y^2 = 3$ 2018 Exam (10) Question (3) (a)

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [1] : Set Of Zeroes Of A Polynomial Function

Generally

If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by z (f)

i.e. z (f) is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(X)$, z (f) :

- f denotes to the function
- $f(X)$ denotes to the rule of the function or the image of X by the function f
- z (f) denotes to the set of zeroes of the function f and it is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Lesson [2] : Algebraic Fractional Function

The algebraic fractional function is a function whose rule is in the form of an algebraic fraction whose numerator and denominator are a polynomial functions and the domain of the algebraic fraction function = $\mathbb{R}$ – the set of zeroes of the denominator.

Definition

If p and k are two polynomial functions , z (k) is the set of zeroes of the function k ,

then the function n where $n : \mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(X) = \frac{p(X)}{k(X)}$

n is called a real algebraic fractional function or briefly it is called an algebraic fraction.

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero.

i.e. The set of zeroes of the algebraic fractional function

= the set of zeroes of the numerator – the set of zeroes of the denominator.

For example:

- If the function n : $n(X) = \frac{X^2 + 3X}{X^2 - 9}$, then $n(X) = \frac{X(X+3)}{(X-3)(X+3)}$

i.e. $z(n) = \{0 , -3\} - \{3 , -3\} = \{0\}$

• If the function n : n $(X) = \frac{3X+6}{X^2+X-2}$, then n $(X) = \frac{3(X+2)}{(X-1)(X+2)}$

i.e. z (n) = $\{-2\} - \{1, -2\} = \varnothing$

The common domain of two algebraic fractions or more

The common domain of two algebraic fractions is the set of real numbers that makes the two algebraic fractions identified together (at the same time)

Assume that we have the two algebraic fractions n_1 and n_2 where :

$n_1(X) = \frac{3}{X-2}$ and $n_2(X) = \frac{5X}{X^2-1}$,

then the domain of n_1 (say) $m_1 = \mathbb{R} - \{2\}$ (because n_1 is undefined when $X = 2$)

and the domain of n_2 (say) $m_2 = \mathbb{R} - \{1, -1\}$ (because n_2 is undefined when $X = 1$ or $X = -1$)

According to that :

$= \mathbb{R} - \{2, 1, -1\}$

Lesson [3] : Equality Of Two Algebraic Fractions

Reducing the algebraic fraction

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator.

If n is an algebraic fraction where n $(X) = \frac{X^2-X-6}{X^2-9}$,

then factorizing each of the numerator and denominator , we find that :

n $(X) = \frac{(X-3)(X+2)}{(X-3)(X+3)}$ where the domain of n = $\mathbb{R} - \{3, -3\}$

since $(X-3)$ is a common factor between the numerator and denominator of the fraction ,

is n $(X) = \frac{X+2}{X+3}$ where the domain of n stays as it was $\mathbb{R} - \{3, -3\}$

From the previous , to reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly.
2. Identify the domain of the algebraic fraction before removing the common factors between the numerator and denominator.
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction.

Equality of two algebraic fractions

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

Examples :

1	**Determine the set of zeroes of the polynomial functions which are defined by the following rules in $\mathbb{R}$:** 📖 $f(X) = (X-2)(X+3)+4$ *(El-Monofia 15)*
2	📖 If the function $f : f(X) = X^3 - 2X^2 - 75$ **Prove that :** The number 5 is the one of the zeroes of the function f *(Beni Suef 15)*
3	If the set of zeroes of the function : $f(X) = aX^2 + X + b$ is $\{0, 1\}$ **Find the value of each two constants a and b** *(Alex. 17)* « – 1 , 0 »
4	**Find the common domain of the following algebraic fractions :** $\frac{3X}{X-2}$, $\frac{X+3}{X^2-9}$ *(North Sinai 09)*
5	Determine the domain of the function $n : n(X) = \frac{2X+1}{X^2-5X+6}$ **, then find n (0) , n (2)** *(New Valley 08)*
6	If the domain of the function f where $f(X) = \frac{X}{X^2-5X+m}$ is $\mathbb{R} - \{2, c\}$ **, then find the value of each m and c** *(El-Sharkia 16)* « 6 , 3 »
7	📖 If the domain of the function f where $f(X) = \frac{X+b}{X+a}$ is $\mathbb{R} - \{-2\}$ and $f(0) = 3$ **, then find the value of each a and b** *(El-Fayoum 16)* « 2 , 6 »
8	**Reduce each of the following algebraic fractions to the simplest form showing the domain of each of them :** $n(X) = \frac{X + \frac{1}{X}}{4X + \frac{4}{X}}$ *(Damietta 17)*

9	**Prove that : n1(X) = n2(X) , and find the common domain :** $n_1(x) = \frac{x^2 - 3x + 9}{x^3 + 27}$, $n_2(x) = \frac{2}{2x + 6}$	*(El-Sharkia 17)*
10	**Prove that : n1(X) = n2(X) , and find the common domain :** $n_1(x) = \frac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \frac{x^3 - x^2 - 6x}{x^3 - 9x}$	*(El-Monofia 17)*
11	**Prove that : n1(X) = n2(X)** $n_1(x) = \frac{x^2 - x}{x^3 - 2x^2}$, $n_2(x) = \frac{x^2 - 3x + 2}{x^3 - 4x^2 + 4x}$	*(Beni Suef 08)*
12	**Prove that : n1(X) = n2(X)** 🕮 $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$	*(El-Beheira 17)*
13	**Show whether n1(X) = n2(X) or not (give reason) :** $n_1(x) = \frac{x + 5}{x^2 - 25}$, $n_2(x) = \frac{2}{2x - 10}$	*(Ismailia 02)*
14	**Show whether n1(X) = n2(X) or not (give reason) :** $n_1(x) = \frac{x^2 - 9}{x^2 + 4x + 3}$, $n_2(x) = \frac{x - 3}{x + 1}$	*(Giza 16)*

Solutions

1	$f(x) = (x + 2)(x - 1)$ $\therefore z(f) = \{-2, 1\}$
2	$\because f(5) = (5)^3 - 2(5)^2 - 75 = 125 - 50 - 75 = 0$ $\therefore$ the number 5 is one of zeroes of the function f
3	$\because z(f) = \{0, 1\}$ $\therefore f(0) = 0$ $\therefore b = 0$ $\therefore f(x) = a x^2 + x$ $, \because f(1) = 0$ $\therefore a \times 1^2 + 1 = 0$ $\therefore a + 1 = 0$ $\therefore a = -1$
4	The domain of $n_1 = \mathbb{R} - \{2\}$ $, \because n_2(x) = \frac{x + 3}{(x + 3)(x - 3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$ $\therefore$ The common domain $= \mathbb{R} - \{2, -3, 3\}$

5	$n(x) = \frac{2x + 1}{(x - 3)(x - 2)}$ $\therefore$ The domain of $n = \mathbb{R} - \{3, 2\}$, $n(0) = \frac{1}{6}$ $n(2)$ is meaningless because $2 \notin$ the domain of n
6	$\because$ The domain of $f = \mathbb{R} - \{2, c\}$ $\therefore$ When $x = 2$ $\therefore x^2 - 5x + m = 0$ $\therefore 4 - 5 \times 2 + m = 0$ $\therefore m = 6$ $\therefore f(x) = \frac{x}{x^2 - 5x + 6}$ $\therefore f(x) = \frac{x}{(x - 2)(x - 3)}$ $\therefore$ The domain of $f = \mathbb{R} - \{2, 3\}$ $\therefore c = 3$
7	$\because$ The domain $= \mathbb{R} - \{-2\}$ $\therefore$ When $x = -2$ $\therefore x + a = 0$ $\therefore -2 + a = 0$ $\therefore a = 2$ $\therefore f(x) = \frac{x + b}{x + 2}$ $, \because f(0) = 3$ $\therefore \frac{0 + b}{0 + 2} = 3$ $\therefore \frac{b}{2} = 3$ $\therefore b = 6$

8	$n(x) = \dfrac{\frac{x^2+1}{x}}{\frac{4x^2+4}{x}}$ $\therefore$ The domain of $n = \mathbb{R} - \{0\}$ $, n(x) = \dfrac{x^2+1}{4x^2+4} = \dfrac{x^2+1}{4(x^2+1)} = \dfrac{1}{4}$
9	$\because n_1(x) = \dfrac{x^2-3x+9}{(x+3)(x^2-3x+9)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-3\}$ $, n_1(x) = \dfrac{1}{x+3}$ $, \because n_2(x) = \dfrac{2}{2(x+3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-3\}$ $, n_2(x) = \dfrac{1}{x+3}$ $\therefore n_1(x) = n_2(x)$ for all the values of $x \in \mathbb{R} - \{-3\}$
10	$\because n_1(x) = \dfrac{(x-2)(x+2)}{(x-2)(x+3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{2, -3\}$ $, n_1(x) = \dfrac{x+2}{x+3}$ $, \because n_2(x) = \dfrac{x(x-3)(x+2)}{x(x-3)(x+3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 3, -3\}$ $, n_2(x) = \dfrac{x+2}{x+3}$ $\therefore n_1(x) = n_2(x)$ for all the values of $x \in \mathbb{R} - \{0, 2, 3, -3\}$
11	$\because n_1(x) = \dfrac{x(x-1)}{x^2(x-2)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 2\}$ $, n_1(x) = \dfrac{x-1}{x(x-2)}$ (1) $\because n_2(x) = \dfrac{(x-2)(x-1)}{x(x-2)^2}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 2\}$ $, n_2(x) = \dfrac{x-1}{x(x-2)}$ (2) From (1) and (2) : $\therefore n_1 = n_2$
12	$\because n_1(x) = \dfrac{x^2}{x^2(x-1)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$ $, n_1(x) = \dfrac{1}{x-1}$ (1) $\because n_2(x) = \dfrac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$ $, n_2(x) = \dfrac{1}{x-1}$ (2) From (1) and (2) : $\therefore n_1 = n_2$
13	$\because n_1(x) = \dfrac{x+5}{(x-5)(x+5)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{5, -5\}$ $, n_1(x) = \dfrac{1}{x-5}$ (1) $\because n_2(x) = \dfrac{2}{2(x-5)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{5\}$ $\therefore n_2(x) = \dfrac{1}{x-5}$ (2) From (1) and (2) : $\therefore n_1 \neq n_2$ because the domain of $n_1 \neq$ the domain of n_2
14	$\because n_1(x) = \dfrac{(x-3)(x+3)}{(x+1)(x+3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-1, -3\}$ $, n_1(x) = \dfrac{x-3}{x+1}$ (1) $\because n_2(x) = \dfrac{x-3}{x+1}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-1\}$ (2) From (1) and (2) : $\therefore n_1 \neq n_2$ because the domain of $n_1 \neq$ the domain of n_2

Exercises

[A] : Choose The Correct Answer : -

1	The degree of the polynomial function f where $f(X) = X^3 + 2X - 3$ is …………… (a) fourth. (b) third. (c) first. (d) zero.
2	The degree of the function $f : f(X) = 2X^3 + 3X^2 - 5$ is …………… (a) frouth. (b) fifth. (c) third. (d) zero.
3	If the function f is a function from set X to set Y , then the domain of the function is …………… (a) X (b) Y (c) $X \times Y$ (d) $Y \times X$
4	The set of zeroes of f where $f(X) = 9$ is …………… (a) $\{9\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$
5	If $f(X) = 2X$, then $f(1) - f(-1) =$ …………… (a) zero (b) 4 (c) 2 (d) -2
6	If $n(X^2) = 9$, then $n(X) =$ …………… (a) 81 (b) 3 (c) ± 3 (d) -3
7	The set of zeroes of the function $f : f(X) = 4$ is …………… (a) $\{-4\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\{2\}$
8	The set of zeroes of the function f where $f(X) = X^2 + 4$ in $\mathbb{R}$ is …………… (a) $\{2\}$ (b) $\{2 , -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$
9	The set of zeroes of f where $f(X) = X^2 + 9$ is …………… (a) $\{3 , -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$
10	The set of zeroes of the function $f : f(X) = X^2 + 1$ is …………… (a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1 , 1\}$ (d) $\varnothing$
11	The set of zeroes of the function $f : f(X) = X^2 + 3$ is …………… (a) $\{0\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{3 , -3\}$
12	The set of zeroes of f where $f(X) = -3X$ is …………… (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3 , 0\}$ (d) $\mathbb{R}$
13	The set of zeroes of f where $f(X) = X(X - 1)$ is …………… (a) $\{1\}$ (b) $\{0 , -1\}$ (c) $\{0 , 1\}$ (d) $\{0\}$

14	The set of zeroes of the function $f(x) = \frac{2-x}{7}$ is .... (a) $\{7\}$ (b) $\{2, 7\}$ (c) $\{2\}$ (d) $\varnothing$
15	The set of zeroes of the function f where $f(x) = \frac{x-3}{x+2}$ is .... (a) $\{zero\}$ (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3, -2\}$
16	The set of zeroes of the function f where $f(x) = \frac{x-1}{x+2}$ is .... (a) $\{-2\}$ (b) $\{-2, 1\}$ (c) $\{1\}$ (d) $\{zero\}$
17	The set of zeroes of the function $f : f(x) = x + 3$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\{-3\}$ (d) 3
18	The set of zeroes of the function f where $f(x) = \frac{x+7}{x-2}$ is .... (a) $\{-7\}$ (b) $\{7\}$ (c) $\{2\}$ (d) $\{7, 2\}$
19	The set of zeroes of f where $f(x) = (x-1)^2 (x+2)$ is .... (a) $\{1, -2\}$ (b) $\{-1, 2\}$ (c) $\{-1, -2\}$ (d) $\{1, 2\}$
20	The set of zeroes of f where $f(x) = x^2 - 25$ is .... (a) $\{5\}$ (b) $\{-5\}$ (c) $\{5, -5\}$ (d) $\varnothing$
21	The set of zeroes of the function $f : f(x) = \frac{x^2-9}{x-3}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\varnothing$
22	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\varnothing$
23	The set of zeroes of f where $f(x) = \frac{x^2-9}{x-2}$ is .... (a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3, -3\}$ (d) $\{3, -3, 2\}$
24	The set of zeroes of the function $f : f(x) = \frac{x^2+x-2}{x^2-4}$ is .... (a) $\{-2, 1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\{-1\}$ (d) $\{1\}$
25	The set of zero is of f where $f(x) = x(x^2 - 2x + 1)$ is .... (a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1, -1\}$
26	The set of zeroes of f where : $f(x) = \frac{x^3+x}{x^3-x}$ is .... (a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$

27	If $f(x) = \frac{7+x}{7-x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2) =$ .... (a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$
28	If z $(f) = \{3\}$, $f(x) = 2x + a$, then a = ... (a) zero. (b) 6 (c) -6 (d) 3
29	The set of zeroes of the function $f : f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is .... (a) $\{2, -2\}$ (b) $\{-2, -1\}$ (c) $\{2, -1\}$ (d) $\{1, -1\}$
30	The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction .... (a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{x}{x-5}$ (d) $\frac{x-5}{x-3}$
31	The domain of the function $f : f(x) = \frac{x-3}{4}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4, 3\}$ (d) $\varnothing$
32	The domain of the function $f : f(x) = \frac{x-2}{7}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{7\}$ (d) $\mathbb{R} - \{2, 7\}$
33	The domain of the function f where $f(x) = \frac{x+2}{5x}$ is .... (a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{\text{zero}\}$
34	The simplest form of $f(x) = \frac{4x^2 - 2x}{2x}$, $x \neq 0$ is .... (a) $4x^2$ (b) $2x - 1$ (c) $2x$ (d) 2
35	The domain of the fraction n : n $(x) = \frac{x+2}{x-1}$ is .... (a) $\mathbb{R} - \{-2\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -2\}$ (d) $\mathbb{R} - \{2\}$
36	The domain of the multiplicative inverse of the fraction $\frac{x-3}{x+1} =$ .... (a) $\mathbb{R} - \{3, 1\}$ (b) $\mathbb{R} - \{3, -1\}$ (c) $\mathbb{R} - \{-3, -1\}$ (d) $\mathbb{R}$
37	The domain of the multiplicative inverse of the function $f : f(x) = \frac{x+2}{x-3}$ is .... (a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2, 3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R}$
38	The domain of the additive inverse of the fraction n : n $(x) = \frac{x-2}{x-5}$ is .... (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2, 5\}$ (d) $\{2, 5\}$
39	The domain of the function f where $f(x) = \frac{7}{x-5}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0, 5\}$

40 The domain of the multiplicative inverse of the algebraic fraction $\frac{x+2}{x+5}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{-5\}$ (c) $\mathbb{R}-\{-2\}$ (d) $\mathbb{R}-\{-2,-5\}$

41 If the domain of function n : $n(x)=\frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a 0

(a) $=$ (b) $>$ (c) $\le$ (d) $<$

42 The domain of the function f where $f(x)=\frac{x-2}{x^2+1}$ is ....

(a) $\mathbb{R}-\{-1\}$ (b) $\mathbb{R}-\{1,-1\}$ (c) $\mathbb{R}-\{1\}$ (d) $\mathbb{R}$

43 The simplest of $f(x)=\frac{3-x}{x-3}$, $x\ne 3$ is ....

(a) 3 (b) 1 (c) -1 (d) zero.

44 The domain of the function f where $f(x)=\frac{x-7}{3(x+1)}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{-1\}$ (c) $\mathbb{R}-\{1\}$ (d) $\mathbb{R}-\{-1,3\}$

45 The common domain of the two fractions : $\frac{2}{x-3}$, $\frac{7}{2x-6}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{0,3\}$ (c) $\mathbb{R}-\{3\}$ (d) $\mathbb{R}-\{3,-3\}$

46 The domain of the function f where $f(x)=\frac{x-3}{5(x-1)}$ is ....

(a) $\mathbb{R}-\{5,1\}$ (b) $\mathbb{R}$ (c) $\mathbb{R}-\{1\}$ (d) $\mathbb{R}-\{3\}$

47 If $2x^2=5$, then $6x^2=$ ....

(a) 5 (b) 10 (c) 15 (d) 20

48 If $f(x)=6x^2+3x(1-2x)$ is a polynomial function , then its degree is ....

(a) first. (b) second. (c) third. (d) fourth.

49 The solution set of the equation : $x^2+1=0$ in $\mathbb{R}$ is ....

(a) $\{1\}$ (b) $\{1,-1\}$ (c) $\{-1\}$ (d) $\varnothing$

50 The solution set of the equation : $x^2-4x+4=0$ in $\mathbb{R}$ is ....

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{4,1\}$ (d) $\varnothing$

51 In equation : $ax^2+bx+c=0$ if $b^2-4ac>0$, then this equation has .... roots.

(a) 1 (b) 2 (c) zero. (d) infinit

52 If the solution set of the equation $x^2-ax+4=0$ is $\{-2\}$, then a = ...

(a) -2 (b) -4 (c) 2 (d) 4

53 If the solution set of the equation : $x^2+mx+9=0$ is $\{-3\}$, then m = ...

(a) 5 (b) 6 (c) ± 6 (d) zero

[C] : Essay Problems : -

1	If $f(x) = x^3 - 3x^2 - 16$ **Prove that the number 4 is one of the zeroes of this function.** 2017 Exam (14) Question (4) (a)
2	Reduce $n(x) = \frac{2x-6}{x^2-5x+6}$, then find : n (– 2) and n (2) 2017 Exam (17) Question (2) (b)
3	If the domain of the function n : $n(x) = \frac{x-1}{x^2 - ax + 9}$ is $\mathbb{R} - \{3\}$, then find the value of a 2018 Exam (15) Question (5) (b)
4	**If the domain of the function n :** $n(x) = \frac{b}{x-2} + \frac{6}{2x+a}$ is $\mathbb{R} - \{2\}$, n (5) = 8 Find the value of each a and b 2017 Exam (6) Question (5) (a)
5	If $n_1(x) = \frac{1}{x}$, $n_2(x) = \frac{x^2+4}{x^3+4x}$ **Prove that :** $n_1 = n_2$ 2017 Exam (19) Question (4) (b)
6	If $n_1(x) = \frac{2x}{2x+6}$, $n_2(x) = \frac{x^2+3x}{x^2+6x+9}$, **then prove that :** $n_1 = n_2$ 2018 Exam (1) Question (3) (b)
7	If $n_1(x) = \frac{x^2}{x^3 - x^2}$ and $n_2(x) = \frac{x^3+x^2+x}{x^4-x}$, **then prove that :** $n_1 = n_2$ 2018 Model Exam (1) Question (5) (a)
8	If $n_1(x) = \frac{x^2-9}{x^2+4x+3}$, $n_2(x) = \frac{x-3}{x+1}$ Does $n_1 = n_2$? Explain your answer. 2017 Exam (2) Question (4) (a)
9	If $n_1(x) = \frac{x^2-2x+4}{x^3+8}$, $n_2(x) = \frac{3}{3x+6}$ **Prove that :** $n_1 = n_2$ 2018 Exam (15) Question (4) (a)
10	**Find the common domain of** $n_1(x)$ **,** $n_2(x)$ **to be equal such that :** $n_1(x) = \frac{x^2+9x+20}{x^2-16}$, $n_2(x) = \frac{x^2+5x}{x^2-4x}$ 2018 Exam (10) Question (2) (b)
11	If $n(x) = \frac{x + \frac{1}{x}}{4x + \frac{4}{x}}$ Find n (x) in the simplest form showing the domain. 2018 Exam (11) Question (5) (b)

12	If the domain of the function f where $f(x) = \frac{x+b}{x+a}$ is $\mathbb{R} - \{-2\}$, and $f(0) = 3$, then find the value of a and b 2017 Exam (13) Question (3) (b)
13	If the domain of the function n where : $n(x) = \frac{4}{x+a} + \frac{b}{2x}$ is $\mathbb{R} - \{0 , -5\}$ and $n(3) = 1$, find the values of a and b 2018 Exam (5) Question (3) (a)
14	**Find the set of zeroes of the function f :** $f(x) = x^3 - x$ 2018 Exam (15) Question (2) (a)
15	If $n_1(x) = \frac{1}{x+1}$, $n_2(x) = \frac{x^2 - x + 1}{x^3 + 1}$ **, then prove that :** $n_1 = n_2$ 2018 Exam (22) Question (5) (b)
16	**Prove that $n_1 = n_2$ where :** $n_1(x) = \frac{2x}{2x+8}$, $n_2(x) = \frac{x^2 + 4x}{x^2 + 8x + 16}$ 2017 Exam (12) Question (4) (b)
17	**Find the common domain of n_1 and n_2 to be equal such that :** $n_1(x) = \frac{x^2 + 2x}{x^2 + 3x + 2}$, $n_2(x) = \frac{x^2 - x}{x^2 - 1}$ 2018 Exam (14) Question (5) (a)
18	If $n_1(x) = \frac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \frac{x^3 - x^2 - 6x}{x^3 - 9x}$ **Prove that :** $n_1(x) = n_2(x)$ for all values of x which belong to the common domain and find this domain. 2018 Exam (5) Question (5) (a)
19	**Find the common domain of n_1 , n_2 to be equal such that :** $n_1(x) = \frac{x^2 + 3x + 2}{x^2 - 4}$, $n_2(x) = \frac{x^2 - 1}{x^2 - 3x + 2}$ 2018 Exam (2) Question (3) (a)
20	**If the set of zeros of the function f where :** $f(x) = a x^2 + b x + 8$ is $\{2 , 4\}$ **Find the value of a and b** 2018 Exam (24) Question (5) (a)
21	If $f(x) = \frac{x^2 - 9}{x + b}$, $f(4) = 1$ **Find :** b 2018 Exam (13) Question (3) (a)
22	If the domain of n : $n(x) = \frac{\ell}{x} + \frac{9}{x+m}$ is $\mathbb{R} - \{0 , -2\}$, $n(4) = 1$ **Find :** ℓ , m 2018 Exam (16) Question (5) (a)
23	If the set of zeroes of the function f where $f(x) = \frac{a x^2 - 6x + 8}{b x - 4}$ is $\{4\}$ and its domain is $\mathbb{R} - \{2\}$ **, then find :** a , b 2018 Exam (4) Question (5) (b)

Homework

[A] : Choose The Correct Answer : -

1	The degree of the polynomial function f where $f(x) = x^3 + 2x - 3$ is .... (a) fourth. (b) third. (c) first. (d) zero.
2	The set of zeroes of the function $f : f(x) = x^2 + 1$ is .... (a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1, 1\}$ (d) $\varnothing$
3	The set of zeroes of f where $f(x) = x^2 - 25$ is .... (a) $\{5\}$ (b) $\{-5\}$ (c) $\{5, -5\}$ (d) $\varnothing$
4	The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction .... (a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{x}{x-5}$ (d) $\frac{x-5}{x-3}$
5	The domain of the multiplicative inverse of the algebraic fraction $\frac{x+2}{x+5}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{-2, -5\}$
6	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4, 1\}$ (d) $\varnothing$
7	The set of zeroes of f where $f(x) = x^2 + 9$ is .... (a) $\{3, -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$
8	The set of zeroes of f where $f(x) = (x-1)^2 (x+2)$ is .... (a) $\{1, -2\}$ (b) $\{-1, 2\}$ (c) $\{-1, -2\}$ (d) $\{1, 2\}$
9	The set of zeroes of the function $f : f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is .... (a) $\{2, -2\}$ (b) $\{-2, -1\}$ (c) $\{2, -1\}$ (d) $\{1, -1\}$
10	The domain of the function f where $f(x) = \frac{7}{x-5}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0, 5\}$
11	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\varnothing$
12	The set of zeroes of the function f where $f(x) = x^2 + 4$ in $\mathbb{R}$ is .... (a) $\{2\}$ (b) $\{2, -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$

13 The set of zeroes of the function f where $f(\mathcal{X}) = \frac{\mathcal{X}+7}{\mathcal{X}-2}$ is ....

(a) $\{-7\}$ (b) $\{7\}$ (c) $\{2\}$ (d) $\{7, 2\}$

14 If z $(f) = \{3\}$, $f(\mathcal{X}) = 2\mathcal{X} + a$, then a = ...

(a) zero. (b) 6 (c) -6 (d) 3

15 The domain of the additive inverse of the fraction n : n $(\mathcal{X}) = \frac{\mathcal{X}-2}{\mathcal{X}-5}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2, 5\}$ (d) $\{2, 5\}$

16 If $f(\mathcal{X}) = 6\mathcal{X}^2 + 3\mathcal{X}(1 - 2\mathcal{X})$ is a polynomial function , then its degree is ....

(a) first. (b) second. (c) third. (d) fourth.

17 The set of zeroes of the function $f : f(\mathcal{X}) = 4$ is ....

(a) $\{-4\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\{2\}$

18 The set of zeroes of the function $f : f(\mathcal{X}) = \mathcal{X} + 3$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\{-3\}$ (d) 3

19 If $f(\mathcal{X}) = \frac{7+\mathcal{X}}{7-\mathcal{X}}$, $\mathcal{X} \in \mathbb{R} - \{7, -7\}$, then $f(-2)$ = ...

(a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$

20 The domain of the multiplicative inverse of the function $f : f(\mathcal{X}) = \frac{\mathcal{X}+2}{\mathcal{X}-3}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2, 3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R}$

21 If $2\mathcal{X}^2 = 5$, then $6\mathcal{X}^2$ = ...

(a) 5 (b) 10 (c) 15 (d) 20

22 If n $(X^2) = 9$, then n (X) = ...

(a) 81 (b) 3 (c) ± 3 (d) -3

23 The set of zeroes of the function f where $f(\mathcal{X}) = \frac{\mathcal{X}-1}{\mathcal{X}+2}$ is ....

(a) $\{-2\}$ (b) $\{-2, 1\}$ (c) $\{1\}$ (d) $\{\text{zero}\}$

24 The set of zeroes of f where : $f(\mathcal{X}) = \frac{\mathcal{X}^3 + \mathcal{X}}{\mathcal{X}^3 - \mathcal{X}}$ is ....

(a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$

25 The domain of the multiplicative inverse of the fraction $\frac{\mathcal{X}-3}{\mathcal{X}+1}$ = ...

(a) $\mathbb{R} - \{3, 1\}$ (b) $\mathbb{R} - \{3, -1\}$ (c) $\mathbb{R} - \{-3, -1\}$ (d) $\mathbb{R}$

26 The domain of the function f where $f(\mathcal{X}) = \frac{\mathcal{X}-3}{5(\mathcal{X}-1)}$ is ....

(a) $\mathbb{R} - \{5, 1\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{3\}$

27 If $f(X) = 2X$, then $f(1) - f(-1) =$ ....

(a) zero (b) 4 (c) 2 (d) -2

28 The set of zeroes of the function f where $f(X) = \frac{X-3}{X+2}$ is ....

(a) $\{zero\}$ (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3, -2\}$

29 The set of zero is of f where $f(X) = X(X^2 - 2X + 1)$ is ....

(a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1, -1\}$

30 The domain of the fraction n : $n(X) = \frac{X+2}{X-1}$ is ....

(a) $\mathbb{R} - \{-2\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -2\}$ (d) $\mathbb{R} - \{2\}$

31 The common domain of the two fractions : $\frac{2}{X-3}$, $\frac{7}{2X-6}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0, 3\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{3, -3\}$

32 The set of zeroes of f where $f(X) = 9$ is ....

(a) $\{9\}$ (b) $\{zero\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$

33 The set of zeroes of the function $f(X) = \frac{2-X}{7}$ is ....

(a) $\{7\}$ (b) $\{2, 7\}$ (c) $\{2\}$ (d) $\varnothing$

34 The set of zeroes of the function $f : f(X) = \frac{X^2 + X - 2}{X^2 - 4}$ is ....

(a) $\{-2, 1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\{-1\}$ (d) $\{1\}$

35 The simplest form of $f(X) = \frac{4X^2 - 2X}{2X}$, $X \neq 0$ is ....

(a) $4X^2$ (b) $2X - 1$ (c) $2X$ (d) 2

36 The domain of the function f where $f(X) = \frac{X-7}{3(X+1)}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{-1, 3\}$

37 If the function f is a function from set X to set Y , then the domain of the function is ....

(a) X (b) Y (c) X × Y (d) Y × X

38 The set of zeroes of f where $f(X) = X(X - 1)$ is ....

(a) $\{1\}$ (b) $\{0, -1\}$ (c) $\{0, 1\}$ (d) $\{0\}$

39 The set of zeroes of f where $f(X) = \frac{X^2 - 9}{X - 2}$ is ....

(a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3, -3\}$ (d) $\{3, -3, 2\}$

40	The domain of the function f where $f(x) = \frac{x+2}{5x}$ is .... (a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{\text{zero}\}$
41	The simplest of $f(x) = \frac{3-x}{x-3}$, $x \neq 3$ is .... (a) 3 (b) 1 (c) – 1 (d) zero.
42	If the solution set of the equation : $x^2 + m x + 9 = 0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero
43	The degree of the function $f : f(x) = 2x^3 + 3x^2 - 5$ is .... (a) frouth. (b) fifth. (c) third. (d) zero.
44	The set of zeroes of f where $f(x) = -3x$ is .... (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3, 0\}$ (d) $\mathbb{R}$
45	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\varnothing$
46	The domain of the function $f : f(x) = \frac{x-2}{7}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{7\}$ (d) $\mathbb{R} - \{2, 7\}$
47	The domain of the function f where $f(x) = \frac{x-2}{x^2+1}$ is .... (a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1, -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$
48	If the solution set of the equation $x^2 - a x + 4 = 0$ is $\{-2\}$, then a = ... (a) – 2 (b) – 4 (c) 2 (d) 4
49	The set of zeroes of the function $f : f(x) = x^2 + 3$ is .... (a) $\{0\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{3, -3\}$
50	The set of zeroes of the function $f : f(x) = \frac{x^2-9}{x-3}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\varnothing$
51	The domain of the function $f : f(x) = \frac{x-3}{4}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4, 3\}$ (d) $\varnothing$
52	If the domain of function n : $n(x) = \frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a .... 0 (a) = (b) > (c) ≤ (d) <
53	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit

[B] : Essay Problems : -

1	**Find the common domain of n_1 and n_2 to be equal such that :** $n_1(x) = \frac{x^2+2x}{x^2+3x+2}$, $n_2(x) = \frac{x^2-x}{x^2-1}$ 2018 Exam (14) Question (5) (a)
2	**If n_1 , n_2 are two functions such that :** $n_1(x) = \frac{x^2+5x}{x^2+10x+25}$, $n_2(x) = \frac{2x}{2x+10}$, then prove that : $n_1 = n_2$ 2017 Exam (8) Question (5) (b)
3	n_1 , n_2 are two algebraic fractions such that : $n_1(x) = \frac{x^2-4}{x^2+x-6}$ and $n_2(x) = \frac{x^2-x-6}{x^2-9}$ **Prove that :** $n_1(x) = n_2(x)$ for all values of x which belong to the common domain and find this domain. 2018 Exam (7) Question (3) (a)
4	If $n_1(x) = \frac{3x}{3x+3}$, $n_2(x) = \frac{x^2+x}{x^2+2x+1}$ **Prove that :** $n_1 = n_2$ 2018 Exam (9) Question (4) (b)
5	If $n_1(x) = \frac{x^2+2x+4}{x^3-8}$ and $n_2(x) = \frac{1}{x-2}$ **Prove that :** $n_1 = n_2$ 2017 Exam (1) Question (3) (b)
6	If $n_1(x) = \frac{x}{x+2}$, $n_2(x) = \frac{2x}{2x+4}$ **, then prove that :** $n_1 = n_2$ 2017 Exam (5) Question (3) (b)
7	If $n_1(x) = \frac{x^2+2x}{x^2+4x+4}$, $n_2(x) = \frac{2x}{2x+4}$ **, prove that :** $n_1 = n_2$ 2018 Exam (3) Question (5) (a)
8	If the set of zeroes of the function f where $f(x) = \frac{a x^2 - 6x + 8}{b x - 4}$ is $\{4\}$ and its domain is $\mathbb{R} - \{2\}$ **, then find :** a , b 2018 Exam (4) Question (5) (b)
9	If $n_1(x) = \frac{5x}{5x^2+20}$, $n_2(x) = \frac{x}{x^2+4}$ **Prove that :** $n_1 = n_2$ 2017 Exam (18) Question (3) (b)
10	If the domain of n : $n(x) = \frac{\ell}{x} + \frac{9}{x+m}$ is $\mathbb{R} - \{0, -2\}$, n (4) = 1 **Find :** ℓ , m 2018 Exam (16) Question (5) (a)
11	If $n_1(x) = \frac{2x}{2x+4}$, $n_2(x) = \frac{x^2+2x}{x^2+4x+4}$ **Prove that :** $n_1 = n_2$ 2018 Exam (16) Question (3) (b)

12 If $f(x) = \dfrac{x^2 - 9}{x + b}$, $f(4) = 1$ **Find :** b

2018 Exam (13) Question (3) (a)

13 If $f_1(x) = \dfrac{x - a}{x + b}$, and the set of zeroes of f_1 is $\{5\}$, and the domain of f_1 is $\mathbb{R} - \{3\}$,

then find the values of a and b

If $f_2(x) = \dfrac{x - 1}{x - 3}$, then find $f_1(x) + f_2(x)$ in the simplest form.

2018 Exam (7) Question (5) (a)

14 **If the set of zeros of the function f where :**

$f(x) = a x^2 + b x + 8$ is $\{2 , 4\}$ **Find the value of a and b**

2018 Exam (24) Question (5) (a)

15 If the domain of the function n where $n(x) = \dfrac{b}{x} + \dfrac{9}{x + a}$ is $\mathbb{R} - \{0 , 3\}$

, n (6) = 7 find the values of a , b

2018 Exam (22) Question (5) (a)

16 **Find the common domain of n_1 , n_2 to be equal such that :**

$n_1(x) = \dfrac{x^2 + 3x + 2}{x^2 - 4}$, $n_2(x) = \dfrac{x^2 - 1}{x^2 - 3x + 2}$

2018 Exam (2) Question (3) (a)

17 If the domain of the function f , where $f(x) = \dfrac{x}{x^2 - 5x + m}$ is $\mathbb{R} - \{2 , c\}$

, then find the value of : m and c

2017 Exam (5) Question (5) (a)

18 If $n_1(x) = \dfrac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \dfrac{x^3 - x^2 - 6x}{x^3 - 9x}$

Prove that : $n_1(x) = n_2(x)$ for all values of x which belong to the common domain and find this domain.

2018 Exam (5) Question (5) (a)

19 If set of zeroes of the function $f : f(x) = a x^2 + x + b$ is $\{0 , 1\}$

find the value of each two constants a and b

2018 Exam (2) Question (5) (b)

20 If $n_1(x) = \dfrac{x^2 - 3x + 9}{x^3 + 27}$, $n_2(x) = \dfrac{2}{2x + 6}$ **Prove that :** $n_1 = n_2$

2018 Exam (4) Question (3) (b)

21 **Find the common domain of f_1 , f_2 to be equal such that :**

$f_1(x) = \dfrac{x^2 + x - 12}{x^2 + 5x + 4}$, $f_2(x) = \dfrac{x^2 - 2x - 3}{x^2 + 2x + 1}$

2018 Exam (11) Question (3) (b)

Lesson [4] : Operations On Algebraic Fractions : Part [1]

1 Adding and subtracting two algebraic fractions having the same denominator :

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(X) = \frac{f(X)}{k(X)}$ and $n_2(X) = \frac{p(X)}{k(X)}$, then :

- $n_1(X) + n_2(X) = \frac{f(X)}{k(X)} + \frac{p(X)}{k(X)} = \frac{f(X) + p(X)}{k(X)}$
- $n_1(X) - n_2(X) = \frac{f(X)}{k(X)} - \frac{p(X)}{k(X)} = \frac{f(X) - p(X)}{k(X)}$

For example:

If $n_1(X) = \frac{X}{X-2}$ and $n_2(X) = \frac{X-1}{X-2}$, then :

- $n_1(X) + n_2(X) = \frac{X}{X-2} + \frac{X-1}{X-2} = \frac{X+X-1}{X-2} = \frac{2X-1}{X-2}$

  where the domain of the sum is $\mathbb{R} - \{2\}$
- $n_1(X) - n_2(X) = \frac{X}{X-2} - \frac{X-1}{X-2} = \frac{X-(X-1)}{X-2} = \frac{X-X+1}{X-2} = \frac{1}{X-2}$

  where the domain of the result is $\mathbb{R} - \{2\}$

2 Adding and subtracting two algebraic fractions having different denominators :

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(X) = \frac{f(X)}{r(X)}$ and $n_2(X) = \frac{p(X)}{k(X)}$, then :

- $n_1(X) + n_2(X) = \frac{f(X)}{r(X)} + \frac{p(X)}{k(X)} = \frac{f(X) \times k(X) + p(X) \times r(X)}{r(X) \times k(X)}$
- $n_1(X) - n_2(X) = \frac{f(X)}{r(X)} - \frac{p(X)}{k(X)} = \frac{f(X) \times k(X) - p(X) \times r(X)}{r(X) \times k(X)}$

For example:

If $n_1(X) = \frac{5}{X-3}$ and $n_2(X) = \frac{3}{X+2}$, then :

- $n_1(X) + n_2(X) = \frac{5}{X-3} + \frac{3}{X+2} = \frac{5(X+2) + 3(X-3)}{(X-3)(X+2)} = \frac{5X + 10 + 3X - 9}{(X-3)(X+2)} = \frac{8X+1}{(X-3)(X+2)}$

  where the domain of the sum is $\mathbb{R} - \{3 , -2\}$
  which is the common domain of the two algebraic fractions n_1 and n_2
- $n_1(X) - n_2(X) = \frac{5}{X-3} - \frac{3}{X+2} = \frac{5(X+2) - 3(X-3)}{(X-3)(X+2)} = \frac{5X + 10 - 3X + 9}{(X-3)(X+2)} = \frac{2X+19}{(X-3)(X+2)}$

  where the domain of the result is $\mathbb{R} - \{3 , -2\}$
  which is the common domain of the two algebraic fractions n_1 and n_2

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain which will be the domain of the result.
4. Reduce each fraction separately to make the operations of addition or subtraction easier.
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible.

The properties of the operations of the addition and subtraction of the algebraic fractions :

- **The addition operation of the algebraic fractions has the following properties :**

1. Commutation.
2. Association.
3. Zero is the additive neutral (additive identity) of any algebraic fraction.
4. The additive inverse of any algebraic fraction is available.

i.e. the additive inverse of the algebraic fraction : $\frac{g(X)}{k(X)}$ is $-\frac{g(X)}{k(X)}$, $\frac{-g(X)}{k(X)}$ or $\frac{g(X)}{-k(X)}$

For example:

The additive inverse of the algebraic fraction $\frac{2}{X-1}$

is $-\frac{2}{X-1}$ or $\frac{-2}{X-1}$ or $\frac{2}{1-X}$

Note that :

The domain of the algebraic fraction is the same domain of its additive inverse.

- **Subtraction operation of algebraic fractions has no property of the previous properties.**

Examples :

1 **In each of the following find n (X) in the simplest form showing the domain of n :**

$$n(X) = \frac{X^2+X-6}{X+3} + \frac{X^2-4}{X+2}$$

(El-Kalyoubia 16)

2 **In each of the following find n (X) in the simplest form showing the domain of n :**

$$n(X) = \frac{X^2+3X}{X^2+2X-3} - \frac{X-2}{X^2-3X+2}$$

(El-Dakahlia 17)

3	**In each of the following find n (x) in the simplest form showing the domain of n :** $n(x)=\frac{x^2+x-2}{x^2-1}-\frac{x+5}{x^2+6x+5}$ *(Damietta 14)*
4	**In each of the following find n (x) in the simplest form showing the domain of n :** $n(x)=\frac{x}{x^2+2x}+\frac{x+2}{x^2-4}$ *(El-Sharkia 14 , Souhag 15)*
5	**In each of the following find n (x) in the simplest form showing the domain of n :** 🕮 $n(x)=\frac{x^2+2x+4}{x^3-8}-\frac{9-x^2}{x^2+x-6}$ *(Alex. 17 , El-Beheira 15)*
6	**In each of the following find n (x) in the simplest form showing the domain of n :** $n(x)=\frac{x^2+x}{x^2-1}-\frac{5-x}{x^2-6x+5}$ *(Luxor 17 , El-Sharkia 13)*
7	If $n(x)=\frac{x^2-5x}{x^2-8x+15}-\frac{x^2+3x+9}{x^3-27}$ **, then find n (x) in the simplest form and calculate the value of n (1) , n (5) if it is possible.** *(El-Sharkia 17)*
8	**Find n (x) in the simplest form showing the domain of n where :** $n(x)=\frac{x+3}{x^2+6x+9}+\frac{x+2}{x+3}$, then find n (– 3) and n (2016) if it is possible. *(El-Sharkia 16)*
9	**Find n (x) in the simplest form showing the domain where :** $n(x)=\frac{x^2+x+1}{x^4-x}+\frac{x+3}{3-2x-x^2}$, and if n (a) = – 2 , find the value of a *(El-Monofia 17)* « $\frac{1}{2}$ »
10	If $f_1(x)=\frac{x-a}{x+b}$, and the set of zeroes of f_1 is $\{5\}$, and the domain of f_1 is $\mathbb{R}-\{3\}$, then find the values of a and b. If $f_2(x)=\frac{x-1}{x-3}$, then find $f_1(x)+f_2(x)$ in the simplest form. *(El-Dakahlia 17)* « 5 , – 3 »
11	🕮 If the domain of the function n where $n(x)=\frac{b}{x}+\frac{9}{x+a}$ is $\mathbb{R}-\{0,4\}$, n (5) = 2 **Find the value of a and b** *(Kafr El-Sheikh 16 , El-Beheira 15 , El-Menia 14)* « – 4 , – 35 »

Solutions

1

$n(x) = \frac{(x-2)(x+3)}{x+3} + \frac{(x-2)(x+2)}{x+2}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3 , -2\}$

$\therefore n(x) = (x-2) + (x-2) = 2x - 4$

2

$\because n(x) = \frac{x(x+3)}{(x+3)(x-1)} - \frac{x-2}{(x-2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3 , 1 , 2\}$

, $n(x) = \frac{x}{x-1} - \frac{1}{x-1} = \frac{x-1}{x-1} = 1$

3

$\because n(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} - \frac{x+5}{(x+5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , -5\}$

$\therefore n(x) = \frac{x+2}{x+1} - \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

4

$\because n(x) = \frac{x}{x(x+2)} + \frac{x+2}{(x+2)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 2 , -2\}$

$\therefore n(x) = \frac{1}{x+2} + \frac{1}{x-2} = \frac{x-2+x+2}{(x+2)(x-2)}$

$= \frac{2x}{(x+2)(x-2)}$

5

$\because n(x) = \frac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \frac{(x+3)(x-3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

$\therefore n(x) = \frac{1}{x-2} + \frac{x-3}{x-2} = \frac{x-2}{x-2} = 1$

6

$\because n(x) = \frac{x(x+1)}{(x-1)(x+1)} + \frac{x-5}{(x-5)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1 , 1 , 5\}$

$\therefore n(x) = \frac{x}{x-1} + \frac{1}{x-1} = \frac{x+1}{x-1}$

7

$\because n(x) = \frac{x(x-5)}{(x-3)(x-5)} - \frac{x^2+3x+9}{(x-3)(x^2+3x+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , 5\}$

$\therefore n(x) = \frac{x}{x-3} - \frac{1}{x-3} = \frac{x-1}{x-3}$

, $n(1) = 0$, $n(5)$ is undefined

8

$n(x) = \frac{x+3}{(x+3)^2} + \frac{x+2}{x+3}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3\}$

$\therefore n(x) = \frac{1}{x+3} + \frac{x+2}{x+3} = \frac{x+3}{x+3} = 1$

, $n(-3)$ is undefined because $-3 \notin$ the domain of n

, $n(2016) = 1$

9

$\because n(x) = \frac{x^2+x+1}{x^4-x} - \frac{x+3}{x^2+2x-3}$

$= \frac{x^2+x+1}{x(x-1)(x^2+x+1)} - \frac{x+3}{(x+3)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 1 , -3\}$

, $n(x) = \frac{1}{x(x-1)} - \frac{1}{x-1} = \frac{1-x}{x(x-1)}$

$= \frac{-(x-1)}{x(x-1)} = \frac{-1}{x}$

, $\because n(a) = -2$ $\qquad \therefore \frac{-1}{a} = -2$

$\therefore -2a = -1$ $\qquad \therefore a = \frac{1}{2}$

10

$\because z(f_1) = \{5\}$ $\qquad \therefore$ at $x = 5$

$\therefore x - a = 0$ $\qquad \therefore 5 - a = 0$ $\qquad \therefore a = 5$

, $\because$ the domain of $f_1 = \mathbb{R} - \{3\}$

$\therefore$ at $x = 3$ $\qquad \therefore x + b = 0$

$\therefore 3 + b = 0$ $\qquad \therefore b = -3$ $\qquad f_1(x) = \frac{x-5}{x-3}$

$\therefore f_1(x) + f_2(x) = \frac{x-5}{x-3} + \frac{x-1}{x-3}$

$\therefore$ The domain $= \mathbb{R} - \{3\}$

, $f_1(x) + f_2(x) = \frac{x-5+x-1}{x-3} = \frac{2x-6}{x-3} = \frac{2(x-3)}{x-3} = 2$

11

$\because$ The domain of $n = \mathbb{R} - \{0 , 4\}$ $\qquad \therefore a = -4$

$\therefore n(x) = \frac{b}{x} + \frac{9}{x-4}$, $\because n(5) = 2$

$\therefore \frac{b}{5} + 9 = 2$ $\qquad \therefore \frac{b}{5} = -7$ $\qquad \therefore b = -35$

Exercises

[A] : Choose The Correct Answer : -

1 The additive inverse of the fraction $\frac{3}{x^2+1}$ is ....

(a) $\frac{-3}{x^2+1}$ (b) $\frac{x^2+1}{3}$ (c) $\frac{x^2+1}{-3}$ (d) $\frac{3}{x^2-1}$

2 The function f where $f(x) = \frac{x-2}{x-5}$ has an additive inverse if the domain is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{-2 , 5\}$ (d) $\mathbb{R} - \{2 , 5\}$

3 The function f where $f(x) = \frac{x-3}{x-4}$ has additive inverse in the domain ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{-4\}$ (d) $\mathbb{R} - \{-3\}$

4 The degree of the polynomial function f where $f(x) = x^3 + 2x - 3$ is ....

(a) fourth. (b) third. (c) first. (d) zero.

5 The domain of the function f where $f(x) = \frac{7}{x-5}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0 , 5\}$

6 The domain of the multiplicative inverse of the function $f : f(x) = \frac{x+2}{x-3}$ is ....

(a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2 , 3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R}$

7 The domain of the fraction n : n $(x) = \frac{x+2}{x-1}$ is ....

(a) $\mathbb{R} - \{-2\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1 , -2\}$ (d) $\mathbb{R} - \{2\}$

8 The domain of the function f where $f(x) = \frac{x+2}{5x}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{\text{zero}\}$

9 The set of zeroes of the function $f : f(x) = \frac{x^2-9}{x-3}$ is ....

(a) $\{3\}$ (b) $\{-3\}$ (c) $\{3 , -3\}$ (d) $\varnothing$

10 The set of zeroes of the function $f : f(x) = \frac{x^2-x-2}{x^2+4}$ is ....

(a) $\{2 , -2\}$ (b) $\{-2 , -1\}$ (c) $\{2 , -1\}$ (d) $\{1 , -1\}$

11 The set of zero is of f where $f(x) = x(x^2 - 2x + 1)$ is ....

(a) $\{0 , 1\}$ (b) $\{0 , -1\}$ (c) $\{-1 , 1\}$ (d) $\{0 , 1 , -1\}$

12 The set of zeroes of f where $f(x) = \frac{x^2-9}{x-2}$ is ....

(a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3 , -3\}$ (d) $\{3 , -3 , 2\}$

13	The set of zeroes of the function $f : f(X) = X^2 + 3$ is .... (a) $\{0\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{3 , -3\}$
14	The set of zeroes of f where $f(X) = (X-1)^2 (X+2)$ is .... (a) $\{1 , -2\}$ (b) $\{-1 , 2\}$ (c) $\{-1 , -2\}$ (d) $\{1 , 2\}$
15	The set of zeroes of the function $f : f(X) = X + 3$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\{-3\}$ (d) 3
16	The set of zeroes of the function f where $f(X) = \frac{X-3}{X+2}$ is .... (a) $\{\text{zero}\}$ (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3 , -2\}$
17	The set of zeroes of f where $f(X) = X(X-1)$ is .... (a) $\{1\}$ (b) $\{0 , -1\}$ (c) $\{0 , 1\}$ (d) $\{0\}$
18	If the solution set of the equation $X^2 - aX + 4 = 0$ is $\{-2\}$, then a = ... (a) – 2 (b) – 4 (c) 2 (d) 4
19	The set of zeroes of f where $f(X) = X^2 + 9$ is .... (a) $\{3 , -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$
20	The set of zeroes of the function $f : f(X) = 4$ is .... (a) $\{-4\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\{2\}$
21	If $f(X) = 2X$, then $f(1) - f(-1) =$ .... (a) zero (b) 4 (c) 2 (d) – 2
22	If the function f is a function from set X to set Y , then the domain of the function is .... (a) X (b) Y (c) X × Y (d) Y × X
23	The domain of the function f where $f(X) = \frac{X-2}{X^2+1}$ is .... (a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1 , -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$
24	The solution set of the equation : $X^2 - 4X + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4 , 1\}$ (d) $\varnothing$
25	If $f(X) = 6X^2 + 3X(1 - 2X)$ is a polynomial function , then its degree is .... (a) first. (b) second. (c) third. (d) fourth.
26	If $f(X) = \frac{7+X}{7-X}$, $X \in \mathbb{R} - \{7 , -7\}$, then $f(-2) =$ .... (a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$
27	If $z(f) = \{3\}$, $f(X) = 2X + a$, then a = ... (a) zero. (b) 6 (c) – 6 (d) 3

28	The set of zeroes of f where : $f(x) = \frac{x^3 + x}{x^3 - x}$ is .... (a) $\{0 , 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$
29	The domain of the function f where $f(x) = \frac{x-3}{5(x-1)}$ is .... (a) $\mathbb{R} - \{5 , 1\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{3\}$
30	The domain of the function f where $f(x) = \frac{x-7}{3(x+1)}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{-1 , 3\}$
31	The domain of the function f : $f(x) = \frac{x-2}{7}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{7\}$ (d) $\mathbb{R} - \{2 , 7\}$
32	The domain of the multiplicative inverse of the algebraic fraction $\frac{x+2}{x+5}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{-2 , -5\}$
33	The domain of the additive inverse of the fraction n : $n(x) = \frac{x-2}{x-5}$ is .... (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2 , 5\}$ (d) $\{2 , 5\}$
34	The domain of the multiplicative inverse of the fraction $\frac{x-3}{x+1}$ = ... (a) $\mathbb{R} - \{3 , 1\}$ (b) $\mathbb{R} - \{3 , -1\}$ (c) $\mathbb{R} - \{-3 , -1\}$ (d) $\mathbb{R}$
35	The simplest form of $f(x) = \frac{4x^2 - 2x}{2x}$, $x \neq 0$ is .... (a) $4x^2$ (b) $2x - 1$ (c) $2x$ (d) 2
36	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2} , -\sqrt{2}\}$ (d) $\varnothing$
37	The set of zeroes of the function f : $f(x) = \frac{x^2 + x - 2}{x^2 - 4}$ is .... (a) $\{-2 , 1\}$ (b) $\mathbb{R} - \{2 , -2\}$ (c) $\{-1\}$ (d) $\{1\}$
38	The set of zeroes of f where $f(x) = -3x$ is .... (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3 , 0\}$ (d) $\mathbb{R}$
39	The set of zeroes of f where $f(x) = x^2 - 25$ is .... (a) $\{5\}$ (b) $\{-5\}$ (c) $\{5 , -5\}$ (d) $\varnothing$
40	The degree of the function f : $f(x) = 2x^3 + 3x^2 - 5$ is .... (a) frouth. (b) fifth. (c) third. (d) zero.
41	The set of zeroes of the function f where $f(x) = \frac{x-1}{x+2}$ is .... (a) $\{-2\}$ (b) $\{-2 , 1\}$ (c) $\{1\}$ (d) $\{\text{zero}\}$

42 The set of zeroes of the function $f(x) = \frac{2-x}{7}$ is ....

(a) $\{7\}$ (b) $\{2, 7\}$ (c) $\{2\}$ (d) $\varnothing$

43 In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots.

(a) 1 (b) 2 (c) zero. (d) infinit

44 The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction ....

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{x}{x-5}$ (d) $\frac{x-5}{x-3}$

45 The set of zeroes of the function $f : f(x) = x^2 + 1$ is ....

(a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1, 1\}$ (d) $\varnothing$

46 The set of zeroes of the function f where $f(x) = x^2 + 4$ in $\mathbb{R}$ is ....

(a) $\{2\}$ (b) $\{2, -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$

47 If $n(X^2) = 9$, then $n(X) =$ ....

(a) 81 (b) 3 (c) ± 3 (d) -3

48 The set of zeroes of f where $f(x) = 9$ is ....

(a) $\{9\}$ (b) $\{zero\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$

49 If the solution set of the equation : $x^2 + m x + 9 = 0$ is $\{-3\}$, then m = ...

(a) 5 (b) 6 (c) ± 6 (d) zero

50 If the domain of function $n : n(x) = \frac{x-2}{x^2 + a}$ is $\mathbb{R}$, then a 0

(a) = (b) > (c) ≤ (d) <

51 The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

(a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\varnothing$

52 If $2x^2 = 5$, then $6x^2 =$ ....

(a) 5 (b) 10 (c) 15 (d) 20

53 The common domain of the two fractions : $\frac{2}{x-3}$, $\frac{7}{2x-6}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{0, 3\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{3, -3\}$

54 The simplest of $f(x) = \frac{3-x}{x-3}$, $x \neq 3$ is ....

(a) 3 (b) 1 (c) -1 (d) zero.

55 The domain of the function $f : f(x) = \frac{x-3}{4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4, 3\}$ (d) $\varnothing$

[C] : Essay Problems : -

1	Find n (x) in the simplest form , showing the domain where : $n(x) = \frac{3}{x+1} + \frac{2x+1}{1-x^2}$ 2018 Exam (6) Question (3) (b)
2	Find n (x) in its simplest form showing the domain of n where : $n(x) = \frac{x^2-x-2}{x^2-4} + \frac{x^2-2x+4}{x^3+8}$ 2017 Exam (15) Question (3) (b)
3	If $n(x) = \frac{x^2+x}{x^2-1} - \frac{x-5}{x^2-6x+5}$ Find n (x) in its simplest form , showing the domain of n 2018 Exam (3) Question (4) (a)
4	Find n in its simplest form , showing its domain where : $n(x) = \frac{x^2+x}{x^2-1} + \frac{x-5}{x^2-6x+5}$ 2018 Exam (10) Question (4) (a)
5	Find n (x) in the simplest form showing the domain where : $n(x) = \frac{x-3}{x^2-7x+12} - \frac{4}{x^2-4x}$ 2018 Model Exam (1) Question (2) (b)
6	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x}{x+1} + \frac{x^2}{x^3+x^2}$, then calculate n (3) 2018 Exam (9) Question (2) (b)
7	Find n (x) in the simplest form where : $n(x) = \frac{x^2+2x+4}{x^3-8} + \frac{1}{x+2}$ 2018 Exam (8) Question (5) (a)
8	If $n(x) = \frac{x^2+x}{x^2-x-2} - \frac{2x+4}{x^2-4}$, find n (x) in the simplest form showing the domain of n 2018 Exam (14) Question (2) (a)
9	Find n (x) in the simplest form showing the domain where : $n(x) = \frac{x^2+x+1}{x^4-x} + \frac{x+3}{3-2x-x^2}$ and if n (a) = – 2 , find the value of a 2018 Exam (5) Question (4) (a)
10	Find n (x) in its simplest form , showing the domain of n : $n(x) = \frac{x^2+x}{x^2-1} - \frac{x+5}{x^2+4x-5}$ 2018 Exam (1) Question (2) (b)
11	If $n(x) = \frac{x}{x^2+2x} + \frac{x-2}{x^2-4}$, find n (x) in the simplest form showing the domain of n 2018 Exam (21) Question (4) (b)
12	Find n (x) in the simplest form where : $n(x) = \frac{x}{x-4} - \frac{4x+16}{x^2-16}$ 2018 Exam (8) Question (3) (b)

13	**Put n (x) in its simplest form showing the domain of n where :** $n(x) = \frac{x^2 - 3x + 2}{x^2 - 1} + \frac{3x - 15}{x^2 - 4x - 5}$ 2017 Exam (8) Question (3) (a)
14	**Find n (x) in the simplest form showing the domain of n , where :** $n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} - \frac{9 - x^2}{x^2 + x - 6}$ 2018 Exam (2) Question (2) (b)
15	**Find n (x) in its simplest form showing the domain of n where :** $n(x) = \frac{x^2 + 3x}{x^2 + 4x + 3} + \frac{x - 5}{x^2 - 4x - 5}$ 2017 Exam (17) Question (4) (b)
16	**Find n (x) in its simplest form showing the domain of n where :** $n(x) = \frac{x^2 - x}{x^2 - 1} - \frac{-x - 5}{x^2 + 6x + 5}$ 2017 Exam (6) Question (3) (b)
17	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{3x - 4}{x^2 - 5x + 6} + \frac{2x + 6}{x^2 + x - 6}$ 2018 Exam (19) Question (4) (b)
18	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x}{x + 1} + \frac{2x^2}{x^3 - x}$ 2018 Exam (18) Question (3) (b)
19	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} + \frac{x^2 + x - 2}{x^2 - 4}$ 2018 Exam (17) Question (2) (b)
20	If $n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{5 - x}{x^2 - 6x + 5}$ **Find n (x) in the simplest form showing the domain of n.** 2018 Exam (20) Question (4) (b)
21	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x - 3}{x^2 - 2x - 3}$ 2017 Exam (9) Question (2) (b)
22	**Simplify the function n (x) where :** $n(x) = \frac{3x}{x^2 - 2x} - \frac{12}{x^2 - 4}$ showing the domain of n 2018 Exam (13) Question (2) (b)

Homework

[A] : Choose The Correct Answer : -

1	The additive inverse of the fraction $\frac{3}{x^2+1}$ is .... (a) $\frac{-3}{x^2+1}$ (b) $\frac{x^2+1}{3}$ (c) $\frac{x^2+1}{-3}$ (d) $\frac{3}{x^2-1}$
2	The domain of the function f where $f(x) = \frac{x+2}{5x}$ is .... (a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{\text{zero}\}$
3	The set of zeroes of the function $f : f(x) = x + 3$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\{-3\}$ (d) 3
4	The domain of the function f where $f(x) = \frac{x-2}{x^2+1}$ is .... (a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1, -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$
5	The domain of the multiplicative inverse of the fraction $\frac{x-3}{x+1}$ = ... (a) $\mathbb{R} - \{3, 1\}$ (b) $\mathbb{R} - \{3, -1\}$ (c) $\mathbb{R} - \{-3, -1\}$ (d) $\mathbb{R}$
6	The set of zeroes of f where $f(x) = x^2 - 25$ is .... (a) $\{5\}$ (b) $\{-5\}$ (c) $\{5, -5\}$ (d) $\varnothing$
7	If n $(X^2) = 9$, then n (X) = ... (a) 81 (b) 3 (c) ± 3 (d) -3
8	The domain of the function $f : f(x) = \frac{x-3}{4}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4, 3\}$ (d) $\varnothing$
9	The domain of the fraction n : n $(x) = \frac{x+2}{x-1}$ is .... (a) $\mathbb{R} - \{-2\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -2\}$ (d) $\mathbb{R} - \{2\}$
10	The set of zeroes of f where $f(x) = (x-1)^2 (x+2)$ is .... (a) $\{1, -2\}$ (b) $\{-1, 2\}$ (c) $\{-1, -2\}$ (d) $\{1, 2\}$
11	If the function f is a function from set X to set Y , then the domain of the function is .... (a) X (b) Y (c) X × Y (d) Y × X
12	The domain of the additive inverse of the fraction n : n $(x) = \frac{x-2}{x-5}$ is .... (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2, 5\}$ (d) $\{2, 5\}$

13	The set of zeroes of f where $f(X) = -3X$ is …………… (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3, 0\}$ (d) $\mathbb{R}$	
14	The set of zeroes of the function f where $f(X) = X^2 + 4$ in $\mathbb{R}$ is …………… (a) $\{2\}$ (b) $\{2, -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$	
15	The simplest of $f(X) = \frac{3 - X}{X - 3}$, $X \neq 3$ is …………… (a) 3 (b) 1 (c) -1 (d) zero.	
16	The domain of the multiplicative inverse of the function $f : f(X) = \frac{X + 2}{X - 3}$ is …………… (a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{-2, 3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $\mathbb{R}$	
17	The set of zeroes of the function $f : f(X) = X^2 + 3$ is …………… (a) $\{0\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{3, -3\}$	
18	If $f(X) = 2X$, then $f(1) - f(-1) =$ …………… (a) zero (b) 4 (c) 2 (d) -2	
19	The domain of the multiplicative inverse of the algebraic fraction $\frac{X + 2}{X + 5}$ is …………… (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R} - \{-2\}$ (d) $\mathbb{R} - \{-2, -5\}$	
20	The set of zeroes of the function $f : f(X) = \frac{X^2 + X - 2}{X^2 - 4}$ is …………… (a) $\{-2, 1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\{-1\}$ (d) $\{1\}$	
21	The set of zeroes of the function $f : f(X) = X^2 + 1$ is …………… (a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1, 1\}$ (d) $\varnothing$	
22	The common domain of the two fractions : $\frac{2}{X - 3}$, $\frac{7}{2X - 6}$ is …………… (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0, 3\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{3, -3\}$	
23	The domain of the function f where $f(X) = \frac{7}{X - 5}$ is …………… (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0, 5\}$	
24	The set of zeroes of f where $f(X) = \frac{X^2 - 9}{X - 2}$ is …………… (a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3, -3\}$ (d) $\{3, -3, 2\}$	
25	The set of zeroes of f where : $f(X) = \frac{X^3 + X}{X^3 - X}$ is …………… (a) $\{0, 1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$	
26	In equation : $aX^2 + bX + c = 0$ if $b^2 - 4ac > 0$, then this equation has …………… roots. (a) 1 (b) 2 (c) zero. (d) infinit	

27	If $2x^2 = 5$, then $6x^2 =$ .... (a) 5 (b) 10 (c) 15 (d) 20
28	The degree of the polynomial function f where $f(x) = x^3 + 2x - 3$ is .... (a) fourth. (b) third. (c) first. (d) zero.
29	The set of zero is of f where $f(x) = x(x^2 - 2x + 1)$ is .... (a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1, -1\}$
30	The set of zeroes of f where $f(x) = x^2 + 9$ is .... (a) $\{3, -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$
31	The domain of the function f where $f(x) = \frac{x - 7}{3(x + 1)}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{-1, 3\}$
32	If $z(f) = \{3\}$, $f(x) = 2x + a$, then a = ... (a) zero. (b) 6 (c) -6 (d) 3
33	The degree of the function $f : f(x) = 2x^3 + 3x^2 - 5$ is .... (a) frouth. (b) fifth. (c) third. (d) zero.
34	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\varnothing$
35	The function f where $f(x) = \frac{x - 3}{x - 4}$ has additive inverse in the domain .... (a) $\mathbb{R} - \{3\}$ (b) $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{-4\}$ (d) $\mathbb{R} - \{-3\}$
36	If $f(x) = \frac{7 + x}{7 - x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2) =$ .... (a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$
37	If the solution set of the equation $x^2 - ax + 4 = 0$ is $\{-2\}$, then a = ... (a) -2 (b) -4 (c) 2 (d) 4
38	The domain of the function f where $f(x) = \frac{x - 3}{5(x - 1)}$ is .... (a) $\mathbb{R} - \{5, 1\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{3\}$
39	The domain of the algebraic fraction $\frac{x - 5}{3}$ equals the domain of the algebraic fraction .... (a) $\frac{x}{x^2 + 1}$ (b) $\frac{x}{x - 3}$ (c) $\frac{x}{x - 5}$ (d) $\frac{x - 5}{x - 3}$
40	The set of zeroes of the function $f(x) = \frac{2 - x}{7}$ is .... (a) $\{7\}$ (b) $\{2, 7\}$ (c) $\{2\}$ (d) $\varnothing$

41	If the domain of function n : $n(x) = \frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a .... 0 (a) = (b) > (c) ≤ (d) <
42	The function f where $f(x) = \frac{x-2}{x-5}$ has an additive inverse if the domain is .... (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{-2 , 5\}$ (d) $\mathbb{R} - \{2 , 5\}$
43	The set of zeroes of the function $f : f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is .... (a) $\{2 , -2\}$ (b) $\{-2 , -1\}$ (c) $\{2 , -1\}$ (d) $\{1 , -1\}$
44	The set of zeroes of f where $f(x) = x(x-1)$ is .... (a) $\{1\}$ (b) $\{0 , -1\}$ (c) $\{0 , 1\}$ (d) $\{0\}$
45	If $f(x) = 6x^2 + 3x(1 - 2x)$ is a polynomial function , then its degree is .... (a) first. (b) second. (c) third. (d) fourth.
46	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2} , -\sqrt{2}\}$ (d) $\varnothing$
47	The set of zeroes of the function f where $f(x) = \frac{x-1}{x+2}$ is .... (a) $\{-2\}$ (b) $\{-2 , 1\}$ (c) $\{1\}$ (d) {zero}
48	If the solution set of the equation : $x^2 + mx + 9 = 0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero
49	The set of zeroes of the function $f : f(x) = \frac{x^2 - 9}{x - 3}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3 , -3\}$ (d) $\varnothing$
50	The set of zeroes of the function f where $f(x) = \frac{x-3}{x+2}$ is .... (a) {zero} (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3 , -2\}$
51	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4 , 1\}$ (d) $\varnothing$
52	The simplest form of $f(x) = \frac{4x^2 - 2x}{2x}$, $x \neq 0$ is .... (a) $4x^2$ (b) $2x - 1$ (c) $2x$ (d) 2
53	The set of zeroes of the function f where $f(x) = \frac{x+7}{x-2}$ is .... (a) $\{-7\}$ (b) $\{7\}$ (c) $\{2\}$ (d) $\{7 , 2\}$
54	The set of zeroes of f where $f(x) = 9$ is .... (a) $\{9\}$ (b) {zero} (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$

[B] : Essay Problems : -

1	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x^2+2x+4}{x^3-8} + \frac{x^2+x-2}{x^2-4}$ 2018 Exam (17) Question (2) (b)
2	Find in the simplest form showing the domain : $n(x) = \frac{x^2+x-6}{x+3} + \frac{x^2-4}{x+2}$ 2017 Exam (4) Question (4) (b)
3	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x^2-2x}{x^2-4} + \frac{2x+6}{x^2+5x+6}$ 2017 Exam (1) Question (4) (b)
4	Find n (x) in its simplest form : $n(x) = \frac{x^2-x}{x^2-1} - \frac{x-5}{x^2-6x+5}$ 2017 Exam (10) Question (3) (b)
5	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x}{x+1} + \frac{2x^2}{x^2+x}$ 2017 Exam (3) Question (2) (b)
6	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x}{x-4} - \frac{x+4}{x^2-16}$ 2018 Exam (23) Question (2) (b)
7	Find n (x) in the simplest form where : $n(x) = \frac{x^2-9}{x^2-x-6} + \frac{4x-x^2}{x^2-2x-8}$, then find if possible n (3) 2017 Exam (8) Question (4) (b)
8	Simplify the function n (x) where : $n(x) = \frac{3x}{x^2-2x} - \frac{12}{x^2-4}$ showing the domain of n 2018 Exam (13) Question (2) (b)
9	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x+3}{x^2+6x+9} + \frac{x+2}{x+3}$, then find n (– 3) , n (2016) if possible. 2017 Exam (5) Question (4) (a)
10	Find n (x) in the simplest form showing the domain of n where : $n(x) = \frac{x^2+2x+1}{2x-8} - \frac{x-4}{x+1}$ 2018 Exam (9) Question (5) (a)

11 **Find n (x) in the simplest form showing the domain of n where :**

$$n(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x - 3}{x^2 - 2x - 3}$$

2017 Exam (9) Question (2) (b)

12 **Find n (x) in the simplest form showing the domain of n where :**

$$n(x) = \frac{x + 3}{x^2 - 9} - \frac{2x + 2}{x^2 - 2x - 3}$$

2017 Exam (2) Question (2) (b)

13 **Find n (x) in its simplest form showing the domain of n where :**

$$n(x) = \frac{x^2 + 3x + 9}{x^3 - 27} + \frac{x^2 - x - 12}{x^2 - 9}$$

2017 Exam (13) Question (2) (b)

14 If $n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{5 - x}{x^2 - 6x + 5}$

Find n (x) in the simplest form showing the domain of n.

2018 Exam (20) Question (4) (b)

15 **Find f (x) in the simplest form , showing the domain of f where :**

$$f(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x + 5}{x^2 + 6x + 5}$$

2018 Exam (24) Question (3) (a)

16 **Find n (x) in the simplest form , showing the domain of n :**

$$n(x) = \frac{3x + 1}{x + 1} + \frac{2x - 2}{x^2 - 1}$$

2017 Exam (20) Question (4) (b)

17 **Find n (x) in the simplest form showing the domain of n where :**

$$n(x) = \frac{2x + 6}{x^2 + x - 6} - \frac{3x - 4}{5x - x^2 - 6}$$

2017 Exam (11) Question (3) (a)

18 If $n(x) = \frac{x^2 + 3x}{x^2 + 2x - 3} - \frac{x - 2}{x^2 - 3x + 2}$

Find n (x) in simplest form showing the domain of n

2018 Exam (7) Question (4) (a)

19 **Find n (x) in its simplest form showing the domain of n , where :**

$$n(x) = \frac{3x + 6}{x^2 + x - 2} - \frac{x + 1}{1 - x^2}$$

2017 Exam (16) Question (3) (a)

20 **Simplify :**

$n(x) = \frac{x^2 + 2x}{x^2 - 4} + \frac{x + 3}{x^2 - 5x + 6}$, showing the domain of n.

2018 Model Exam (2) Question (5) (a)

Lesson [4] : Operations On Algebraic Fractions : Part [2]

Multiplying two algebraic fractions

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$

, then : $n_1(x) \times n_2(x) = \frac{f(x)}{r(x)} \times \frac{p(x)}{k(x)} = \frac{f(x) \times p(x)}{r(x) \times k(x)}$

For example:

If : $n_1(x) = \frac{2}{x}$, $n_2(x) = \frac{x}{x-1}$,

then : $n_1(x) \times n_2(x) = \frac{2}{x} \times \frac{x}{x-1}$

$= \frac{2 \times \cancel{x}}{\cancel{x}(x-1)}$

where the domain of the product $= \mathbb{R} - \{0 , 1\}$

, $n_1(x) \times n_2(x) = \frac{2}{x-1}$

Notice that :

The domain of the product is the common domain of the two algebraic fractions before reduction.

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction alone if it is possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

The properties of the operation of multiplying the algebraic fractions :

The operation of multiplying the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity).

4 Existing the multiplicative inverses.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(x) = \frac{p(x)}{k(x)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(x) = \frac{k(x)}{p(x)}$

and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

For example:

If $n(x) = \frac{x+1}{x-5}$, then : $n^{-1}(x) = \frac{x-5}{x+1}$

where the domain of $n = \mathbb{R} - \{5\}$

and the domain of $n^{-1} = \mathbb{R} - \{5, -1\}$

Note that :

$n(x)$ and $n^{-1}(x)$ each of them is the reciprocal of the other

i.e. the numerator of each of them is a denominator for the other.

2 Dividing an algebraic fraction by another :

The rule of dividing two algebraic fractions is similar to the rule of dividing two rational numbers, therefore it is better to remember together how to divide two rational numbers.

Remember that :

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers and $\frac{c}{d} \neq 0$

, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ the multiplicative inverse of the number $\frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

For example:

- $\frac{1}{4} \div \frac{3}{5} = \frac{1}{4} \times \frac{5}{3} = \frac{5}{12}$
- $\frac{5}{8} \div \frac{-15}{4} = \frac{\cancel{5}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{-\cancel{15}_{3}} = \frac{1}{2} \times \frac{1}{-3} = -\frac{1}{6}$

Regarding that the multiplicative inverses of the algebraic fractions exist , then the operation of division is possible and it is defined as follows :

Dividing an algebraic fraction by another :

If n_1 and n_2 are two algebraic fractions where :

$n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$, then : $n_1(x) \div n_2(x) = n_1(x) \times n_2^{-1}(x) = \frac{f(x)}{r(x)} \times \frac{k(x)}{p(x)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1 and n_2^{-1}

$= \mathbb{R}$ – the set of zeroes of denominator of n_1 or denominator of n_2 or numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

For example:

If $n_1(x) = \frac{x}{x-1}$, $n_2(x) = \frac{2x}{x-1}$,

then $n_1(x) \div n_2(x) = \frac{x}{x-1} \div \frac{2x}{x-1} = \frac{\cancel{x}}{\cancel{x-1}} \times \frac{\cancel{x-1}}{2\cancel{x}} = \frac{1}{2}$ where $x \notin \{1, 0\}$

Examples :

1 **In each of the following find n (x) in the simplest form showing the domain of n :**

$n(x) = \frac{x^2+2x+1}{2x-8} \times \frac{x-4}{x+1}$ *(Suez 17 , Cairo 16 , Ismailia 15)*

2 **In each of the following find n (x) in the simplest form showing the domain of n :**

📖 $n(x) = \frac{x^3-1}{x^2-2x+1} \times \frac{2x-2}{x^2+x+1}$ *(Luxor 17 , Souhag 12)*

3 **In each of the following find n (x) in the simplest form showing the domain of n :**

$n(x) = \frac{x^2-3x-4}{x^2-1} \times \frac{x^2-x}{x^2+3x}$ *(El-Kalyoubia 16 , El-Gharbia 04)*

4 **In each of the following find n (x) in the simplest form showing the domain of n :**

$n(x) = \frac{x^2+2x}{x^3-27} \times \frac{x^2+3x+9}{x+2}$, then find n (6) , n (– 2) if it is possible. *(South Sinai 17)*

5 **In each of the following find n (x) in the simplest form showing the domain of n :**

$n(x) = \frac{x-1}{x^2-1} \div \frac{x^2-5x}{x^2-4x-5}$ *(El-Menia 16 , El-Beheira 15 , Aswan 14)*

6 **In each of the following find n (x) in the simplest form showing the domain of n :**

📖 $n(x) = \frac{x^2-2x+1}{x^3-1} \div \frac{x-1}{x^2+x+1}$ *(El-Gharbia 17 , Alexandria 11)*

7	🕮 If $n(x) = \dfrac{x^2 - 2x}{(x-2)(x^2+2)}$ **First :** find $n^{-1}(x)$ and identify its domain. **Second :** if $n^{-1}(x) = 3$ what is the value of x ? *(El-Gharbia 17 , Aswan 16 , El-Beheira 14)* « 1 »
8	🕮 If $f(x) = \dfrac{x^2-49}{x^3-8} \div \dfrac{x+7}{x-2}$, then find n (x) in the simplest form and identify its domain and find $f(1)$ *(El-Beheira 17 , El-Gharbia 12)* « $-\frac{6}{7}$ »

Solutions

1	$n(x) = \dfrac{(x+1)^2}{2(x-4)} \times \dfrac{x-4}{x+1}$ $\therefore$ The domain of $n = \mathbb{R} - \{4 , -1\}$ $, n(x) = \dfrac{x+1}{2}$
2	$n(x) = \dfrac{(x-1)(x^2+x+1)}{(x-1)^2} \times \dfrac{2(x-1)}{(x^2+x+1)}$ $\therefore$ The domain of $n = \mathbb{R} - \{1\}$ $, n(x) = 2$
3	$n(x) = \dfrac{(x-4)(x+1)}{(x-1)(x+1)} \times \dfrac{x(x-1)}{x(x+3)}$ $\therefore$ The domain of $n = \mathbb{R} - \{0 , 1 , -1 , -3\}$ $, n(x) = \dfrac{x-4}{x+3}$
4	$n(x) = \dfrac{x(x+2)}{(x-3)(x^2+3x+9)} \times \dfrac{x^2+3x+9}{x+2}$ $\therefore$ The domain of $n = \mathbb{R} - \{3 , -2\}$ $, n(x) = \dfrac{x}{x-3}$ $, n(6) = \dfrac{6}{6-3} = 2$ $, n(-2)$ is undefined because $-2 \notin$ the domain of n
5	$n(x) = \dfrac{x-1}{(x-1)(x+1)} \times \dfrac{(x-5)(x+1)}{x(x-5)}$ $\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , 0 , 5\}$ $, n(x) = \dfrac{1}{x}$

6	$n(x) = \dfrac{(x-1)^2}{(x-1)(x^2+x+1)} \times \dfrac{x^2+x+1}{x-1}$ $\therefore$ The domain of $n = \mathbb{R} - \{1\}$ $, n(x) = 1$
7	**First :** $n(x) = \dfrac{x(x-2)}{(x-2)(x^2+2)}$ $\therefore$ The domain of $n = \mathbb{R} - \{2\}$ $, n(x) = \dfrac{x}{x^2+2}$ $\therefore n^{-1}(x) = \dfrac{x^2+2}{x}$ $\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2 , 0\}$ **Second :** $\dfrac{x^2+2}{x} = 3$ $\therefore x^2 - 3x + 2 = 0$ $\therefore (x-2)(x-1) = 0$ $\therefore x = 2$ (refused) or $x = 1$
8	$n(x) = \dfrac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \times \dfrac{x-2}{x+7}$ $\therefore$ The domain of $n = \mathbb{R} - \{2 , -7\}$ $, n(x) = \dfrac{x-7}{x^2+2x+4}$ $, n(1) = \dfrac{1-7}{1+2+4} = -\dfrac{6}{7}$

Exercises

[A] : Choose The Correct Answer : -

1 If $x \neq 0$, then $\frac{5x}{x^2+1} \div \frac{x}{x^2+1} =$ ....

(a) -5 (b) -1 (c) 1 (d) 5

2 If $n(x) = \frac{x-2}{2}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{0 , 2\}$

3 If the function f where $f(x) = \frac{x^2-9}{x}$ has a multiplicative inverse , then their common domain is ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{0 , 3\}$ (c) $\mathbb{R} - \{0 , -3 , 3\}$ (d) $\mathbb{R}$

4 If $n(x) = \frac{x}{x-1}$, then the domain of $n^{-1} =$ ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{0 , 1\}$ (d) $\mathbb{R} - \{-1\}$

5 If $n(x) = \frac{x+1}{x-2}$ is an algebraic fraction , then the domain in which the fraction has multiplicative inverse is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-1 , 2\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\{-1 , 2\}$

6 If $n(x) = \frac{x-1}{x-2}$, then the domain of $n^{-1} =$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{1 , 2\}$

7 If $n(x) = \frac{x-1}{x+2}$, then $n^{-1}(1)$ is ....

(a) -1 (b) zero (c) 3 (d) undefined.

8 If $n(x) = \frac{x-7}{x+3}$, then the domain of n^{-1} is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-3\}$ (c) $\mathbb{R} - \{7\}$ (d) $\mathbb{R} - \{-3 , 7\}$

9 The function f where $f(x) = \frac{x-1}{x-3}$ has a multiplicative inverse if its domain is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{8\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{1 , 3\}$

10 If the fraction $\frac{x-a}{x+3}$ is the multiplicative inverse of $\frac{x+3}{x+5}$, then a = ...

(a) -5 (b) -3 (c) 3 (d) 5

11 If $n(x) = \frac{x-1}{x+4}$, then the domain of $n^{-1} =$ ....

(a) $\mathbb{R} - \{-4\}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{1 , 4\}$ (d) $\mathbb{R} - \{-4 , 1\}$

12 The degree of the polynomial function f where $f(x) = x^3 + 2x - 3$ is ....

(a) fourth. (b) third. (c) first. (d) zero.

13 The domain of the function n^{-1} : $n(x) = \frac{x+4}{x-4}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{4, -4\}$

14 The function f where $f(x) = \frac{x-2}{x-5}$ has a multiplicative inverse if its domain is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{2, 5\}$

15 If $n(x) = \frac{x+2}{x-5}$, then the domain of n^{-1} is ....

(a) $\{2, -5\}$ (b) $\{-2, 5\}$ (c) $\mathbb{R} - \{-2, 5\}$ (d) $\mathbb{R} - \{2, -5\}$

16 If the algebraic fraction $\frac{x-a}{x+5}$ have a multiplicative inverse which is $\frac{x+5}{x+3}$, then a = ...

(a) 3 (b) -5 (c) -3 (d) 5

17 The multiplicative inverse of the algebraic fraction $\frac{3}{x^2+1}$ is ....

(a) $\frac{-3}{x^2+1}$ (b) $\frac{x^2+1}{-3}$ (c) $\frac{x^2+1}{3}$ (d) $\frac{x^2-1}{3}$

18 If $n(x) = \frac{x}{x^2+9}$, then the domain of n^{-1} is ....

(a) $\varnothing$ (b) $\mathbb{R} - \{-3, 3\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0\}$

19 If $n(x) = \frac{x-2}{x^2-x-6}$, then the domain of n^{-1} is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2, 3\}$ (c) $\mathbb{R} - \{-2, 2\}$ (d) $\mathbb{R} - \{-2, 2, 3\}$

20 The domain of the multiplicative inverse of the fraction : $\frac{x-2}{x^3+27}$ is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-3, 2\}$ (c) $\mathbb{R} - \{2, -3, 3\}$ (d) $\mathbb{R} - \{3, -3\}$

21 The set of zeroes of f where $f(x) = x^2 + 9$ is ....

(a) $\{3, -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$

22 The set of zeroes of the function f where $f(x) = \frac{x+7}{x-2}$ is ....

(a) $\{-7\}$ (b) $\{7\}$ (c) $\{2\}$ (d) $\{7, 2\}$

23 If $f(x) = \frac{7+x}{7-x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2) =$ ....

(a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$

24 The domain of the multiplicative inverse of the fraction $\frac{x-3}{x+1}$ = ...

(a) $\mathbb{R} - \{3, 1\}$ (b) $\mathbb{R} - \{3, -1\}$ (c) $\mathbb{R} - \{-3, -1\}$ (d) $\mathbb{R}$

25	The common domain of the two fractions : $\frac{2}{x-3}$, $\frac{7}{2x-6}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R}-\{0,3\}$ (c) $\mathbb{R}-\{3\}$ (d) $\mathbb{R}-\{3,-3\}$
26	The set of zeroes of the function f where $f(x)=x^2+4$ in $\mathbb{R}$ is .... (a) $\{2\}$ (b) $\{2,-2\}$ (c) $\mathbb{R}$ (d) $\varnothing$
27	The set of zeroes of the function $f : f(x)=x+3$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R}-\{3\}$ (c) $\{-3\}$ (d) 3
28	The set of zeroes of f where : $f(x)=\frac{x^3+x}{x^3-x}$ is .... (a) $\{0,1\}$ (b) $\{1\}$ (c) $\{0\}$ (d) $\varnothing$
29	The domain of the fraction n : $n(x)=\frac{x+2}{x-1}$ is .... (a) $\mathbb{R}-\{-2\}$ (b) $\mathbb{R}-\{1\}$ (c) $\mathbb{R}-\{1,-2\}$ (d) $\mathbb{R}-\{2\}$
30	The domain of the function f where $f(x)=\frac{x-7}{3(x+1)}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R}-\{-1\}$ (c) $\mathbb{R}-\{1\}$ (d) $\mathbb{R}-\{-1,3\}$
31	If the solution set of the equation : $x^2+mx+9=0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero
32	The set of zeroes of the function $f : f(x)=4$ is .... (a) $\{-4\}$ (b) {zero} (c) $\varnothing$ (d) $\{2\}$
33	The set of zeroes of the function f where $f(x)=\frac{x-1}{x+2}$ is .... (a) $\{-2\}$ (b) $\{-2,1\}$ (c) $\{1\}$ (d) {zero}
34	The set of zero is of f where $f(x)=x(x^2-2x+1)$ is .... (a) $\{0,1\}$ (b) $\{0,-1\}$ (c) $\{-1,1\}$ (d) $\{0,1,-1\}$
35	The simplest form of $f(x)=\frac{4x^2-2x}{2x}$, $x\neq 0$ is .... (a) $4x^2$ (b) $2x-1$ (c) $2x$ (d) 2
36	The simplest of $f(x)=\frac{3-x}{x-3}$, $x\neq 3$ is .... (a) 3 (b) 1 (c) -1 (d) zero.
37	If the solution set of the equation $x^2-ax+4=0$ is $\{-2\}$, then a = ... (a) -2 (b) -4 (c) 2 (d) 4
38	If $n(X^2)=9$, then $n(X)=$ .... (a) 81 (b) 3 (c) ± 3 (d) -3

39	The set of zeroes of the function f where $f(x) = \frac{x-3}{x+2}$ is .... (a) $\{zero\}$ (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3, -2\}$
40	The set of zeroes of the function $f : f(x) = \frac{x^2+x-2}{x^2-4}$ is .... (a) $\{-2, 1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\{-1\}$ (d) $\{1\}$
41	The domain of the function f where $f(x) = \frac{x+2}{5x}$ is .... (a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{zero\}$
42	The domain of the function f where $f(x) = \frac{x-2}{x^2+1}$ is .... (a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1, -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$
43	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit
44	If $f(x) = 2x$, then $f(1) - f(-1) =$ .... (a) zero (b) 4 (c) 2 (d) -2
45	The set of zeroes of the function $f(x) = \frac{2-x}{7}$ is .... (a) $\{7\}$ (b) $\{2, 7\}$ (c) $\{2\}$ (d) $\varnothing$
46	The set of zeroes of f where $f(x) = \frac{x^2-9}{x-2}$ is .... (a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3, -3\}$ (d) $\{3, -3, 2\}$
47	The domain of the function $f : f(x) = \frac{x-2}{7}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{7\}$ (d) $\mathbb{R} - \{2, 7\}$
48	If the domain of function $n : n(x) = \frac{x-2}{x^2+a}$ is $\mathbb{R}$, then a .... 0 (a) = (b) > (c) ≤ (d) <
49	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4, 1\}$ (d) $\varnothing$
50	The set of zeroes of f where $f(x) = 9$ is .... (a) $\{9\}$ (b) $\{zero\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$
51	The set of zeroes of f where $f(x) = x(x-1)$ is .... (a) $\{1\}$ (b) $\{0, -1\}$ (c) $\{0, 1\}$ (d) $\{0\}$
52	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\varnothing$

[C] : Essay Problems : -

1	**Find n (x) in the simplest form showing the domain of n where :** (1) $n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{x - 5}{x^2 - 6x + 5}$ (2) $n(x) = \frac{x^2 + 2x}{x^3 - 27} \times \frac{x^2 + 3x + 9}{x + 2}$ 2018 Exam (22) Question (2)
2	If $n(x) = \frac{x^2 - 3x}{x^2 - 9} \div \frac{2x}{x + 3}$, find n (x) in its simplest form showing the domain of n 2018 Exam (18) Question (4) (b)
3	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 - 2x - 15}{x^2 - 9} \div \frac{2x - 10}{x^2 - 6x + 9}$ 2017 Exam (3) Question (5) (a)
4	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 + x + 1}{x} \times \frac{x^2 - x}{x^3 - 1}$ 2018 Exam (19) Question (3) (b)
5	If $f(x) = \frac{x^2 - 49}{x^3 - 8} \div \frac{x + 7}{x - 2}$ **Find :** f (x) in its simplest form showing the domain of f 2018 Exam (13) Question (5) (b)
6	If $n(x) = \frac{x^2 + 3x}{x^3 + 27}$, find $n^{-1}(x)$ in its simplest form showing the domain of n^{-1} 2018 Exam (17) Question (4) (b)
7	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 - x - 6}{x^3 - 27} \times \frac{x^2 + 3x + 9}{x + 2}$ 2017 Exam (9) Question (3) (b)
8	If $f(x) = \frac{x^3 - 8}{x^2 - 9} \div \frac{x - 2}{x + 3}$ find f (x) in its simplest form , showing the domain of f 2017 Exam (16) Question (4) (a)
9	If $n(x) = \frac{x^2 - 16}{x + 4}$ **Find :** (1) $n^{-1}(x)$ showing the domain of n^{-1} (2) $n^{-1}(4)$ (3) n (4) 2018 Exam (20) Question (2) (b)
10	**Find n (x) in the simplest form showing the domain of n :** $n(x) = \frac{x^2 + 2x + 1}{2x - 8} \times \frac{x - 4}{x + 1}$ 2018 Exam (16) Question (4) (a)
11	If $n(x) = \frac{x + 3}{x^2 + 5x - 14} \div \frac{x^2 + 3x}{2x + 14}$ **Find :** n (x) in its simplest form , showing the domain of n 2018 Exam (10) Question (3) (b)

12	**Find the set of zeroes of the function f : $f(x) = \frac{x-1}{x+1}$** , then find $f^{-1}(2)$ 2018 Exam (14) Question (4) (b)
13	If $n(x) = \frac{x^2 - 3x + 2}{x^2 - 1} \times \frac{x^2 - 4x - 5}{3x - 15}$ **Find n (x) in its simplest form showing the domain of n** 2017 Exam (12) Question (3) (b)
14	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^3 - 1}{x^2 - 2x + 1} \div \frac{x^2 + x + 1}{2x - 2}$ 2018 Exam (14) Question (3) (b)
15	If $n(x) = \frac{x-1}{x+3}$ find $n^{-1}(x)$ and identify the domain of n^{-1} 2018 Exam (21) Question (2) (b)
16	**Find n (x) in the simplest form showing the domain of n where :** $n(x) = \frac{x^2 - 3x - 4}{x^2 - 1} \times \frac{x^2 - x}{x^2 + 3x}$ 2017 Exam (4) Question (2) (b)
17	If $n(x) = \frac{x^3 - 8}{x^2 - x - 2} \div \frac{x^2 + 2x + 4}{2x^2 - x - 3}$ Find n (x) in its simplest form showing the domain of n 2018 Exam (3) Question (2) (a)
18	If $n(x) = \frac{x+2}{2x-6}$ **Find :** (1) $n^{-1}(x)$ showing the domain of n^{-1} (2) $n^{-1}(2)$ 2017 Exam (15) Question (4) (b)
19	**Find n (x) in simplest form showing the domain of n , such that :** $n(x) = \frac{x^3 - x^2 - 2x}{x^2 - 5x + 6} \times \frac{x^2 + 2x - 15}{x^3 + 6x^2 + 5x}$, then find n (7) , n (3) if possible. 2018 Exam (7) Question (4) (b)
20	If $n(x) = \frac{x^3 - 8}{x^2 - 3x + 2} \div \frac{x^3 + 2x^2 + 4x}{2x^2 + x - 3}$ put n (x) in the simplest form showing the domain of n 2017 Exam (7) Question (2) (b)
21	If $n(x) = \frac{x-2}{x+1}$ **Find :** (1) The domain of n^{-1} (2) $n^{-1}(3)$ 2018 Exam (3) Question (5) (b)

Homework

[A] : Choose The Correct Answer : -

1	The domain of the function $f : f(x) = \frac{x-3}{4}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{-4, 3\}$ (d) $\varnothing$
2	If $f(x) = 6x^2 + 3x(1 - 2x)$ is a polynomial function , then its degree is .... (a) first. (b) second. (c) third. (d) fourth.
3	If $z(f) = \{3\}$, $f(x) = 2x + a$, then $a =$ .... (a) zero. (b) 6 (c) -6 (d) 3
4	If $n(x) = \frac{x-1}{x+2}$, then $n^{-1}(1)$ is .... (a) -1 (b) zero (c) 3 (d) undefined.
5	If the algebraic fraction $\frac{x-a}{x+5}$ have a multiplicative inverse which is $\frac{x+5}{x+3}$, then $a =$ .... (a) 3 (b) -5 (c) -3 (d) 5
6	The common domain of the two fractions : $\frac{2}{x-3}$, $\frac{7}{2x-6}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0, 3\}$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{3, -3\}$
7	The set of zero is of f where $f(x) = x(x^2 - 2x + 1)$ is .... (a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1, -1\}$
8	In equation : $a x^2 + b x + c = 0$ if $b^2 - 4ac > 0$, then this equation has .... roots. (a) 1 (b) 2 (c) zero. (d) infinit
9	The set of zeroes of f where $f(x) = x^2 - 2$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{\sqrt{2}, -\sqrt{2}\}$ (d) $\varnothing$
10	The domain of the function f where $f(x) = \frac{7}{x-5}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R} - \{5\}$ (d) $\mathbb{R} - \{0, 5\}$
11	The set of zeroes of f where $f(x) = (x-1)^2 (x+2)$ is .... (a) $\{1, -2\}$ (b) $\{-1, 2\}$ (c) $\{-1, -2\}$ (d) $\{1, 2\}$
12	If $n(x) = \frac{x-1}{x-2}$, then the domain of $n^{-1} =$ .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{1, 2\}$
13	The domain of the multiplicative inverse of the fraction $\frac{x-3}{x+1} =$ .... (a) $\mathbb{R} - \{3, 1\}$ (b) $\mathbb{R} - \{3, -1\}$ (c) $\mathbb{R} - \{-3, -1\}$ (d) $\mathbb{R}$

14 If $n(x) = \frac{x+2}{x-5}$, then the domain of n^{-1} is ....

(a) $\{2 , -5\}$ (b) $\{-2 , 5\}$ (c) $\mathbb{R} - \{-2 , 5\}$ (d) $\mathbb{R} - \{2 , -5\}$

15 The set of zeroes of the function f where $f(x) = \frac{x-1}{x+2}$ is ....

(a) $\{-2\}$ (b) $\{-2 , 1\}$ (c) $\{1\}$ (d) $\{\text{zero}\}$

16 The domain of the function f where $f(x) = \frac{x-2}{x^2+1}$ is ....

(a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1 , -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$

17 The set of zeroes of f where $f(x) = x(x-1)$ is ....

(a) $\{1\}$ (b) $\{0 , -1\}$ (c) $\{0 , 1\}$ (d) $\{0\}$

18 The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction ....

(a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{x}{x-5}$ (d) $\frac{x-5}{x-3}$

19 The set of zeroes of the function $f : f(x) = x^2 + 1$ is ....

(a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1 , 1\}$ (d) $\varnothing$

20 If $n(x) = \frac{x+1}{x-2}$ is an algebraic fraction , then the domain in which the fraction has multiplicative inverse is ....

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-1 , 2\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\{-1 , 2\}$

21 The function f where $f(x) = \frac{x-2}{x-5}$ has a multiplicative inverse if its domain is ....

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{2 , 5\}$

22 If $f(x) = \frac{7+x}{7-x}$, $x \in \mathbb{R} - \{7 , -7\}$, then $f(-2) =$ ....

(a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$

23 The set of zeroes of the function $f : f(x) = 4$ is ....

(a) $\{-4\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\{2\}$

24 The domain of the function f where $f(x) = \frac{x+2}{5x}$ is ....

(a) $\mathbb{R} - \{5\}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{\text{zero}\}$

25 The set of zeroes of f where $f(x) = 9$ is ....

(a) $\{9\}$ (b) $\{\text{zero}\}$ (c) $\varnothing$ (d) $\mathbb{R} - \{9\}$

26 The set of zeroes of the function f where $f(x) = \frac{x+7}{x-2}$ is ....

(a) $\{-7\}$ (b) $\{7\}$ (c) $\{2\}$ (d) $\{7 , 2\}$

27	The set of zeroes of the function $f : f(x) = \frac{x^2 - 9}{x - 3}$ is .... (a) $\{3\}$ (b) $\{-3\}$ (c) $\{3, -3\}$ (d) $\varnothing$
28	If $2x^2 = 5$, then $6x^2 =$ .... (a) 5 (b) 10 (c) 15 (d) 20
29	If $n(x) = \frac{x}{x-1}$, then the domain of $n^{-1} =$ .... (a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{0, 1\}$ (d) $\mathbb{R} - \{-1\}$
30	The domain of the function $n^{-1} : n(x) = \frac{x+4}{x-4}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-4\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{4, -4\}$
31	If the solution set of the equation : $x^2 + mx + 9 = 0$ is $\{-3\}$, then m = ... (a) 5 (b) 6 (c) ± 6 (d) zero
32	The set of zeroes of the function $f : f(x) = \frac{x^2 + x - 2}{x^2 - 4}$ is .... (a) $\{-2, 1\}$ (b) $\mathbb{R} - \{2, -2\}$ (c) $\{-1\}$ (d) $\{1\}$
33	The solution set of the equation : $x^2 - 4x + 4 = 0$ in $\mathbb{R}$ is .... (a) $\{-2\}$ (b) $\{2\}$ (c) $\{4, 1\}$ (d) $\varnothing$
34	The set of zeroes of f where $f(x) = -3x$ is .... (a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3, 0\}$ (d) $\mathbb{R}$
35	The domain of the additive inverse of the fraction n : $n(x) = \frac{x-2}{x-5}$ is .... (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{2, 5\}$ (d) $\{2, 5\}$
36	If the function f where $f(x) = \frac{x^2 - 9}{x}$ has a multiplicative inverse , then their common domain is .... (a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{0, 3\}$ (c) $\mathbb{R} - \{0, -3, 3\}$ (d) $\mathbb{R}$
37	The degree of the polynomial function f where $f(x) = x^3 + 2x - 3$ is .... (a) fourth. (b) third. (c) first. (d) zero.
38	The set of zeroes of f where $f(x) = x^2 + 9$ is .... (a) $\{3, -3\}$ (b) $\varnothing$ (c) $\{3\}$ (d) $\{-3\}$
39	The domain of the function f where $f(x) = \frac{x-7}{3(x+1)}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R} - \{-1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R} - \{-1, 3\}$
40	The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is .... (a) $\{1\}$ (b) $\{1, -1\}$ (c) $\{-1\}$ (d) $\varnothing$

41	The set of zeroes of the function f where $f(X) = \frac{X-3}{X+2}$ is .... (a) $\{zero\}$ (b) $\{3\}$ (c) $\{-2\}$ (d) $\{3, -2\}$
42	If the domain of function n : $n(X) = \frac{X-2}{X^2+a}$ is $\mathbb{R}$, then a .... 0 (a) $=$ (b) $>$ (c) $\leq$ (d) $<$
43	If the function f is a function from set X to set Y , then the domain of the function is .... (a) X (b) Y (c) X × Y (d) Y × X
44	The set of zeroes of the function $f : f(X) = \frac{X^2-X-2}{X^2+4}$ is .... (a) $\{2, -2\}$ (b) $\{-2, -1\}$ (c) $\{2, -1\}$ (d) $\{1, -1\}$
45	If $n(X) = \frac{X-2}{2}$, then the domain of n^{-1} is .... (a) $\mathbb{R}$ (b) $\mathbb{R}-\{2\}$ (c) $\mathbb{R}-\{0\}$ (d) $\mathbb{R}-\{0, 2\}$
46	If $n(X) = \frac{X-1}{X+4}$, then the domain of n^{-1} = ... (a) $\mathbb{R}-\{-4\}$ (b) $\mathbb{R}-\{-1\}$ (c) $\mathbb{R}-\{1, 4\}$ (d) $\mathbb{R}-\{-4, 1\}$
47	The domain of the multiplicative inverse of the fraction : $\frac{X-2}{X^3+27}$ is .... (a) $\mathbb{R}-\{2\}$ (b) $\mathbb{R}-\{-3, 2\}$ (c) $\mathbb{R}-\{2, -3, 3\}$ (d) $\mathbb{R}-\{3, -3\}$
48	The domain of the fraction n : $n(X) = \frac{X+2}{X-1}$ is .... (a) $\mathbb{R}-\{-2\}$ (b) $\mathbb{R}-\{1\}$ (c) $\mathbb{R}-\{1, -2\}$ (d) $\mathbb{R}-\{2\}$
49	If $n(X^2) = 9$, then $n(X)$ = ... (a) 81 (b) 3 (c) ± 3 (d) -3
50	The domain of the function $f : f(X) = \frac{X-2}{7}$ is .... (a) $\mathbb{R}$ (b) $\mathbb{R}-\{2\}$ (c) $\mathbb{R}-\{7\}$ (d) $\mathbb{R}-\{2, 7\}$
51	The set of zeroes of f where $f(X) = X^2 - 25$ is .... (a) $\{5\}$ (b) $\{-5\}$ (c) $\{5, -5\}$ (d) $\varnothing$
52	If $X \neq 0$, then $\frac{5X}{X^2+1} \div \frac{X}{X^2+1}$ = ... (a) -5 (b) -1 (c) 1 (d) 5
53	If the fraction $\frac{X-a}{X+3}$ is the multiplicative inverse of $\frac{X+3}{X+5}$, then a = ... (a) -5 (b) -3 (c) 3 (d) 5

[B] : Essay Problems : -

1 **Put q (x) in its simplest form showing the domain of q where :**

$$q(x) = \frac{x^2 + x + 1}{x^3 - 1} \div \frac{x^2 - x}{x^2 - 2x + 1}$$

2017 Exam (8) Question (4) (a)

2 **Simplify :**

$n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$, showing the domain of n.

2018 Model Exam (2) Question (2) (b)

3 **Find n (x) in the simplest form showing the domain of n :**

$$n(x) = \frac{x^2 - x - 2}{x^2 - 1} \div \frac{x - 5}{x^2 - 6x + 5}$$

2018 Exam (8) Question (4) (b)

4 If $n(x) = \frac{x^2 - 2x}{x^2 - 5x + 6}$

Find : $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}

2018 Exam (8) Question (2) (b)

5 **Find n (x) in the simplest form showing the domain of n , where :**

$$n(x) = \frac{x}{x - 2} \div \frac{x + 3}{x^2 - x - 2}$$

2018 Exam (2) Question (4) (a)

6 **Find n (x) in its simplest form showing the domain of n where :**

$$n(x) = \frac{x^2 - x - 6}{x^2 - 4} \div \frac{3x - 9}{x^2 - x - 2}$$

2017 Exam (13) Question (4) (b)

7 If $n(x) = \frac{x^2 + 3x}{x^2 + x - 6}$

(1) **Find :** $n^{-1}(x)$ and find the domain of n^{-1} (2) If $n^{-1}(x) = 2$, find value of x

2018 Exam (23) Question (3) (b)

8 If $f(x) = \frac{3x + 1}{x - 2} \div \frac{3x^2 + 16x + 5}{x^2 + 5x}$, then find $f(x)$ in the simplest form and identify the domain of f , then find $f(0)$, $f(-1)$ if possible.

2017 Exam (11) Question (4) (b)

9 If $n(x) = \frac{x^2 - 5x}{(x - 5)(x^2 + 1)}$

(1) Find $n^{-1}(x)$ and identify the domain of n^{-1}

(2) If $n^{-1}(x) = 2$, find the value of x

2018 Exam (12) Question (5) (a)

10 **Find n ($\mathcal{X}$) in the simplest form , showing the domain of n where :**

$$n(\mathcal{X}) = \frac{\mathcal{X}^2 - 2\mathcal{X} + 1}{\mathcal{X}^3 - 1} \div \frac{\mathcal{X} - 1}{\mathcal{X}^2 + \mathcal{X} + 1}$$

2018 Exam (6) Question (4) (b)

11 If $n(\mathcal{X}) = \dfrac{\mathcal{X}^2 - 3\mathcal{X}}{(\mathcal{X} - 3)(\mathcal{X}^2 + 2)}$

, then find : $n^{-1}(\mathcal{X})$ in the simplest form , showing the domain of n^{-1}

2018 Exam (1) Question (5) (b)

12 **Find n ($\mathcal{X}$) in its simplest form showing the domain of n where :**

$$n(\mathcal{X}) = \frac{\mathcal{X} - 1}{\mathcal{X}^2 - 1} \div \frac{\mathcal{X}^2 - 5\mathcal{X}}{\mathcal{X}^2 - 4\mathcal{X} - 5}$$

2017 Exam (15) Question (5) (b)

13 **Find n ($\mathcal{X}$) in the simplest form showing the domain :**

$$n(\mathcal{X}) = \frac{\mathcal{X}^2 - 3\mathcal{X} + 2}{\mathcal{X}^2 - 1} \div \frac{3\mathcal{X} - 15}{\mathcal{X}^2 - 4\mathcal{X} - 5}$$

2018 Exam (11) Question (4) (b)

14 If $n(\mathcal{X}) = \dfrac{\mathcal{X}^2 - 2\mathcal{X}}{(\mathcal{X} - 2)(\mathcal{X}^2 + 2)}$

(1) Find $n^{-1}(\mathcal{X})$ in the simplest form and determine the domain of n^{-1}

(2) If $n^{-1}(\mathcal{X}) = 3$ what is the value of $\mathcal{X}$?

2018 Exam (6) Question (5) (a)

15 If $n(\mathcal{X}) = \dfrac{\mathcal{X}^2 + 2\mathcal{X}}{\mathcal{X}^3 - 27} \div \dfrac{\mathcal{X} + 2}{\mathcal{X}^2 + 3\mathcal{X} + 9}$

Find n ($\mathcal{X}$) in its simplest form showing the domain of n

2017 Exam (14) Question (2) (b)

16 **Find n ($\mathcal{X}$) in the simplest form showing the domain where :**

$n(\mathcal{X}) = \dfrac{\mathcal{X}^2 + 4\mathcal{X} + 3}{\mathcal{X}^3 - 27} \div \dfrac{\mathcal{X} + 3}{\mathcal{X}^2 + 3\mathcal{X} + 9}$ then find n (2) , n (– 3) if possible.

2018 Model Exam (1) Question (3) (b)

17 If $n(\mathcal{X}) = \dfrac{\mathcal{X}^2 - 2\mathcal{X}}{(\mathcal{X} - 2)(\mathcal{X} + 2)}$

(1) **Find :** $n^{-1}(\mathcal{X})$ (2) If $n^{-1}(\mathcal{X}) = 3$ what is the value of $\mathcal{X}$?

2018 Exam (10) Question (5) (a)

18 **Simplify :**

$n(\mathcal{X}) = \dfrac{\mathcal{X}^2 + 3\mathcal{X}}{\mathcal{X}^2 - 9} \div \dfrac{2\mathcal{X}}{\mathcal{X} + 3}$, showing the domain of n.

2018 Model Exam (2) Question (4) (b)

Prep [3] - Second Term - Algebra - Unit [3] - Probability

Before studying the operations on events , we shall remember some main concepts which we have studied before in probability.

1 The random experiment :

It is an experiment in which we can specify all its possible outcomes before performing it , but we cannot determine which outcome will occur certainly.

2 The sample space (S) :

It is the set of all possible outcomes of a random experiment.

3 The event :

It is a subset of the sample space.

4 The probability of occurrence of the event :

- It is said that an event occurred if the outcome of the random experiment is an element of this event.
- We can calculate the probability of an event (say A) from the relation :

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

In the experiment of rolling a fair die once and observing the number appears on the upper face , if S is the sample space of the experiment and A is the event of getting an even number , then :

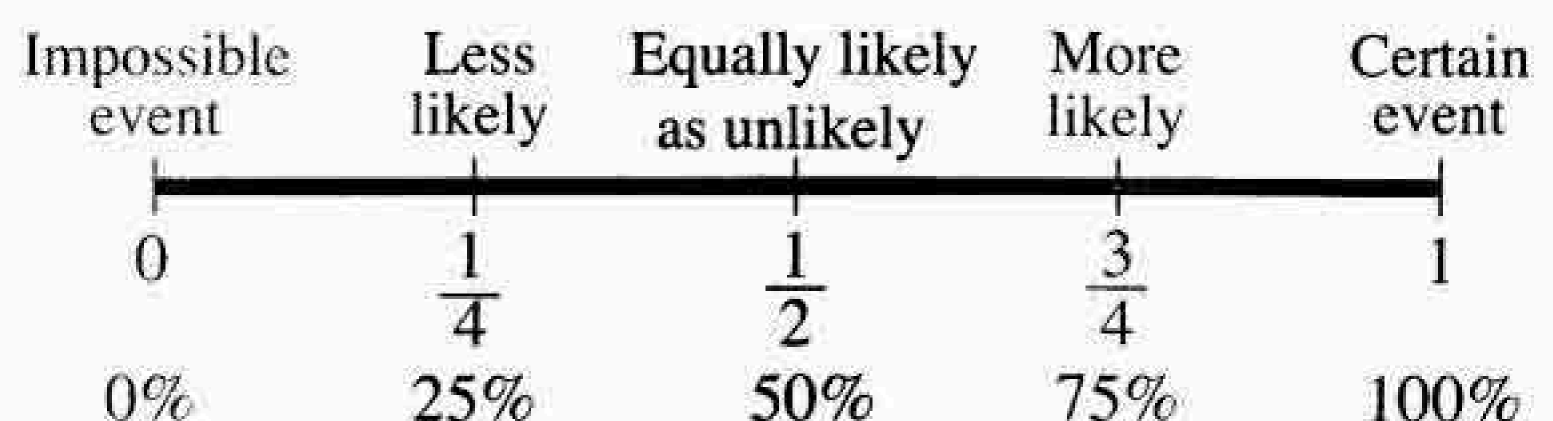
$S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$, $A = \{2, 4, 6\}$, $n(A) = 3$

, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (**i.e.** The probability of occurring the event A $= \frac{1}{2}$)

Remarks

- Zero $\leq$ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage.

The opposite figure shows the possibility of occurring an event due to the value of its probability.



Operations on events

Since the event is a subset of the sample space (S) , then we can carry out on events the same operations which we carry out on sets such as intersection , union , complementary , the difference regarding that the universal set of these events is the sample space. Also we can represent these events by Venn diagrams.

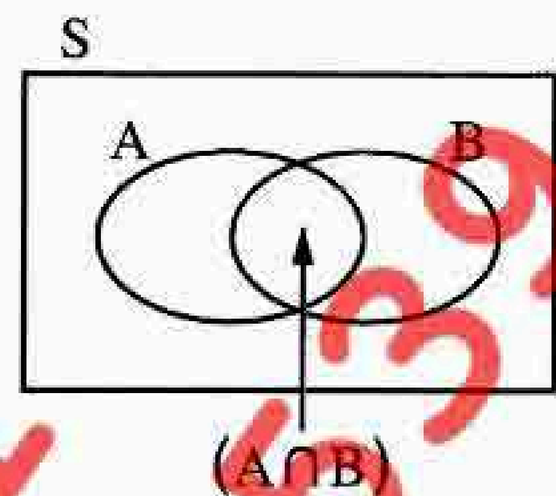
1 Intersection of two events

For any two events A and B of a sample space S :

The event of occurring the two events A and B together = $A \cap B$, then :

The probability of occurring the two events A and B together

$$= P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$



Remarks

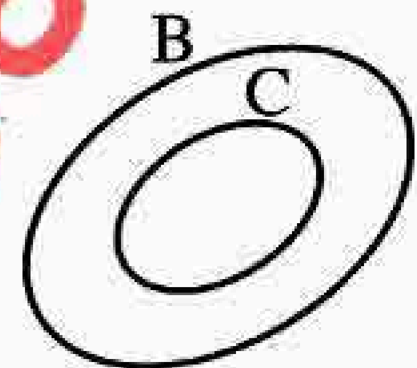
From the previous example we notice that :

1 $C \subset B$ therefore $B \cap C = C$, then we deduce that :

The probability of occurring the two events B and C together

= the probability of occurring the event C

i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$

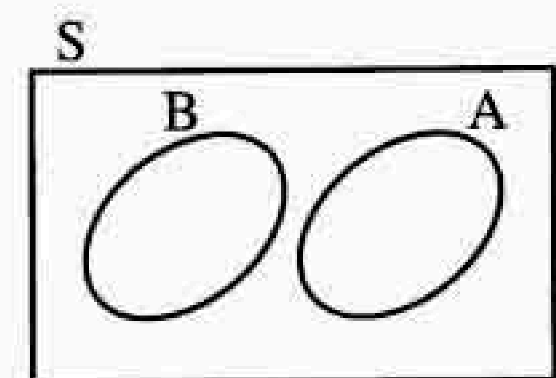


2 $A \cap C = \varnothing$ therefore it is said that the two events A and C are two mutually exclusive events

Mutually exclusive events

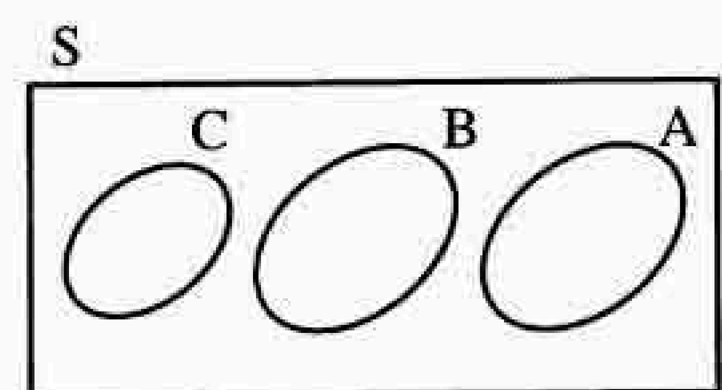
- It is said that the two events A and B are mutually exclusive if

  $A \cap B = \varnothing$, then $P(A \cap B) = 0$



i.e. The probability of their occurring together = the probability of the impossible event = 0

- It is said that some events are mutually exclusive if every pair of them is mutually exclusive.



For example: If $A \cap B = \varnothing$, $B \cap C = \varnothing$, $A \cap C = \varnothing$

, then the events A , B and C are mutually exclusive.

2 Union of two events

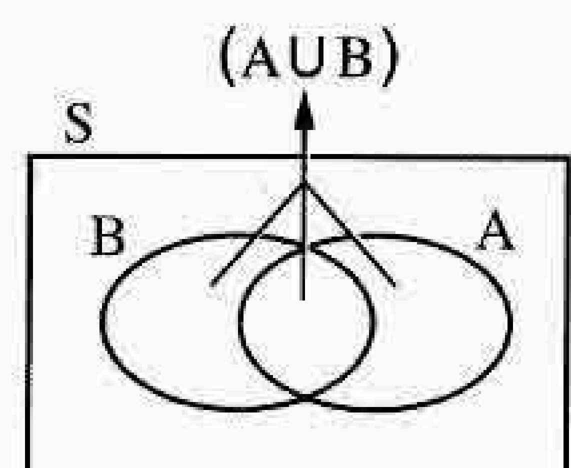
For any two events A and B from a sample space (S) :

The event of occurring the events A or the event B or both of them

(**i.e.** One of them at least occurs) = $A \cup B$, then :

The probability of occurring the events A or the event B or both of them

(**i.e.** The probability of occurring one of them at least) $= P(A \cup B) = \frac{n(A \cup B)}{n(S)}$



Remarks

In the previous example : $C \subset A$ therefore $A \cup C = A$

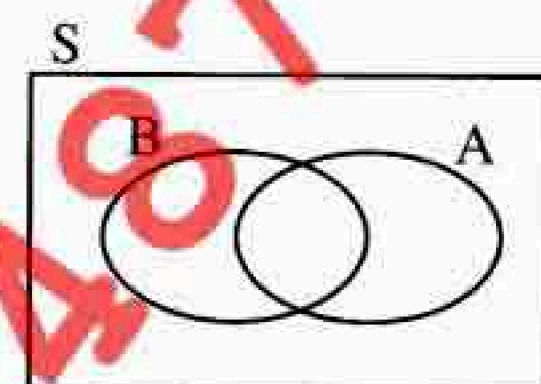
, then we can deduce that :

The probability of occurring the event A or C = $P(A \cup C) = P(A) = \frac{n(A)}{n(S)}$

Rule :

- For any two events from the sample space S of a random experiment :

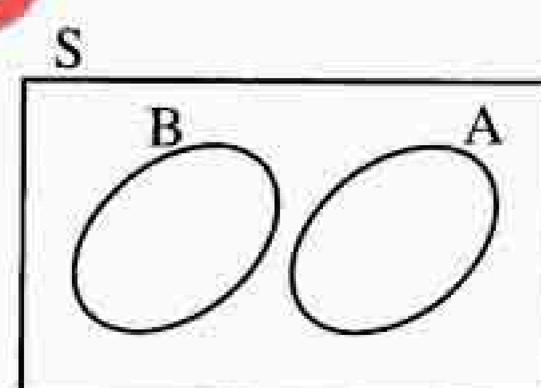
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



- If A and B are two mutually exclusive events , then :

$P(A \cap B)$ = zero , then :

$$P(A \cup B) = P(A) + P(B)$$

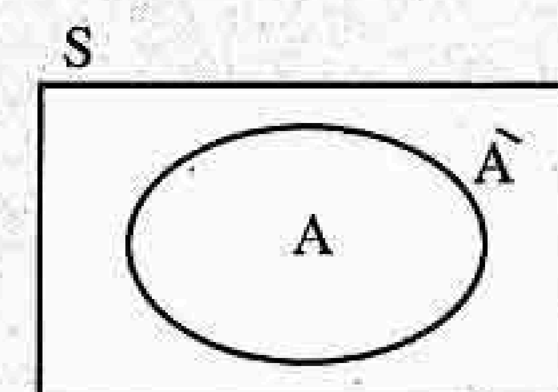


3 The complementary event

If A is an event of the sample space S $(A \subset S)$ then :

the complementary event of A which is denoted by $\grave{A}$ is the event of non occurring A where $A \cup \grave{A} = S$, $A \cap \grave{A} = \varnothing$

, then the probability of non occurrence of the event A = $P(\grave{A}) = \frac{n(\grave{A})}{n(S)}$



Remarks

For any event A of the sample space S it will be :

1 $A \cap \grave{A} = \varnothing$

i.e. The two events A and $\grave{A}$ are two mutually exclusive events

i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A})$ = zero

2 $A \cup \grave{A} = S$

i.e. The union of any event and the complementary event of it = the set of sample space S ,

then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

From that we deduce that :

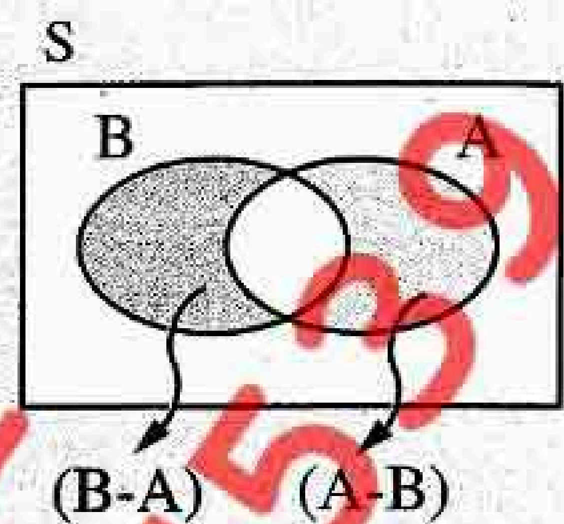
$$P(A) = 1 - P(\grave{A}) \text{ , } P(\grave{A}) = 1 - P(A)$$

Note that :

$$P(S) = \frac{n(S)}{n(S)} = 1$$

4 The difference between events

If A and B are two events of a sample space S then:



- The event of occurrence A and non occurrence B

  (*i.e.* the event of occurrence A only) = A – B

  , then the probability of occurrence the event A and non occurrence the event B = $P(A-B) = \frac{n(A-B)}{n(S)}$

- The event of occurrence B and non occurrence A

  (*i.e.* the event of occurrence B only) = B – A

  , then the probability of occurrence the event B and non occurrence the event A $= P(B-A) = \frac{n(B-A)}{n(S)}$

Remarks

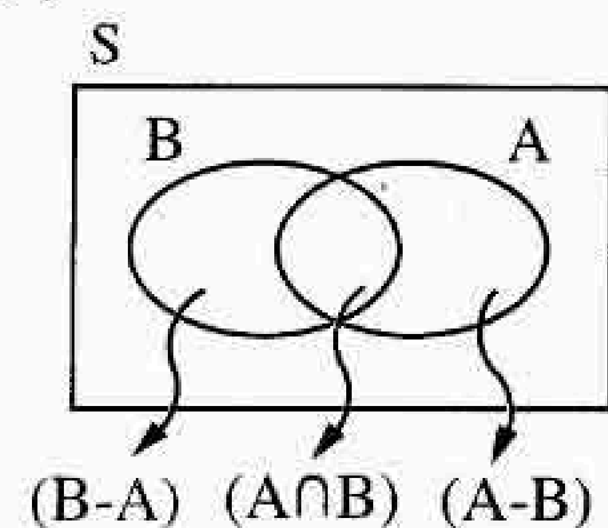
If A and B are two events of a sample space (S) of a random experiment ,

then $(A-B) \cup (A \cap B) = A$

i.e. $P(A-B) + P(A \cap B) = P(A)$

Also : $(B-A) \cup (A \cap B) = B$

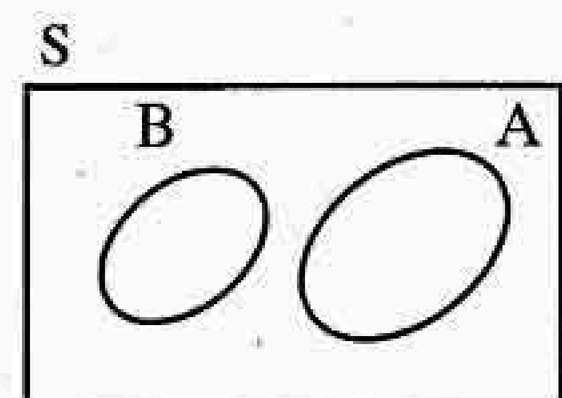
i.e. $P(B-A) + P(A \cap B) = P(B)$



Remarks

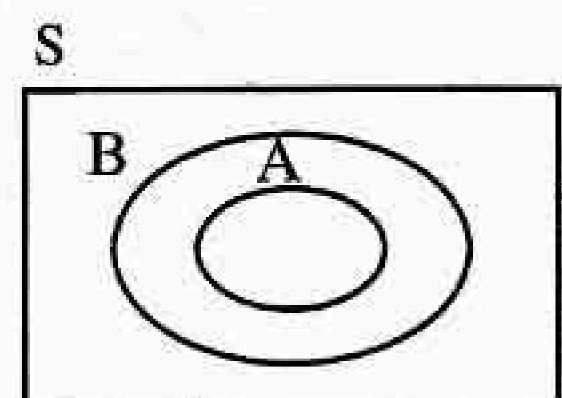
1 If A and B are two mutually exclusive of the sample space (S) , then :

- $A - B = A$ *i.e.* $P(A-B) = P(A)$
- $B - A = B$ *i.e.* $P(B-A) = P(B)$



2 If A and B are two events of the sample space (S) and $A \subset B$, then :

- $A - B = \varnothing$
- $P(A-B) = P(\varnothing) = \frac{n(\varnothing)}{n(S)} =$ zero.



Remember :

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

3) $P(A) = P(A \cup B) + P(A \cap B) - P(B)$

4) $P(B) = P(A \cup B) + P(A \cap B) - P(A)$

5) $P(A - B) = P(A) - P(A \cap B)$

6) $P(B - A) = P(B) - P(A \cap B)$

If A and B are two mutually events then :

1) $P(A \cap B) = 0$

2) $P(A \cup B) = P(A) + P(B)$

3) $P(A) = P(A \cup B) - P(B)$

4) $P(B) = P(A \cup B) - P(A)$

5) $P(A - B) = P(A)$

6) $P(B - A) = P(B)$

Remark [1]

If A is $\subset$ B then :

1) $P(A \cup B) = P(B)$

2) $P(A \cap B) = P(A)$

Remark [2]

If 1) $P(A) = 2P(A')$ then : $P(A) = \frac{2}{3}$, $P(A') = \frac{1}{3}$

If 2) $P(A) = 3P(A')$ then : $P(A) = \frac{3}{4}$, $P(A') = \frac{1}{4}$

If 3) $P(A) = 4P(A')$ then : $P(A) = \frac{4}{5}$, $P(A') = \frac{1}{5}$

Examples :

1 If A and B are two events in the sample space of a random experiment.

Answer the following :

$P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$, then find $P(A \cup B)$ *(Port Said 13)* « $\frac{5}{6}$ »

2 If A and B are two events in the sample space of a random experiment.

Answer the following :

$P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$, then find $P(A \cap B)$ *(Damietta 11)* « $\frac{1}{4}$ »

3 If A and B are two events in the sample space of a random experiment.

Answer the following :

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then find $P(A \cup B)$ in the following cases :

(i) $P(A \cap B) = \frac{1}{8}$

(ii) A and B are mutually exclusive events. *(Aswan 17)* « $\frac{17}{24}$, $\frac{5}{6}$ »

4 **If A and B are two events from a sample space of a random experiment , $P(B) = \frac{1}{12}$ and $P(A \cup B) = \frac{1}{3}$, then find P (A) if :**

(1) A and B are two mutually exclusive events.

(2) $B \subset A$ *(Luxor 17 , North Sinai 14)* « $\frac{1}{4}$, $\frac{1}{3}$ »

5 **If A and B are two events from the sample space of a random experiment , if P (A) = 0.5 , $P(A \cup B) = 0.8$ and $P(B) = 2x$, then calculate the value of x if :**

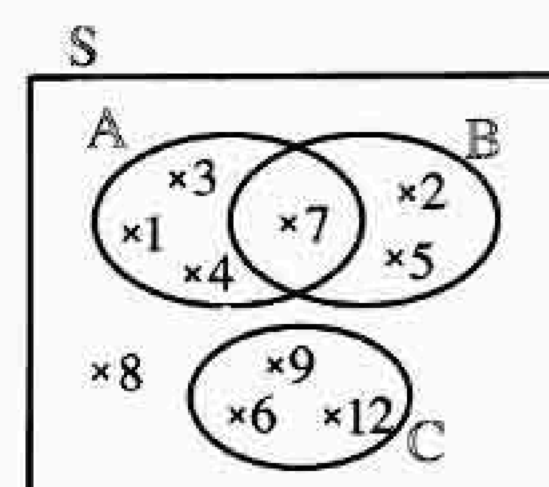
(1) $A \subset B$ (2) $P(A \cap B) = 0.1$ *(Kafr El-Sheikh 16)* « 0.4 , 0.2 »

6 **Use the opposite Venn diagram to find :**

(1) $P(A \cap B)$, $P(A \cup B)$

(2) $P(A \cap C)$, $P(A \cup C)$

(3) $P(B \cap C)$, $P(B \cup C)$ *(Assiut 2011)*



7 S is the sample space of a random experiment where its outcomes are equal , A and B are two events from S

If the number of outcomes that leads to the occurrence of the event A equals 13 and the number of all possible outcomes of the random experiment is 24 , $P(A \cup B) = \frac{5}{6}$ and $P(B) = \frac{5}{12}$ **Find :**

(1) The probability of occurrence of the event A

(2) The probability of occurrence the event A and B together. *(El-Menia 17 , El-Gharbia 16)* « $\frac{13}{24}$, $\frac{1}{8}$ »

8 A box contains 12 balls , 5 of them are blue , 4 are red and the left are white. A ball is randomly drawn from the box. **Find the probability that the drawn ball is :**

(1) blue. (2) not red. (3) blue or red. *(Alexandria 13)* « $\frac{5}{12}$, $\frac{2}{3}$, $\frac{3}{4}$ »

9 A card is randomly drawn from 20 identical cards numbered from 1 to 20

Calculate the probability that the number on the card is :

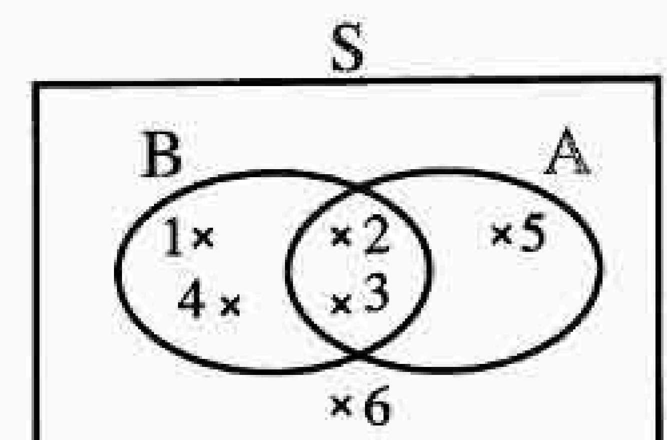
(1) Divisible by 3 (2) Divisible by 5

(3) Divisible by 3 and divisible by 5

(4) Divisible by 3 or divisible by 5 *(Aswan 2011)* « $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{9}{20}$ »

10 **In the opposite figure :**

If A and B are two events of a sample space S of a random experiment then , **Find :**



(1) $P(A \cap B)$

(2) $P(A - B)$

(3) The probability of non-occurrence of the event A *(Cairo 17)* « $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ »

11 If X and Y are two events in a sample space of a random experiment where :

$P(Y) = \frac{2}{5}$, $P(X) = P(X`)$, $P(X \cap Y) = \frac{1}{5}$ **Find :**

(1) $P(X)$ (2) $P(X \cup Y)$ *(El-Kalyoubia 16 , El-Dakahlia 14)* « $\frac{1}{2}$, $\frac{7}{10}$ »

Solutions

1

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$

2

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{5}{8} = \frac{3}{8} + \frac{1}{2} - P(A \cap B)$

$\therefore P(A \cap B) = \frac{3}{8} + \frac{1}{2} - \frac{5}{8} = \frac{1}{4}$

3

[i] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{8} = \frac{17}{24}$

[ii] $\because$ A and B are two mutually exclusive events

$\therefore P(A \cap B) = \text{zero}$

$\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

4

(1) $\because$ A and B are two mutually exclusive events

$\therefore P(A \cap B) = \text{zero}$ $\therefore P(A \cup B) = P(A) + P(B)$

$\therefore \frac{1}{3} = P(A) + \frac{1}{12}$

$\therefore P(A) = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

(2) $B \subset A$ $\therefore P(A) = P(A \cup B) = \frac{1}{3}$

5

(1) $\because A \subset B$ $\therefore P(B) = P(A \cup B)$

$\therefore 2x = 0.8$ $\therefore x = 0.4$

(2) $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.8 = 0.5 + 2x - 0.1$

$\therefore 2x = 0.8 - 0.5 + 0.1 = 0.4$

$\therefore x = 0.2$

6

(1) $P(A \cap B) = \frac{1}{10}$, $P(A \cup B) = \frac{6}{10} = \frac{3}{5}$

(2) $P(A \cap C) = \text{zero}$, $P(A \cup C) = \frac{7}{10}$

(3) $P(B \cap C) = \text{zero}$, $P(B \cup C) = \frac{6}{10} = \frac{3}{5}$

7

(1) $P(A) = \frac{13}{24}$

(2) The probability of occurrence of the two events A and B together $= P(A \cap B)$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore \frac{5}{6} = \frac{13}{24} + \frac{5}{12} - P(A \cap B)$

$\therefore P(A \cap B) = \frac{13}{24} + \frac{5}{12} - \frac{5}{6} = \frac{1}{8}$

8

(1) The probability that the drawn ball is blue $= \frac{5}{12}$

(2) The probability that the drawn ball is not red

= the probability that the drawn ball is blue or white

$= \frac{5}{12} + \frac{3}{12} = \frac{2}{3}$

(3) The probability that the drawn ball is blue or red $= \frac{5}{12} + \frac{4}{12} = \frac{3}{4}$

9

(1) $P(A) = \frac{6}{20} = \frac{3}{10}$

(2) $P(B) = \frac{4}{20} = \frac{1}{5}$

(3) $P(A \cap B) = \frac{1}{20}$

(4) $P(A \cup B) = \frac{9}{20}$

S: B: 5, 10, 20 ; A ∩ B: 15 ; A: 9, 6, 12, 3, 18 ; outside: 19, 17, 16, 14, 13, 11, 8, 7, 4, 2, 1

10

(1) $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

(2) $P(A - B) = \frac{1}{6}$

(3) The probability of non occurrence of the event A

$= P(\bar{A}) = \frac{3}{6} = \frac{1}{2}$

11

(1) $\because P(X) = P(\grave{X})$, $P(X) + P(\grave{X}) = 1$

$\therefore P(X) = \frac{1}{2}$

(2) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{1}{2} + \frac{2}{5} - \frac{1}{5} = \frac{7}{10}$

Exercises

[A] : Choose The Correct Answer : -

1	The probability of the impossible event equals .... (a) zero. (b) $\varnothing$ (c) 1 (d) S
2	If A and B are two mutually exclusive events, then $P(A \cap B)$ equals .... (a) $\varnothing$ (b) zero (c) $\frac{1}{2}$ (d) 1
3	If A and B are two mutually exclusive events of a random experiment, then $P(A \cup B) =$ .... (a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A) + P(B)$
4	If A and B are two mutually exclusive events, then $P(A - B) =$ .... (a) zero (b) $P(A)$ (c) $P(B)$ (d) $P(A \cup B)$
5	If $A \subset B$, then $P(A \cap B) =$ .... (a) $\varnothing$ (b) zero. (c) $P(A)$ (d) $P(B)$
6	If A and B are two events in a random experiment and $B \subset A$, then $P(A \cap B) =$ .... (a) $\varnothing$ (b) zero. (c) $P(A)$ (d) $P(B)$
7	If $A \subset B$, then $P(A \cup B) =$ .... (a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A - B)$
8	If $P(A) = P(\grave{A})$, then $P(A) =$ .... (a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{4}$ (d) 0
9	If $\grave{A}$ is a complement event of A, then $A \cup \grave{A} =$ .... (a) $\varnothing$ (b) $\grave{A}$ (c) sample space. (d) A
10	If $\grave{A}$ is the complement event of the event A in a sample space of a random experiment, then $P(A) + P(\grave{A}) =$ .... (a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 3
11	If A is an event of random experiment, then $P(\grave{A}) =$ .... (a) 1 (b) -1 (c) $1 - P(A)$ (d) $P(A) - 1$
12	If $\frac{P(A)}{P(\grave{A})} = 3$, then $P(A) =$ .... (a) $\frac{3}{4}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

13 Twice the number x subtracted by 3 is ....

(a) $x - 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $3 - 2x$

14 If $x + y = 5$, then $3x + 3y =$ ....

(a) 5 (b) 3 (c) 8 (d) 15

15 The solution set of the two equations : $x = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(-1 , 1)\}$ (b) $\{(1 , -1)\}$ (c) $\{(-1 , -1)\}$ (d) $\{(1 , 1)\}$

16 If $P(A) = 0.6$, then $P(\grave{A}) =$ ....

(a) 0.4 (b) 0.6 (c) 0.5 (d) 1

17 If $P(A) = \frac{3}{4}$, then $P(\grave{A}) =$ ....

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

18 If $A \subset S$, $P(A) = \frac{1}{3}$, then $P(\grave{A}) =$ ....

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

19 If $A \subset S$ of random experiment and $P(\grave{A}) = 2P(A)$, then $P(A) =$ ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

20 If $P(A) = 4P(\grave{A})$, then $P(A) =$ ....

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

21 If the probability that a student succeeded is 95 % , then the probability that he does not succeed is ....

(a) 20 % (b) 5 % (c) 10 % (d) zero

22 If the probability of failure of a student is 0.4 , then the probability of his success is ....

(a) zero (b) 1 (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

23 When a regular coin is tossed once , then the probability of getting a head is ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

24 If A , B are two events from the sample of a random experiment , $P(A) = 0.7$ and $P(A - B) = 0.5$, then $P(A \cap B) =$ ....

(a) 0.6 (b) 0.4 (c) 0.3 (d) 0.2

25 If A and B are two mutually exclusive events , $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{15}$ then $P(B) =$ ....

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{4}{15}$ (d) $\frac{11}{15}$

26	If A , B are two mutually exclusive events , P (B) = 0.5 and P (A ∪ B) = 0.7 , then P (A) = ... (a) 0.02 (b) 0.2 (c) 0.5 (d) 0.13
27	If A and B are two mutually exclusive events of a random experiment , if P (A) = $\frac{1}{3}$, P (A ∪ B) = $\frac{7}{12}$, then P (B) = ... (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
28	If a die is tossed once , then the probability of appearance of on odd number is .... (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3
29	In the experiment of rolling a regular die once , the probability of appearance of an even number on the upper face = ... (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$
30	If a die is rolled once , then the probability of getting an odd number and even number together = ... (a) $\frac{1}{2}$ (b) zero. (c) $\frac{3}{4}$ (d) 1
31	If a regular die is tossed once , the probability of appearance of a number less than 3 equals .... (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$
32	If a die is tossed once , then the probability of appearance of a number greater than 4 is .... (a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
33	A card is drawn randomly from 20 identical cards numbered from 1 to 20 , then the probability that the number of the drawn card multiple of 7 is .... (a) 10 % (b) 15 % (c) 20 % (d) 25 %
34	The point (– 3 , 4) lies in .... quadrant. (a) fourth (b) third (c) second (d) first
35	If 2 X = 1 , then $\frac{1}{5}$ X = ... (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$
36	If (5 , A – 4) = (B + 2 , 3) , then A + B = ... (a) 2 (b) 3 (c) 10 (d) 5
37	If X + 3 y = 7 , then X + 3 (y + 5) = ... (a) 22 (b) 21 (c) 7 (d) 3

38	The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point .... (a) $(-1 , 2)$ (b) $(2 , -1)$ (c) $(1 , -2)$ (d) $(-1 , -2)$
39	If the point $(5 , b - 7)$ lies on the x–axis , then b = ... (a) 2 (b) 3 (c) 5 (d) 7
40	The solution set of the equation : $2x + 4 = 0$ in $\mathbb{N}$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $\varnothing$
41	If $(5 , x - 4) = (y , 3)$, then $x + y$ = ... (a) 25 (b) 12 (c) 8 (d) 6
42	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are .... (a) parallel. (b) intersecting. (c) distant. (d) coincident.
43	The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is .... (a) $(-2 , -3)$ (b) $(-2 , 3)$ (c) $(2 , -3)$ (d) $(2 , 3)$
44	The degree of the equation : $3x + 4y + xy = 5$ is .... (a) zero. (b) first. (c) second. (d) third.
45	If $3x = 1$, then $\frac{1}{5}x$ = ... (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
46	If $(7^{a-2} , 3) = (1 , b + 5)$, then a + b = ... (a) -1 (b) zero (c) 1 (d) 2
47	The two straight lines : $3x = 7$, $2y = 9$ are .... (a) perpendicular. (b) coincide. (c) intersect and non perpendicular. (d) parallel.
48	The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(3 , 4)\}$ (b) $\{(4 , 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$
49	If the age of a man now is x year , then his age after 5 years from now is .... years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
50	If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then x = ... (a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$

[C] : Essay Problems : -

1 If $P(A) = 0.2$, $P(B) = 0.6$, $P(A \cap B) = 0.3$, then find $P(A \cup B)$

2017 Exam (2) Question (4) (b)

2 If A and B are two events in a random experiment , $P(A) = 0.5$, $P(B) = 0.3$, $P(A \cup B) = 0.4$

Find (1) $P(A \cap B)$ (2) $P(A - B)$

2017 Exam (16) Question (5) (b)

3 If A and B are two events from the sample of a random experiment and

$P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.5$

Find : (1) $P(A \cup B)$ (2) $P(B')$

2018 Exam (9) Question (5) (b)

4 If A and B are two events in a sample space for a random experiment , and if

$P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cap B) = 0.6$

Find : (1) The probability of non occurrence of the event A

(2) The probability of occurrence one of the two events at least.

2018 Exam (4) Question (2) (b)

5 If A and B are two events of the sample space of a random experiment and

$P(A') = \frac{2}{3}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{5}$

Find : (1) $P(A)$ (2) $P(A \cup B)$ (3) $P(B - A)$

2017 Exam (17) Question (4) (a)

6 A box contains 30 identical cards numbered from 1 to 30 and a card was drawn randomly.

Calculate the probability that the number on the drawn card is :

(1) Divisible by 4 (2) A prime number.

2017 Exam (4) Question (5) (a)

7 **Use the opposite Venn diagram and find :**

(1) $P(A \cap B)$

(2) $P(A \cup B)$

(3) $P(A - B)$

S
A B
×3 ×1 ×5 ×2 ×4
×6

2018 Exam (22) Question (4) (b)

8 If A and B are two events of a random experiment and

$P(A) = 0.3$, $P(B) = 0.6$, $P(A \cap B) = 0.2$

Find : (1) $P(A \cup B)$ (2) $P(A - B)$

2018 Model Exam (2) Question (3) (b)

9	If A and B are two events from the sample space of a random experiment , if $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(B) = 2x$, then calculate the value of x if : (1) $A \subset B$ (2) $P(A \cap B) = 0.1$ 2017 Exam (11) Question (3) (b)
10	**If A , B are two events in a random experiment where :** $P(A) = 0.7$, $P(B) = 0.6$, $P(A \cap B) = 0.3$ **Calculate the value of :** (1) $P(A')$ (2) $P(A - B)$ (3) $P(A \cup B)$ 2017 Exam (5) Question (2) (b)
11	If A and B are two events in a sample space of a random experiment , and $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{10}$ **, then find :** (1) $P(A \cup B)$ (2) $P(B - A)$ 2017 Exam (8) Question (5) (a)
12	If X , Y are two events of a sample space of a random experiment , $P(Y) = \frac{2}{5}$, $P(X) = P(X')$, $P(X \cap Y) = \frac{1}{5}$ **, then find :** (1) $P(X)$ (2) $P(X \cup Y)$ 2017 Exam (4) Question (3) (b)
13	A bag contains 10 identical cards numbered from 1 to 10 , one card of them is drawn randomly , calculate the probability that the number on the drawn card is : (1) A prime number. (2) A number divisible by 5 2018 Exam (14) Question (5) (b)
14	A classroom consists of 40 students , 30 of them succeeded in math. 24 in science and 20 in both math. and science. If a student is chosen randomly. **Find the probability that this student is :** (1) fail in math. (2) succeeded in math. or science 2018 Exam (8) Question (4) (a)
15	If A and B are two events in a random experiment , and if $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.3$ **Find :** (1) The probability of non occurence of the events A (2) The probability of occurence of at least one of them. 2017 Exam (15) Question (5) (a)

Homework

[A] : Choose The Correct Answer : -

1	If A and B are two events in a random experiment and $B \subset A$, then $P(A \cap B) =$ …………… (a) $\varnothing$ (b) zero. (c) $P(A)$ (d) $P(B)$
2	If A and B are two mutually exclusive events of a random experiment , if $P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{7}{12}$, then $P(B) =$ …………… (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
3	If $\frac{P(A)}{P(A')} = 3$, then $P(A) =$ …………… (a) $\frac{3}{4}$ (b) 1 (c) $\frac{1}{3}$ (d) $\frac{1}{4}$
4	If the age of a man now is x year , then his age after 5 years from now is …………… years. (a) $x - 5$ (b) $5 - x$ (c) $5x$ (d) $x + 5$
5	If a die is tossed once , then the probability of appearance of a number greater than 4 is …………… (a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$
6	If $2x = 1$, then $\frac{1}{5}x =$ …………… (a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{10}$
7	If $(7^{a-2}, 3) = (1, b + 5)$, then $a + b =$ …………… (a) -1 (b) zero (c) 1 (d) 2
8	The two equations : $x = -1$, $y - 2 = 0$ represent two straight lines intersect at the point …………… (a) $(-1, 2)$ (b) $(2, -1)$ (c) $(1, -2)$ (d) $(-1, -2)$
9	If $A \subset B$, then $P(A \cap B) =$ …………… (a) $\varnothing$ (b) zero. (c) $P(A)$ (d) $P(B)$
10	If A and B are two mutually exclusive events , $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{7}{15}$ then $P(B) =$ …………… (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{4}{15}$ (d) $\frac{11}{15}$
11	If A is an event of random experiment , then $P(A') =$ …………… (a) 1 (b) -1 (c) $1 - P(A)$ (d) $P(A) - 1$

12 If $P(A) = 4\,P(\grave{A})$, then $P(A) =$ ....

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

13 If a regular die is tossed once , the probability of appearance of a number less than 3 equals ....

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

14 If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then $x =$ ....

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$

15 If $(5 , x - 4) = (y , 3)$, then $x + y =$ ....

(a) 25 (b) 12 (c) 8 (d) 6

16 The solution set of the two equations : $x = -1$, $y - 1 = 0$ in $\mathbb{R} \times \mathbb{R}$ is ....

(a) $\{(-1 , 1)\}$ (b) $\{(1 , -1)\}$ (c) $\{(-1 , -1)\}$ (d) $\{(1 , 1)\}$

17 If A and B are two mutually exclusive events , then $P(A - B) =$ ....

(a) zero (b) $P(A)$ (c) $P(B)$ (d) $P(A \cup B)$

18 If $\grave{A}$ is the complement event of the event A in a sample space of a random experiment , then $P(A) + P(\grave{A}) =$ ....

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 3

19 If $A \subset S$ of random experiment and $P(\grave{A}) = 2\,P(A)$, then $P(A) =$ ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 1

20 If A , B are two events from the sample of a random experiment , $P(A) = 0.7$ and $P(A - B) = 0.5$, then $P(A \cap B) =$ ....

(a) 0.6 (b) 0.4 (c) 0.3 (d) 0.2

21 If a die is rolled once , then the probability of getting an odd number and even number together = ...

(a) $\frac{1}{2}$ (b) zero. (c) $\frac{3}{4}$ (d) 1

22 The degree of the equation : $3x + 4y + xy = 5$ is ....

(a) zero. (b) first. (c) second. (d) third.

23 The probability of the impossible event equals ....

(a) zero. (b) $\varnothing$ (c) 1 (d) S

24 If $(5 , A - 4) = (B + 2 , 3)$, then $A + B =$ ....

(a) 2 (b) 3 (c) 10 (d) 5

25	The two straight lines : $3x = 7$, $2y = 9$ are .... (a) perpendicular. (b) coincide. (c) intersect and non perpendicular. (d) parallel.
26	If A and B are two mutually exclusive events of a random experiment , then $P(A \cup B) =$ .... (a) $P(A)$ (b) $P(B)$ (c) $P(A \cap B)$ (d) $P(A) + P(B)$
27	If $\grave{A}$ is a complement event of A , then $A \cup \grave{A} =$ .... (a) $\varnothing$ (b) $\grave{A}$ (c) sample space. (d) A
28	If $A \subset S$, $P(A) = \frac{1}{3}$, then $P(\grave{A}) =$ .... (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$
29	When a regular coin is tossed once , then the probability of getting a head is .... (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$
30	In the experiment of rolling a regular die once , the probability of appearance of an even number on the upper face = ... (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{5}{6}$
31	If the point $(5 , b - 7)$ lies on the x-axis , then b = ... (a) 2 (b) 3 (c) 5 (d) 7
32	Twice the number x subtracted by 3 is .... (a) $x - 3$ (b) $2x + 3$ (c) $2x - 3$ (d) $3 - 2x$
33	The two equations of first degree in two variables which have an infinite number of solutions are represented by two straight lines those are .... (a) parallel. (b) intersecting. (c) distant. (d) coincident.
34	If A and B are two mutually exclusive events, then $P(A \cap B)$ equals .... (a) $\varnothing$ (b) zero (c) $\frac{1}{2}$ (d) 1
35	If $P(A) = P(\grave{A})$, then $P(A) =$ .... (a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{4}$ (d) 0
36	If $P(A) = \frac{3}{4}$, then $P(\grave{A}) =$ .... (a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
37	The point $(-3 , 4)$ lies in .... quadrant. (a) fourth (b) third (c) second (d) first

38	If the probability of failure of a student is 0.4 , then the probability of his success is .... (a) zero (b) 1 (c) $\frac{2}{5}$ (d) $\frac{3}{5}$
39	If a die is tossed once , then the probability of appearance of on odd number is .... (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3
40	If $3x = 1$, then $\frac{1}{5}x =$ .... (a) $\frac{3}{5}$ (b) $\frac{1}{15}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$
41	If $x + 3y = 7$, then $x + 3(y + 5) =$ .... (a) 22 (b) 21 (c) 7 (d) 3
42	The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .... (a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$
43	If $A \subset B$, then $P(A \cup B) =$ .... (a) P (A) (b) P (B) (c) $P(A \cap B)$ (d) $P(A - B)$
44	If P (A) = 0.6 , then $P(\grave{A}) =$ .... (a) 0.4 (b) 0.6 (c) 0.5 (d) 1
45	If the probability that a student succeeded is 95 % , then the probability that he does not succeed is .... (a) 20 % (b) 5 % (c) 10 % (d) zero
46	If A , B are two mutually exclusive events , P (B) = 0.5 and $P(A \cup B) = 0.7$, then P (A) = ... (a) 0.02 (b) 0.2 (c) 0.5 (d) 0.13
47	A card is drawn randomly from 20 identical cards numbered from 1 to 20 , then the probability that the number of the drawn card multiple of 7 is .... (a) 10 % (b) 15 % (c) 20 % (d) 25 %
48	The solution set of the equation : $2x + 4 = 0$ in $\mathbb{N}$ is .... (a) $\{2\}$ (b) $\{-2\}$ (c) $\{0\}$ (d) $\varnothing$
49	If $x + y = 5$, then $3x + 3y =$ .... (a) 5 (b) 3 (c) 8 (d) 15
50	The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is .... (a) (– 2 , – 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (2 , 3)

[B] : Essay Problems : -

1

If A and B are two events in a random experiment :

$P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cup B) = 0.9$

Find : (1) $P(A \cap B)$ (2) $P(A - B)$

2017 Exam (19) Question (5) (a)

2

If A and B are two events in a sample space of a random experiment

, $P(A) = 0.7$, $P(B) = 0.6$, $(A \cap B) = 0.4$

Find : (1) The probability of non occurence of event A

(2) The probability of occuring the event A and non occuring of event B

(3) The probability of occuring one of the two events at least.

2017 Exam (6) Question (3) (a)

3

If A and B are two events from the sample space of a random experiment if

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{5}{8}$

Find : $P(A \cap B)$ and $P(B - A)$

2017 Exam (13) Question (5) (b)

4

If A and B are two events of the sample space of a random experiment

$P(A) = \frac{5}{9}$, $P(B) = \frac{2}{9}$, $P(A \cap B) = \frac{1}{9}$

Find : (1) $P(A \cup B)$

(2) The probability of non occurrence any of the two events.

(3) The probability of occurrence of event A only.

2018 Exam (5) Question (5) (b)

5

A bag contains 15 balls numbered from 1 to 15 , if a ball is drawn randomly

, if the event A is getting an odd number and the event B is getting a prime number

Find : (1) $P(A)$ (2) $P(B)$ (3) $P(A - B)$

2018 Exam (23) Question (5) (a)

6

In the opposite figure :

If A and B are two events in a sample space S

of a random experiment , **then find :**

(1) $P(A \cap B)$ (2) $P(A - B)$

(3) The probability of non-occurrence of the event A

S
B A
×1 ×4 ×2 ×3 ×5
×6

2018 Model Exam (1) Question (5) (b)

7

If A and B are two events from a sample space of a random experiment and

$P(A) = 0.4$, $P(B) = 0.5$, $P(A \cup B) = 0.7$

Find : (1) $P(A \cap B)$ (2) $P(B - A)$

	2018 Exam (11) Question (5) (a)
8	If A and B are two events in a sample space for a random experiment , $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$ **, then find :** (1) $P(A \cup B)$ (2) $P(A - B)$ (3) $P(\acute{B})$ 2017 Exam (1) Question (5) (a)
9	If A and B are two events of the sample space (S) of a random experiment such that : $P(A) = 0.7$, $P(A \cap B) = 0.3$ **Find :** $P(A - B)$ 2018 Exam (2) Question (2) (a)
10	If A and B are two events from a sample space of a random experiment and $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ **Find $P(A \cup B)$ if :** (1) $P(A \cap B) = \frac{1}{8}$ (2) A and B are mutually exclusive events. 2018 Exam (21) Question (4) (a)
11	If A , B are two events from a sample space of random experiment , and $P(B) = \frac{1}{12}$, $P(A \cup B) = \frac{1}{3}$ **, then find P (A) if :** (1) A and B are two mutually exclusive events. (2) $B \subset A$ 2018 Exam (12) Question (4) (a)
12	**A bag contains 21 symmetrical balls , 8 white , 6 red and the rest is black , one ball was drawn randomly , find the probability that it was :** (1) White. (2) Not black. (3) Red or black. 2017 Exam (18) Question (5) (a)
13	If S is the sample space of a random experiment where its outcomes are equal , A and B are two events from S , if the number of outcomes that leads to the occurrence of the event A = 13 and the number of all possible outcomes of the random experiment is 24 , $P(A \cup B) = \frac{5}{6}$ and $P(B) = \frac{5}{12}$ **Find :** (1) The probability of occurrence of the event A (2) The probability of occurrence of the events A and B together. 2018 Exam (16) Question (5) (b)
14	**Use the opposite Venn diagram to calculate the probability of :** (1) Non occurrence of the event A (2) The occurrence of the event B only. (3) Occurrence of A or B S, B, A, ×4, ×1, ×3, ×6, ×2, ×5 2018 Exam (10) Question (4) (b)

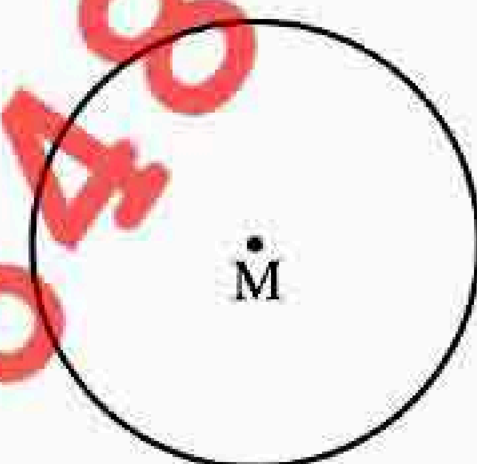
Prep [3] - Second Term - Geometry - Unit [4] - The Circle

Lesson [1] : Basic Definitions And Concepts

The circle

It is the set of points of the plane which are at a constant distance from a fixed point in the same plane.

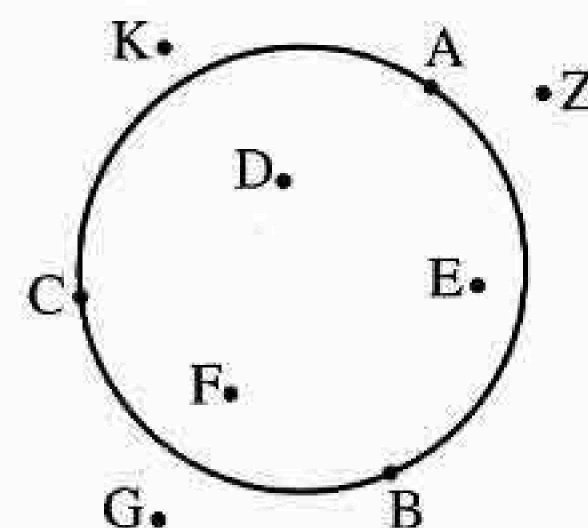
- The fixed point is called **"the centre of the circle"**.
- The constant distance is called **"the radius length of the circle"**.
- The circle is usually denoted by its centre, so we say the circle M to mean the circle whose centre is the point M



Partition of the plane by the circle

- **The drawn circle divides the plane into three sets of points as shown in the opposite figure :**

1. **The set of points on the circle as :** the points A , B , C , ...
2. **The set of points inside the circle as :** the points D , E , F , ...
3. **The set of points outside the circle as :** the points Z , K , G , ...



The radius of the circle

It is a line segment with one endpoint at the centre of the circle and the other endpoint on the circle.

Notice that :

1. Any circle has an infinite number of radii and all of them are equal in length.
2. If two radii of two circles are equal in length , then the two circles are congruent and vice versa.

The chord of the circle

It is a line segment whose endpoints are any two points on the circle.

The diameter of the circle

It is a chord passing through the centre of the circle.

Notice that :

1. Any circle has an infinite number of diameters and all of them are equal in length.
2. The diameter of the circle is the longest chord of the circle , and its length = 2 r

The circumference of the circle and its area

- The circumference of the circle is the length of the closed curve that represents the circle

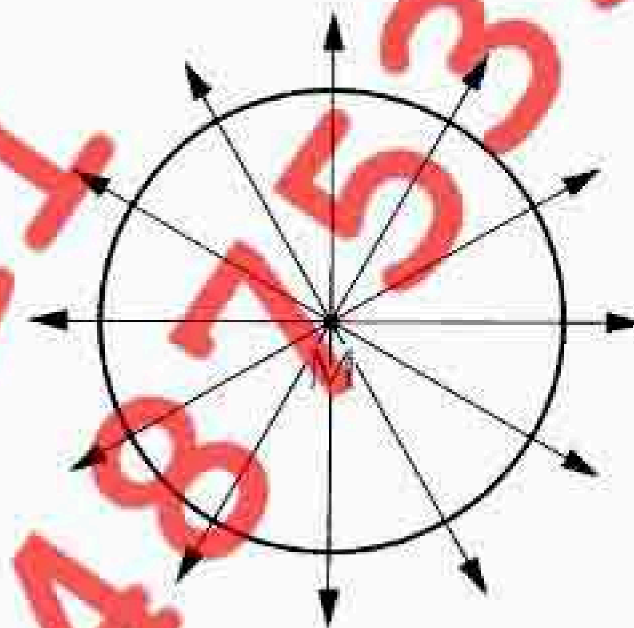
and it equals 2 π r length unit.

- The area of the circle = πr^2 square unit.

(where r is the radius length and π is the approximating ratio).

Symmetry in the circle

- Any straight line passing through the centre of the circle is an axis of symmetry of it.
- Since the number of these straight lines are infinite , then the circle has an infinite number of axes of symmetry.



Important Corollaries

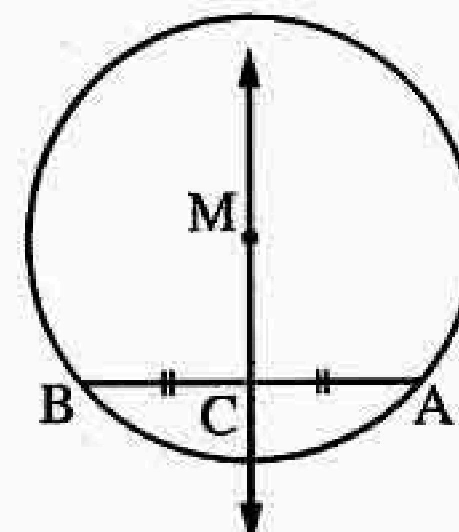
Corollary 1

The straight line passing through the centre of the circle and the midpoint of any chord of it (not passing through the centre) is perpendicular to this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M

and C is the midpoint of $\overline{AB}$, then $\overleftrightarrow{MC} \perp \overline{AB}$

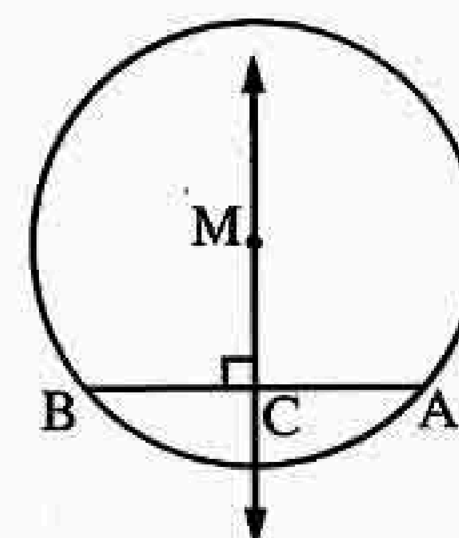


Corollary 2

The straight line passing through the centre of the circle and perpendicular to any chord of it bisects this chord.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M and $\overleftrightarrow{MC} \perp \overline{AB}$, where C $\in \overline{AB}$, then C is the midpoint of $\overline{AB}$



Corollary 3

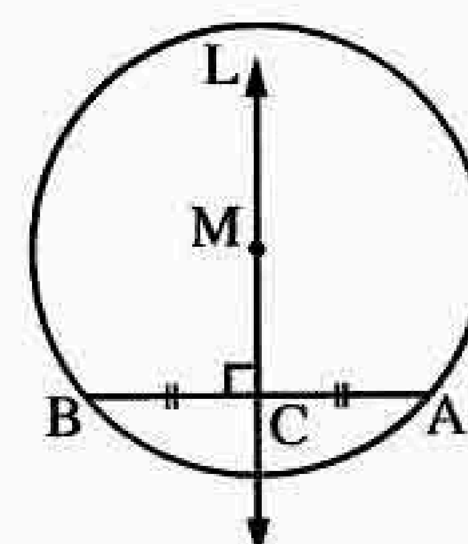
The perpendicular bisector to any chord of a circle passes through the centre of the circle.

In the opposite figure :

If $\overline{AB}$ is a chord of the circle M , C is the midpoint of $\overline{AB}$

and the straight line L $\perp \overline{AB}$ from the point C ,

then M $\in$ the straight line L



From the previous, we deduce that :

The axis of symmetry of any chord of a circle passes through its centre , so this axis is also an axis of symmetry of the circle.

Lesson [2] : Positions Of A Point and A Straight Line With Respect To A Circle

First : Position of a point with respect to a given circle :

If M is a circle of radius length r and A is a point in its plane , then :

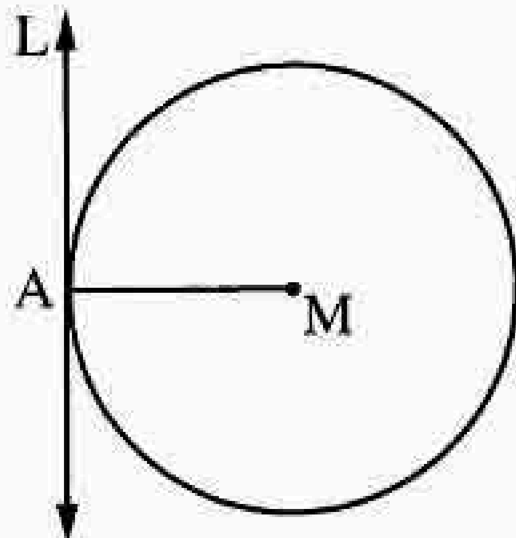
A is outside the circle M	A is on the circle M	A is inside the circle M
(figure: M, A)	(figure: M, A)	(figure: M, A)
If MA > r	If MA = r	If MA < r

Second : Position of a straight line with respect to a circle :

If	Then	The figure	Note that
(1) MA > r	The straight line L lies outside the circle M	(figure: L, M, A)	• L ∩ the circle M = ∅ • L ∩ the surface of the circle M = ∅
(2) MA = r	The straight line L is a tangent to the circle M at A A is called "the point of tangency"	(figure: L, M, A)	• L ∩ the circle M = {A} • L ∩ the surface of the circle M = {A}
(3) MA < r	The straight line L is a secant to the circle M	(figure: L, X, M, A, Y)	• L ∩ the circle M = {X , Y} • L ∩ the surface of the circle M = $\overline{XY}$ $\overline{XY}$ is called the chord of intersection

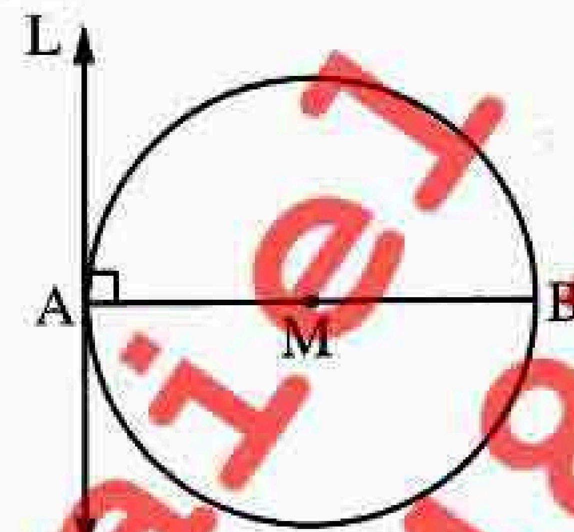
Two important facts

1 The tangent to a circle is perpendicular to the radius drawn from the point of tangency.



i.e. if the straight line L is a tangent to the circle M at the point A ,
then $\overline{MA} \perp L$

2 The straight line which is perpendicular to the diameter of a circle at one of its endpoints is a tangent to the circle.



i.e. if $\overline{AB}$ is a diameter of the circle M and the straight line L ⊥ $\overline{AB}$ at the point A , then L is a tangent to the circle M at the point A

Remark [1]

The two tangents which are drawn from the two endpoints of a diameter of a circle are parallel.

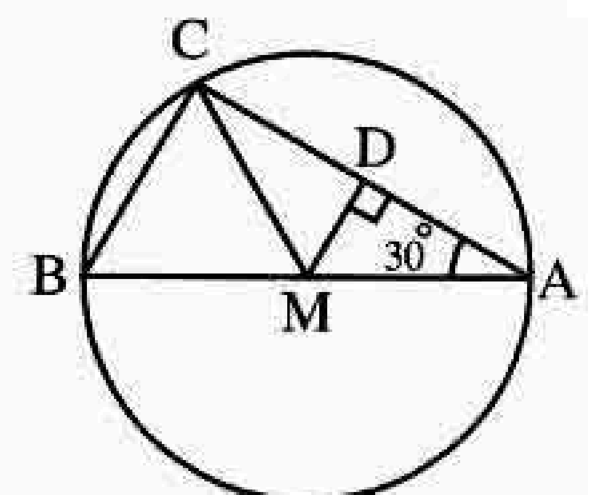
Examples :

1

In the opposite figure :

$\overline{AB}$ is a diameter of a circle M , $\overline{AC}$ is a chord ,
$\overline{MD} \perp \overline{AC}$, m (∠ A) = 30°

Prove that : (1) $\overline{MD}$ // $\overline{BC}$
(2) Δ MBC is an equilateral triangle.



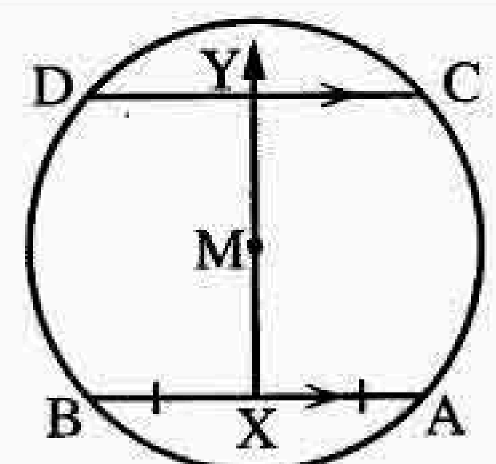
(Fayoum 2012)

2

In the opposite figure :

M is a circle , $\overline{AB}$ // $\overline{CD}$,
X is the midpoint of $\overline{AB}$
and $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$



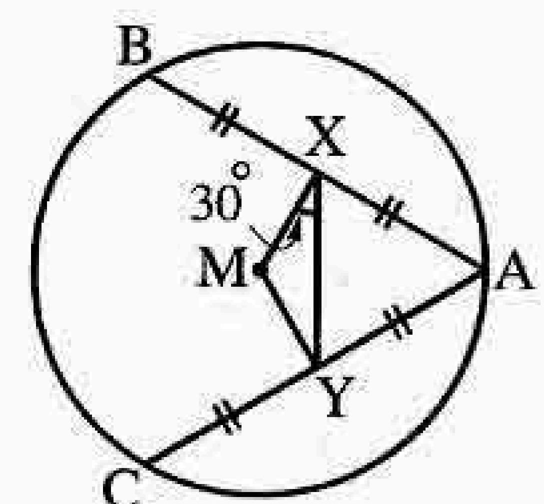
(Aswan 2015 , Alexandria 2013)

3

In the opposite figure :

AC = AB , X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{AC}$,
m (∠ MXY) = 30°

Prove that : The triangle AXY is equilateral.



(Assiut 2014)

4 **In the opposite figure :**

Two concentric circles with centre M ,
$\overline{AB}$ is a chord of the greater circle
and intersects the smaller circle at C , D
and $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

(Red Sea 2012)

5 **In the opposite figure :**

$\overline{AB}$ is a chord of circle M ,
$\overrightarrow{AC}$ bisects $\angle$ BAM and intersects circle M at C
If D is the midpoint of $\overline{AB}$

Prove that : $\overline{DM} \perp \overline{CM}$

(Souhag 2014)

6 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,
$\overline{CD}$ is a chord of it , $\overline{XC} \perp \overline{CD}$
and $\overline{YD} \perp \overline{CD}$

Prove that : AX = BY

(Sharkia 2009)

7 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M at A ,
MA = 8 cm. , m ($\angle$ ABM) = 30° and $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$ and $\overline{AC}$

(El-Monofia 2014 , New Valley 2012) « $8\sqrt{3}$ cm. , $4\sqrt{3}$ cm. »

8 **Prove that :** The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located in circle whose centre is the point M (– 1 , 2) , then find the circumference of the circle.

(El-Beheira 2011) «10 π length units »

9 **In the opposite figure :**

$\overrightarrow{AB}$ touches the circle M at B , $\overline{CD}$ is a diameter of it ,
m ($\angle$ BAM) = x° and m ($\angle$ MDB) = 2 x°

Find : The value of x in degrees.

(Ismailia 2006) «18°»

Solutions

1

$\because \overline{MD} \perp \overline{AC}$

$\therefore$ D is the midpoint of $\overline{AC}$

, $\because$ M is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \parallel \overline{BC}$ (Q.E.D. 1)

$\therefore m(\angle ACB) = m(\angle ADM) = 90^\circ$ "Corresponding angles"

, In Δ ACB : $m(\angle B) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$

, $\because$ MC = MB = r

$\therefore \Delta$ MBC is an equilateral triangle (Q.E.D. 2)

2

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore m(\angle AXY) = 90^\circ$

, $\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{XY}$ is a transversal

$\therefore m(\angle XYD) = m(\angle AXY)$

$= 90^\circ$ (alternate angles)

$\therefore \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ (Q.E.D)

3

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$

$\therefore m(\angle AXY) = 90^\circ - 30^\circ = 60^\circ$

, $\because$ AB = AC $\quad \therefore \frac{1}{2}AB = \frac{1}{2}AC$

$\therefore$ AX = AY $\quad$, $\because m(\angle AXY) = 60^\circ$

$\therefore \Delta$ AXY is an equilateral triangle. (Q.E.D.)

4

In the great circle :

$\because \overline{ME} \perp \overline{AB}$ $\quad \therefore$ E is the midpoint of $\overline{AB}$

$\therefore$ AE = EB (1)

In the small circle :

$\because \overline{ME} \perp \overline{CD}$ $\quad \therefore$ E is the midpoint of $\overline{CD}$

$\therefore$ CE = ED (2)

Subtracting (2) from (1) : $\therefore$ AE – CE = EB – ED

$\therefore$ AC = BD (Q.E.D)

5

In Δ AMC :

$\because$ AM = MC = r $\quad \therefore m(\angle MAC) = m(\angle ACM)$

$\because m(\angle BAC) = m(\angle MAC)$

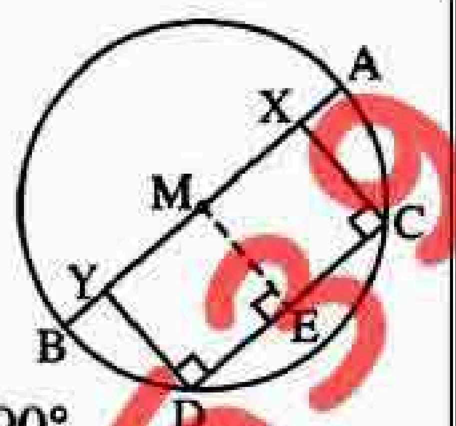
$\therefore m(\angle BAC) = m(\angle ACM)$ and they are alternate angles

$\therefore \overline{AB} \parallel \overline{CM}$

$\because$ D is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MD} \perp \overline{AB}$

$\because \overline{AB} \parallel \overline{CM}$ $\quad \therefore \overline{DM} \perp \overline{CM}$ (Q.E.D)

6



Construction :

Draw $\overrightarrow{ME} \perp \overleftrightarrow{CD}$ to cut it at E

Proof : $\because \overline{ME} \perp \overline{CD}$

$\therefore$ E is the midpoint of $\overline{CD}$

, $m(\angle XCE) = m(\angle MED) = 90^\circ$

but they are corresponding angles

$\therefore \overleftrightarrow{XC} \parallel \overleftrightarrow{ME}$ similarly $\overleftrightarrow{ME} \parallel \overleftrightarrow{YD}$

$\therefore \overleftrightarrow{XC} \parallel \overleftrightarrow{ME} \parallel \overleftrightarrow{YD}$

$\because \overleftrightarrow{XY}$ and $\overleftrightarrow{CD}$ are two transversals to them

, CE = ED $\quad \therefore$ XM = MY

$\because$ AM = BM = r $\quad \therefore$ AM – XM = BM – MY

$\therefore$ AX = BY (Q.E.D.)

7

$\because \overleftrightarrow{AB}$ is a tangent to the circle M at A

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

$\therefore m(\angle MAB) = 90^\circ$

In Δ MAB : $\because m(\angle ABM) = 30^\circ$

$\therefore$ MB = 2 MA = 16 cm.

$\therefore AB = \sqrt{(MB)^2 - (MA)^2} = \sqrt{256 - 64}$

$= \sqrt{192}$

$= 8\sqrt{3}$ cm. (First req.)

In Δ ABC which is right-angled at C

$\because m(\angle ABC) = 30^\circ$

$\therefore AC = \frac{1}{2}AB = \frac{1}{2} \times 8\sqrt{3}$

$= 4\sqrt{3}$ cm. (Second req.)

8

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25}$

= 5 length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} = \sqrt{25}$

= 5 length units

, $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25}$

= 5 length units

$\therefore$ MA = MB = MC

$\therefore$ The points A , B and C lie on the circle M (Q.E.D.1)

, its circumference = 10π length units. (Q.E.D.2)

Exercises

[A] : Choose The Correct Answer :

1	If A $\in$ the circle M of diameter length 6 cm. , then MA = …………… cm. (a) 3 (b) 4 (c) 5 (d) 6
2	Which of the following points does not belong to the circle that its centre is the origin and its radius is 7 cm.? (a) (0 , 7) (b) (0 , – 7) (c) (7 , 0) (d) (7 , 7)
3	If the point A $\in$ the circle M and its diameter length equals 6 cm. , then MA = …………… cm. (a) 4 (b) 6 (c) 3 (d) 8
4	If M is a circle of diameter length 7 cm. , A is a point on its plane and MA = 4 cm. , then the position of A with respect to this circle is …………… (a) inside the circle. (b) outside the circle. (c) on the circle. (d) coincide on the centre M
5	If M is a circle of a diameter length equals 14 cm. , MA = $(2x + 3)$ cm. where A lies on the circle , then x = …………… (a) 5 (b) 3 (c) 2 (d) 1
6	The number of symmetric axes of any circle is …………… (a) zero (b) 1 (c) 2 (d) an infinite number.
7	Number of the axes of symmetry of the semicircle is …………… (a) zero. (b) 1 (c) 2 (d) infinite.
8	The circle has …………… number of axes of symmetry. (a) 1 (b) 2 (c) 3 (d) an infinite
9	The number of the axes of symmetry of the semicircle …………… the number of the axes of symmetry of the isosceles triangle. (a) > (b) < (c) = (d) ≥
10	If the straight line L $\cap$ the circle M = $\varnothing$, then L is …………… of the circle. (a) a secant (b) outside (c) a tangent (d) an axis of symmetry
11	If $\overleftrightarrow{AB}$ $\cap$ the circle M = $\{A , B\}$, then $\overleftrightarrow{AB}$ $\cap$ the surface of the circle M = …………… (a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) $\{A , B\}$ (d) $\overrightarrow{AB}$

12	The number of tangents can be drawn from a point lies on a circle equals .... (a) one. (b) two. (c) four. (d) infinite number.
13	The tangent to a circle whose diameter length is 10 cm. , is at a distance of .... cm. from its centre. (a) 4 (b) 5 (c) 6 (d) 10
14	A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre. (a) 4 (b) 3 (c) 8 (d) 6
15	A tangent to a circle of diameter length 6 cm. is at distance of .... cm. from its centre. (a) 6 (b) 12 (c) 3 (d) 2
16	If the straight line L is outside a circle of radius length 3 cm. and its centre is the origin point M (0 , 0) , if L at distance x from its centre , then $x \in$ .... (a) $[3 , \infty[$ (b) $]3 , \infty[$ (c) $[6 , \infty[$ (d) $]-\infty , -6[$
17	If the diameter length of a circle is 6 cm. and the straight line L is distant from its centre by 6 cm. , then L is .... (a) distant from the circle. (b) intersects the circle. (c) touches the circle. (d) passes through the centre of the circle.
18	If the length of a diameter of a circle is 7 cm. , and the straight line L at a distance of 3.5 cm. from its centre , then L is .... (a) a secant to the circle at two points. (b) lying outside the circle. (c) a tangent to the circle. (d) an axis of symmetry to the circle.
19	If the length of a diameter of a circle is 8 cm. and the straight line L at a distance of 4 cm. from its centre , then L is .... (a) a secant to the circle at two points. (b) lying outside the circle. (c) a tangent to the circle. (d) an axis of symmetry to the circle.
20	A circle of diameter 8 cm. and the straight line "L" is at distance of 3 cm. from its centre , than L .... (a) touches the circle. (b) is a secant to the circle. (c) lies outside the circle. (d) is axis to the circle.
21	A circle , its radius length $(2x + 6)$ cm. and the straight line L is at distance $(x + 2)$ cm. from its centre where $x > 0$, then L is .... (a) outside the circle. (b) a tangent to the circle. (c) a secant to the circle. (d) passing through the centre.

22 A circle with diameter length $(2X + 5)$ cm. , the straight line L is distant from its centre by $(X + 2)$ cm. where $X > 0$, then the straight line is ....

(a) a secant to the circle at two points. (b) lying outside the circle.
(c) a tangent to the circle. (d) an axis of symmetry to the circle.

23 If the length of perpendicular drawn from the centre of circle M on the straight line L equals 6 cm. and its radius length is 6 cm. , then L .... the circle.

(a) intersects (b) touches
(c) lies outside (d) passes through the centre of

24 M is a circle with radius length r , $\overleftrightarrow{MA} \perp$ straight line L where $\overleftrightarrow{MA} \cap L = \{A\}$
If $MA > r$, then L is ....

(a) a tangent to the circle. (b) a diameter in the circle.
(c) outside the circle. (d) a secant to the circle.

25 The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

26 If the point A belongs to the circle M of diameter 6 cm. , then MA equals ....

(a) 3 cm. (b) 4 cm. (c) 5 cm. (d) 6 cm.

27 The chord which passes through the centre of the circle is called .... to the circle.

(a) tangent (b) secant (c) diameter (d) radius

28 If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in a circle M and the area of triangle AMB = 8 cm^2 , then the length of radius of this circle = ...

(a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.

29 The chord which passes through the centre of the circle is called ....

(a) tangent. (b) diameter. (c) radius. (d) side.

30 **In the opposite figure :**
If $\overline{AB}$ is a tangent segment to the circle M
, then DB = ... cm.

(a) 2 (b) 3
(c) 4 (d) 5

31 **In the opposite figure :**
CD = 3 cm. , $\overline{MC} \perp \overline{AB}$
, D is the midpoint of $\overline{MA}$
then the area of the circle M = ... π cm^2

(a) 3 (b) 6 (c) 9 (d) 36

32	**In the opposite figure :** $\overrightarrow{AB}$ is a tangent to the circle M , MB = 6 cm. , AB = 8 cm. , then AM = ... cm. (a) 5 (b) 10 (c) 12 (d) 13
33	**In the opposite figure :** $\overrightarrow{AB}$ is a tangent to the circle M , if MB = 5 cm. , AC = 8 cm. , then AB = ... cm. (a) 5 (b) 10 (c) 12 (d) 13
34	**In the opposite figure :** $\overline{AB}$ is a tangent , AM = 5 cm. , m (∠ B) = 30° , then the length of $\overline{BC}$ equals .... cm. (a) 5 (b) 7 (c) 8 (d) 10
35	**In the opposite figure :** $\overline{AC}$ is a tangent to circle M at C if MC = 5 cm. , AB = 8 cm. , then AC = ... cm. (a) 5 (b) 10 (c) 13 (d) 12

[B] : Essay Problems : -

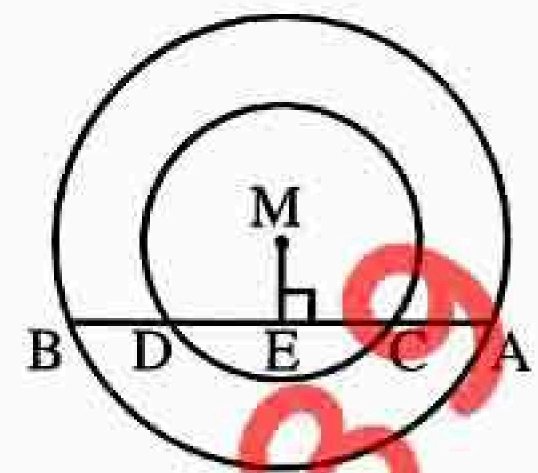
1	**In the opposite figure :** $\overline{AB}$ is a diameter of a circle M , $\overline{AC}$ is a chord , $\overline{MD} \perp \overline{AC}$, m (∠ A) = 30° **Prove that :** (1) $\overline{MD}$ // $\overline{BC}$ (2) Δ MBC is an equilateral triangle. *(Fayoum 2012)*
2	**In the opposite figure :** M is a circle of radius length 13 cm. , $\overline{AB}$ is a chord of length 24 cm. , C is the midpoint of $\overline{AB}$ and $\overrightarrow{MC}$ ∩ circle M = {D} **Find :** The area of the triangle ADB *(El-Dakahlia 2013)* « 96 cm^2 »

3

In the opposite figure :

Two concentric circles with centre M ,
$\overline{AB}$ is a chord of the greater circle
and intersects the smaller circle at C , D
and $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD



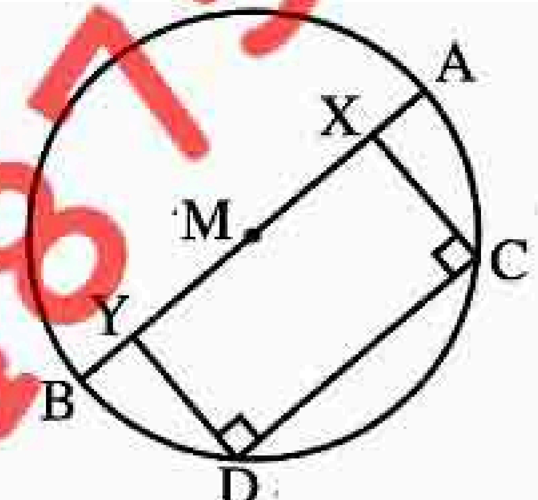
(Red Sea 2012)

4

In the opposite figure :

$\overline{AB}$ is a diameter of the circle M ,
$\overline{CD}$ is a chord of it , $\overline{XC} \perp \overline{CD}$
and $\overline{YD} \perp \overline{CD}$

Prove that : AX = BY

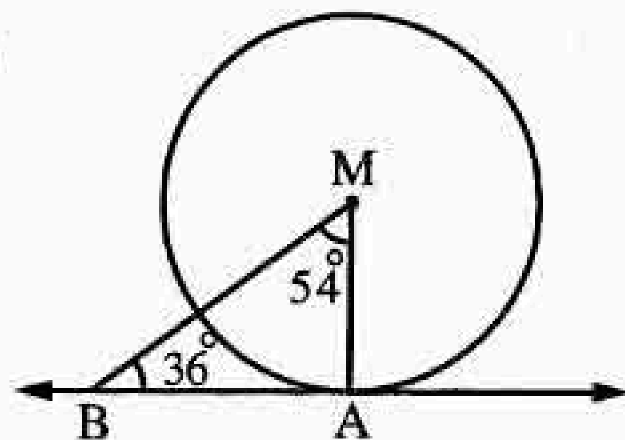


(Sharkia 2009)

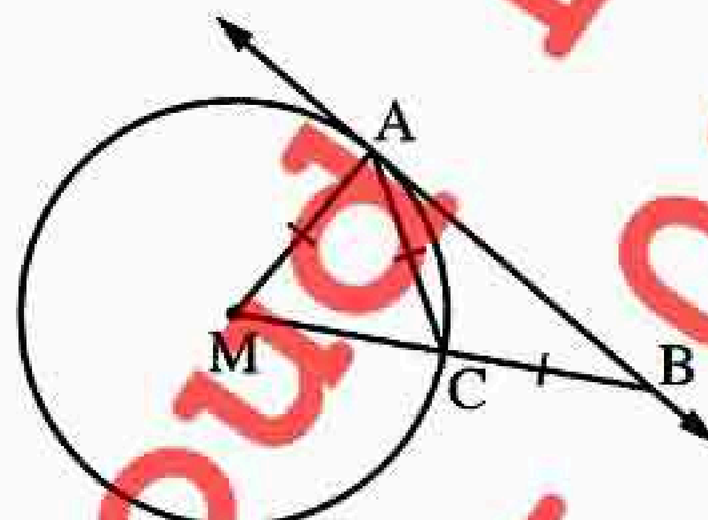
5

In each of the following figures , explain why $\overleftrightarrow{AB}$ is a tangent to circle M :

(1)

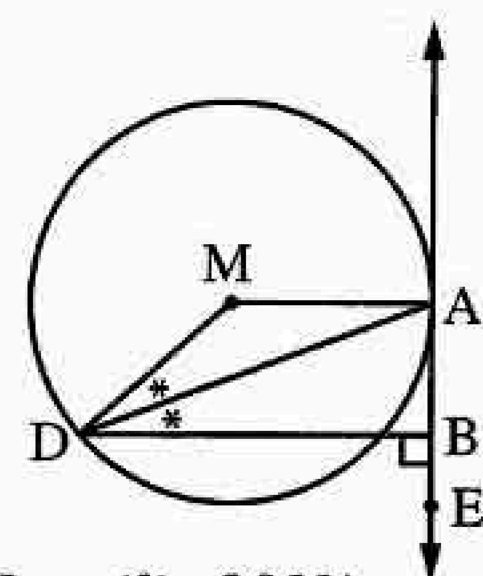


(2)



(El-Gharbia 2016)

(3)



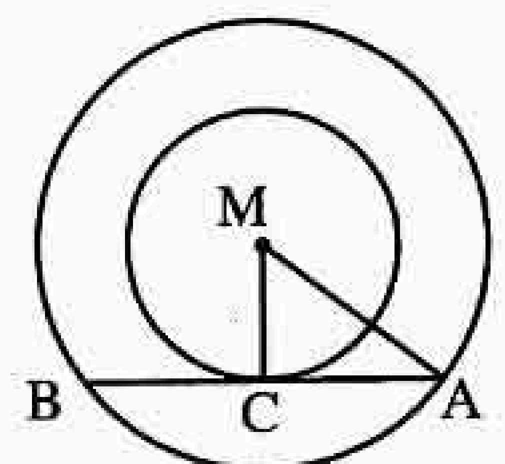
(Ismailia 2011)

6

In the opposite figure :

$\overline{AB}$ is a chord of the great circle and touches the small circle at C , AB = 8 cm. and the radius length of the great circle = 5 cm.

Find : The radius length of the small circle.

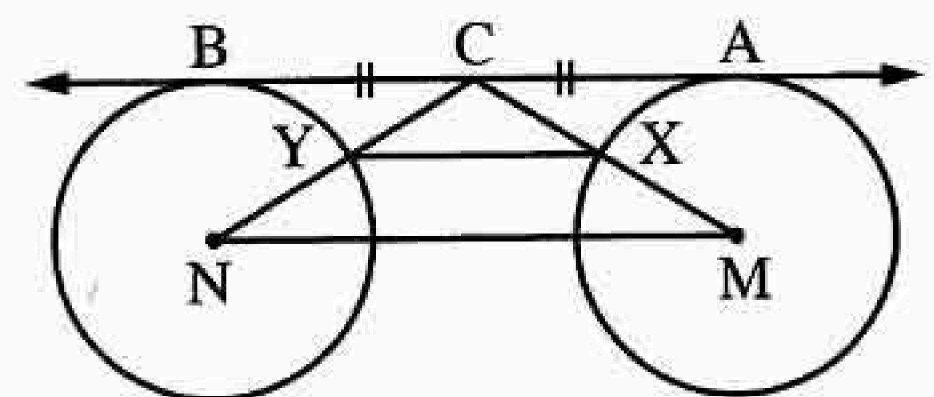


(Souhag 2009) «3 cm.»

7

In the opposite figure :

M and N are two congruent circles ,
$\overleftrightarrow{AB}$ is a common tangent to them ,
C is the midpoint of $\overline{AB}$,
the circle M $\cap$ $\overline{MC} = \{X\}$, the circle N $\cap$ $\overline{NC} = \{Y\}$



Prove that : (1) $\overline{AB}$ // $\overline{MN}$
(2) Δ CMN is an isosceles triangle.
(3) $\overline{XY}$ // $\overline{MN}$

(Kalyoubia 2004)

8

In the opposite figure :

$\overline{AB}$ is a chord of the circle M ,

m (∠ D) = 25°

and m (∠ MAC) = 40°

Prove that :

C is the midpoint of $\overline{AB}$

(Kafr El-Sheikh 2009)

9

In the opposite figure :

M is a circle , $\overline{AB}$ // $\overline{CD}$,

X is the midpoint of $\overline{AB}$

and $\overrightarrow{XM}$ is drawn to cut $\overline{CD}$ at Y

Prove that : Y is the midpoint of $\overline{CD}$

(Aswan 2015 , Alexandria 2013)

10

If $\overline{CD}$ is a diameter of circle M where M (1 , 1) , D (3 , –2)

Find : The equation of the tangent to M at C *(El-Dakahlia 2011)* « $y = \frac{2}{3} x + 4\frac{2}{3}$ »

11

Prove that : The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) are located in circle whose centre is the point M (– 1 , 2) , then find the circumference of the circle.

(El-Beheira 2011) «10 π length units »

12

In the opposite figure :

$\overrightarrow{AB}$ is a tangent to the circle M at A ,

MA = 8 cm. , m (∠ ABM) = 30° and $\overline{AC} \perp \overline{MB}$

Find : The length of each of $\overline{AB}$ and $\overline{AC}$

(El-Monofia 2014 , New Valley 2012) « $8\sqrt{3}$ cm. , $4\sqrt{3}$ cm. »

13

In the opposite figure :

$\overleftrightarrow{DC}$ touches the circle M at C , $\overline{AB}$ // $\overline{MD}$,

m (∠ BAC) = 80° , m (∠ MDC) = 20°

and $\overline{AC} \cap \overline{MD} = \{E\}$

Find : m (∠ ECM)

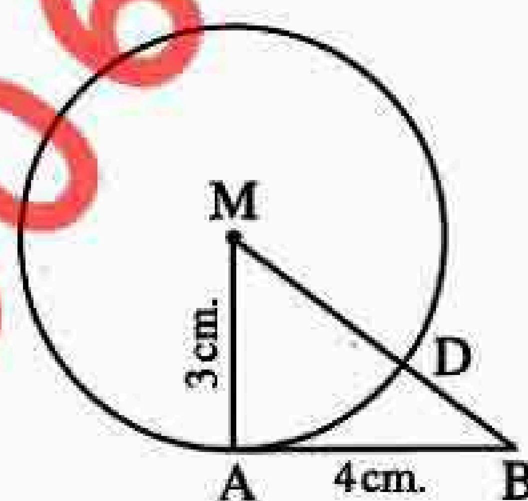
(Beni Suef 2005) «30°»

14

$\overline{AB}$ is a diameter in a circle of area 36 π cm^2 , $\overleftrightarrow{BC}$ is drawn a tangent to the circle at B , if m (∠ ACB) = 60° , then calculate the area of Δ ABC *(El-Dakahlia 2014)* « $24\sqrt{3}$ cm^2.»

Homework

[A] : Choose The Correct Answer : -

1	If the straight line L $\cap$ the circle M = $\varnothing$, then L is .... of the circle. (a) a secant (b) outside (c) a tangent (d) an axis of symmetry
2	A circle of diameter 8 cm. and the straight line "L" is at distance of 3 cm. from its centre , than L .... (a) touches the circle. (b) is a secant to the circle. (c) lies outside the circle. (d) is axis to the circle.
3	**In the opposite figure :** If $\overline{AB}$ is a tangent segment to the circle M , then DB = ... cm. (a) 2 (b) 3 (c) 4 (d) 5 
4	The number of the axes of symmetry of the semicircle .... the number of the axes of symmetry of the isosceles triangle. (a) > (b) < (c) = (d) ≥
5	If the length of a diameter of a circle is 8 cm. and the straight line L at a distance of 4 cm. from its centre , then L is .... (a) a secant to the circle at two points. (b) lying outside the circle. (c) a tangent to the circle. (d) an axis of symmetry to the circle.
6	The chord which passes through the centre of the circle is called .... (a) tangent. (b) diameter. (c) radius. (d) side.
7	The circle has .... number of axes of symmetry. (a) 1 (b) 2 (c) 3 (d) an infinite
8	If the length of a diameter of a circle is 7 cm. , and the straight line L at a distance of 3.5 cm. from its centre , then L is .... (a) a secant to the circle at two points. (b) lying outside the circle. (c) a tangent to the circle. (d) an axis of symmetry to the circle.
9	If $\overline{MA}$ and $\overline{MB}$ are two perpendicular radii in a circle M and the area of triangle AMB = 8 cm^2 , then the length of radius of this circle = ... (a) 8 cm. (b) 16 cm. (c) 4 cm. (d) 2 cm.
10	Number of the axes of symmetry of the semicircle is .... (a) zero. (b) 1 (c) 2 (d) infinite.

11 If the diameter length of a circle is 6 cm. and the straight line L is distant from its centre by 6 cm. , then L is ....

(a) distant from the circle. (b) intersects the circle.

(c) touches the circle. (d) passes through the centre of the circle.

12 The chord which passes through the centre of the circle is called .... to the circle.

(a) tangent (b) secant (c) diameter (d) radius

13 The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

14 If the straight line L is outside a circle of radius length 3 cm. and its centre is the origin point M (0 , 0) , if L at distance x from its centre , then $x \in$ ....

(a) $[3 , \infty[$ (b) $]3 , \infty[$ (c) $[6 , \infty[$ (d) $]-\infty , -6[$

15 If the point A belongs to the circle M of diameter 6 cm. , then MA equals ....

(a) 3 cm. (b) 4 cm. (c) 5 cm. (d) 6 cm.

16 If M is a circle of a diameter length equals 14 cm. , MA = $(2x + 3)$ cm. where A lies on the circle , then x = ...

(a) 5 (b) 3 (c) 2 (d) 1

17 A tangent to a circle of diameter length 6 cm. is at distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

18 The two tangents which are drawn from the two endpoints of a diameter of a circle are ....

(a) parallel. (b) equal in length. (c) congruent. (d) intersecting.

19 **In the opposite figure :**

$\overline{AC}$ is a tangent to circle M at C

if MC = 5 cm. , AB = 8 cm.

, then AC = ... cm.

(a) 5 (b) 10 (c) 13 (d) 12

20 If M is a circle of diameter length 7 cm. , A is a point on its plane and MA = 4 cm. , then the position of A with respect to this circle is ....

(a) inside the circle. (b) outside the circle.

(c) on the circle. (d) coincide on the centre M

21 If A $\in$ the circle M of diameter length 6 cm. , then MA = ... cm.

(a) 3 (b) 4

(c) 5 (d) 6

22 A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre.

(a) 4 (b) 3 (c) 8 (d) 6

23 M is a circle with radius length r , $\overleftrightarrow{MA} \perp$ straight line L where $\overleftrightarrow{MA} \cap L = \{A\}$

If MA > r , then L is ....

(a) a tangent to the circle. (b) a diameter in the circle.

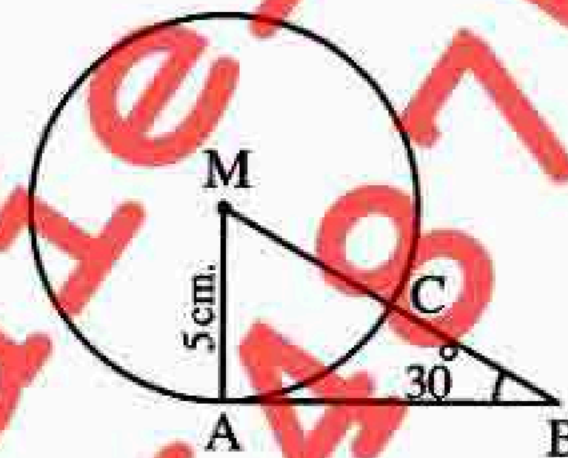
(c) outside the circle. (d) a secant to the circle.

24 **In the opposite figure :**

$\overline{AB}$ is a tangent , AM = 5 cm.

, m (∠ B) = 30°

, then the length of $\overline{BC}$ equals .... cm.



(a) 5 (b) 7 (c) 8 (d) 10

25 If the point A ∈ the circle M and its diameter length equals 6 cm. , then MA = ... cm.

(a) 4 (b) 6 (c) 3 (d) 8

26 The tangent to a circle whose diameter length is 10 cm. , is at a distance of .... cm. from its centre.

(a) 4 (b) 5 (c) 6 (d) 10

27 If the length of perpendicular drawn from the centre of circle M on the straight line L equals 6 cm. and its radius length is 6 cm. , then L .... the circle.

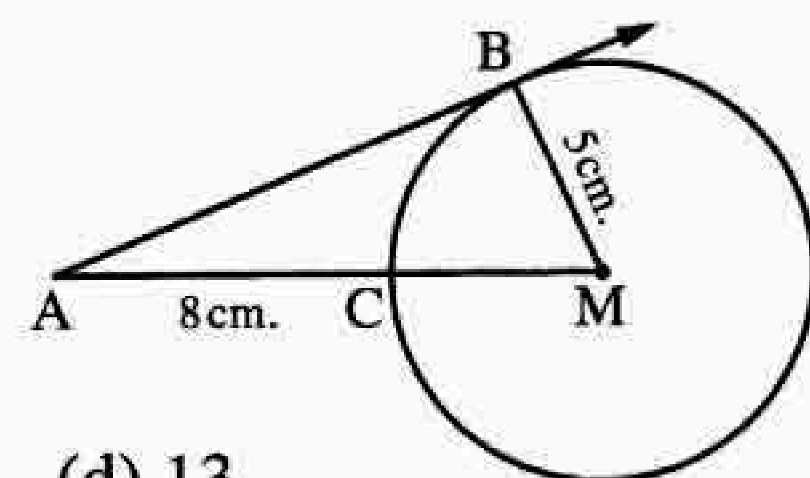
(a) intersects (b) touches

(c) lies outside (d) passes through the centre of

28 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M

, if MB = 5 cm. , AC = 8 cm. , then AB = ... cm.



(a) 5 (b) 10 (c) 12 (d) 13

29 Which of the following points does not belong to the circle that its centre is the origin and its radius is 7 cm.?

(a) (0 , 7) (b) (0 , – 7) (c) (7 , 0) (d) (7 , 7)

30 The number of tangents can be drawn from a point lies on a circle equals ....

(a) one. (b) two. (c) four. (d) infinite number.

31 A circle with diameter length $(2X + 5)$ cm. , the straight line L is distant from its centre by $(X + 2)$ cm. where $X > 0$, then the straight line is ....

(a) a secant to the circle at two points. (b) lying outside the circle.

(c) a tangent to the circle. (d) an axis of symmetry to the circle.

32 **In the opposite figure :**

$\overrightarrow{AB}$ is a tangent to the circle M

, MB = 6 cm. , AB = 8 cm.

, then AM = ……………… cm.

(a) 5 (b) 10 (c) 12 (d) 13

33 If $\overleftrightarrow{AB} \cap$ the circle M = $\{A , B\}$, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ………………

(a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) $\{A , B\}$ (d) $\overrightarrow{AB}$

34 A circle , its radius length $(2x + 6)$ cm. and the straight line L is at distance $(x + 2)$ cm. from its centre where $x > 0$, then L is ………………

(a) outside the circle. (b) a tangent to the circle.

(c) a secant to the circle. (d) passing through the centre.

35 **In the opposite figure :**

CD = 3 cm. , $\overline{MC} \perp \overline{AB}$

, D is the midpoint of $\overline{MA}$

then the area of the circle M = ……………… π cm^2

(a) 3 (b) 6 (c) 9 (d) 36

[B] : Essay Problems : -

1 **In the opposite figure :**

ABC is a triangle drawn inside a circle with centre M (inscribed triangle) , $\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$

Prove that :

(1) $\overline{ED} \parallel \overline{AB}$

(2) The perimeter of Δ CDE = $\frac{1}{2}$ the perimeter of Δ ABC

(Kafr El-Sheikh 2016 , El-Beheira 2013)

2 **In the opposite figure :**

$\overline{AB}$ is a chord of circle M ,

$\overrightarrow{AC}$ bisects $\angle$ BAM and intersects circle M at C

If D is the midpoint of $\overline{AB}$

Prove that : $\overline{DM} \perp \overline{CM}$

(Souhag 2014)

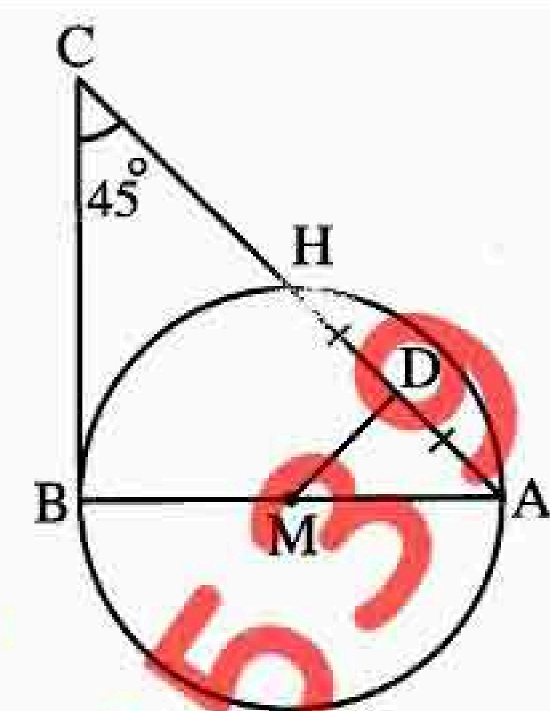
3

In the opposite figure :

$\overrightarrow{BC}$ is a tangent at B , m (∠ C) = 45° ,

D is the midpoint of $\overline{AH}$

Prove that : DA = DM



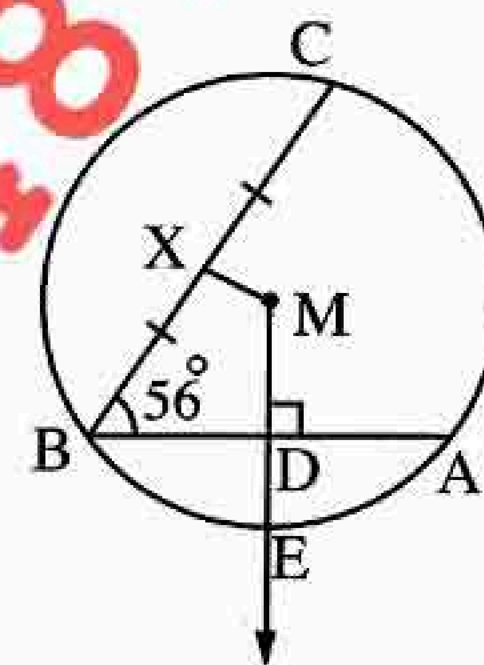
(Aswan 2011)

4

📖 In the opposite figure :

$\overline{AB}$ and $\overline{BC}$ are two chords in circle M ,
which has radius length of 5 cm.,
$\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and inersects the circle M at E ,
X is the midpoint of $\overline{BC}$, AB = 8 cm., m (∠ ABC) = 56°

Find : (1) m (∠ DMX) (2) The length of $\overline{DE}$



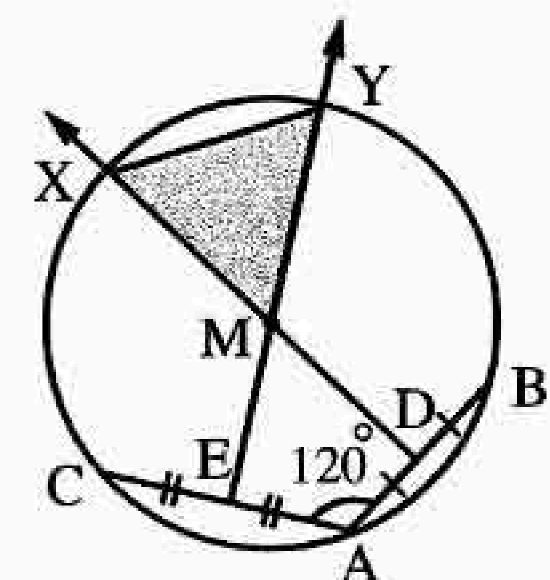
(Souhag 2015 , Alexandria 2011) « 124° , 2 cm.»

5

📖 In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two chords in circle M
that includes an angle of measure 120° ,
D and E are the two midpoints of $\overline{AB}$ and $\overline{AC}$
respectively , $\overrightarrow{DM}$ and $\overrightarrow{EM}$ are drawn to intersect
the circle at X and Y respectively.

Prove that : The triangle XYM is an equilateral triangle.



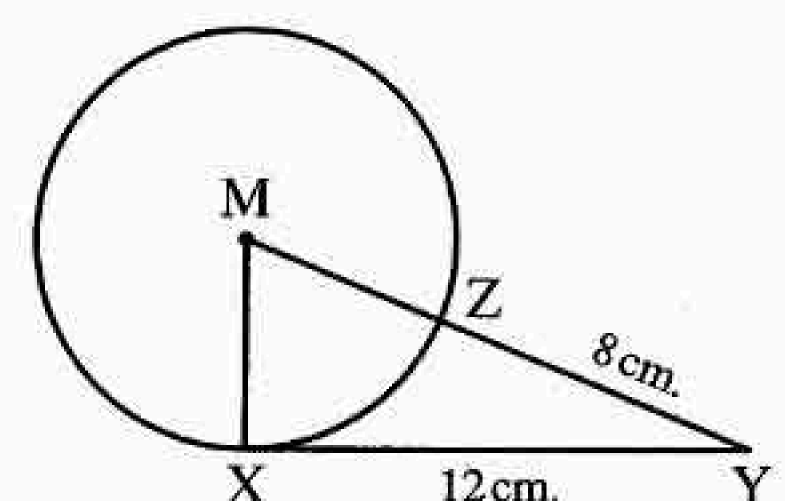
(Aswan 2016 , Beni Suef 2015)

6

In the opposite figure :

M is a circle , $\overleftrightarrow{XY}$ is a tangent to the circle at X
, $\overline{MY}$ ⋂ the circle M = {Z} ,
XY = 12 cm. , YZ = 8 cm.

Find : The radius length of the circle.



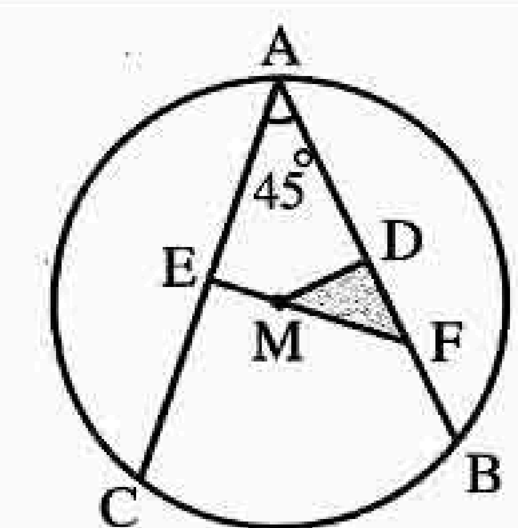
(El-Menia 2013) « 5 cm. »

7

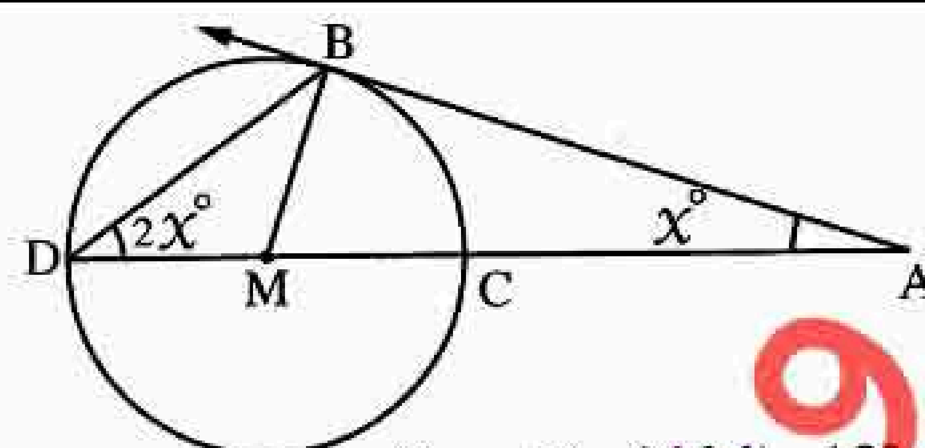
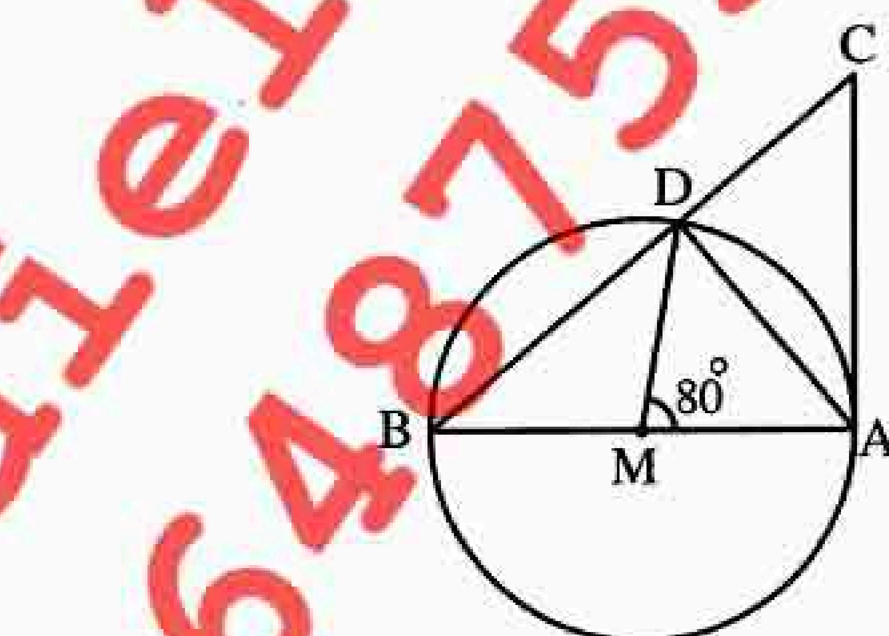
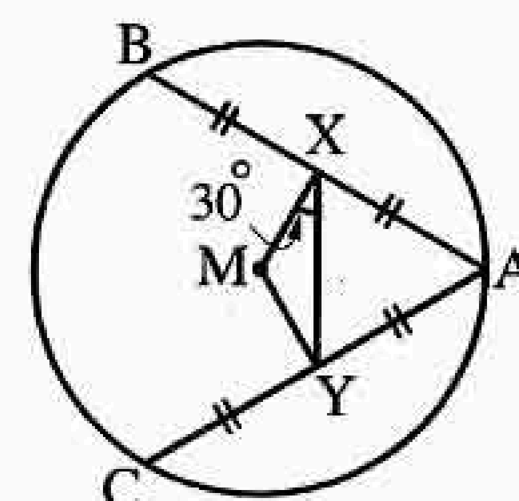
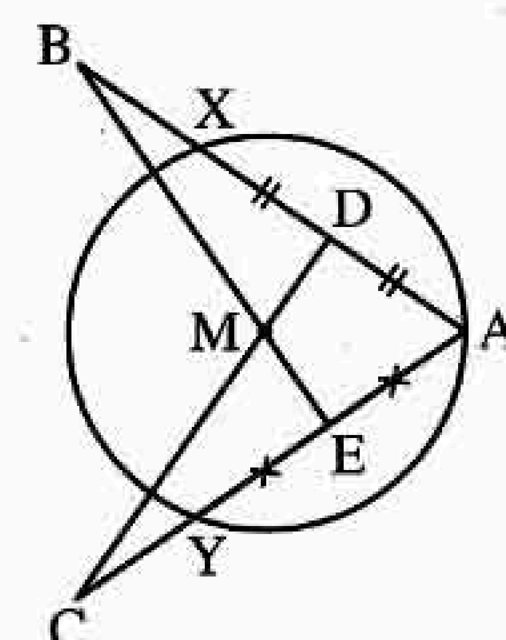
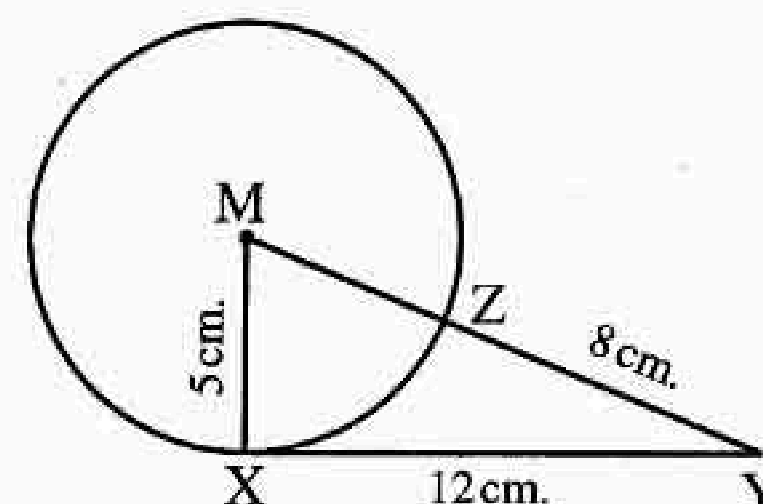
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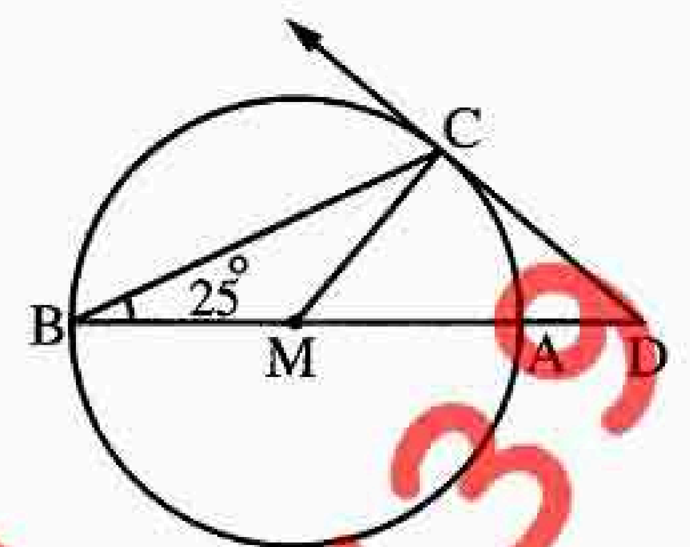
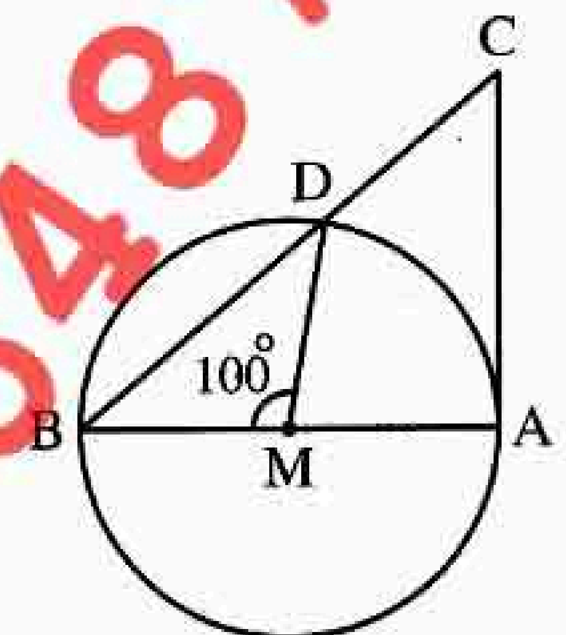
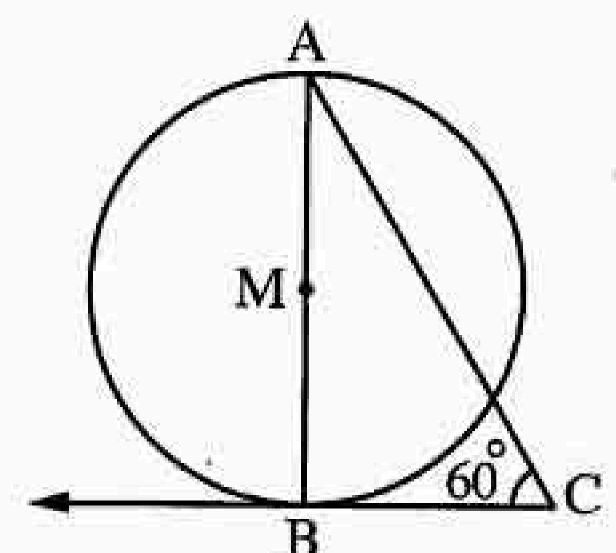
$\overline{AB}$ and $\overline{AC}$ are two chords of the circle M ,
m (∠ BAC) = 45° ,
D and E are the midpoints
of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : Δ DFM is an isosceles triangle.



(New Valley 2005)

8	**In the opposite figure :** $\overrightarrow{AB}$ touches the circle M at B , $\overline{CD}$ is a diameter of it , m (∠ BAM) = $x°$ and m (∠ MDB) = $2x°$ **Find :** The value of x in degrees.  *(Ismailia 2006)* «18°»
9	**In the opposite figure :** $\overline{AB}$ is a diameter of circle M , $\overline{AC}$ is a tangent to it at A and m (∠ AMD) = 80° **Find :** (1) m (∠ CAD) (2) m (∠ ABD) (3) m (∠ ADB)  *(El-Menia 2007)* «40° , 40° , 90°»
10	**In the opposite figure :** AC = AB , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$, m (∠ MXY) = 30° **Prove that :** The triangle AXY is equilateral.  *(Assiut 2014)*
11	**In the opposite figure :** $\overline{AX}$ and $\overline{AY}$ are two chords of the circle M , D is the midpoint of $\overline{AX}$, E is the midpoint of $\overline{AY}$, $\overrightarrow{DM} \cap \overrightarrow{AY} = \{C\}$ and $\overrightarrow{EM} \cap \overrightarrow{AX} = \{B\}$ If AB = AC **Prove that :** AX = AY  *(Damietta 2003)*
12	📖 **In the opposite figure :** M is a circle with radius length 5 cm. , XY = 12 cm. , $\overline{MY} \cap$ circle M = {Z} and ZY = 8 cm. **Prove that :** $\overleftrightarrow{XY}$ is a tangent to the circle M at X  *(South Sinai 2016 , El-Beheira 2014 , Qena 2015)*

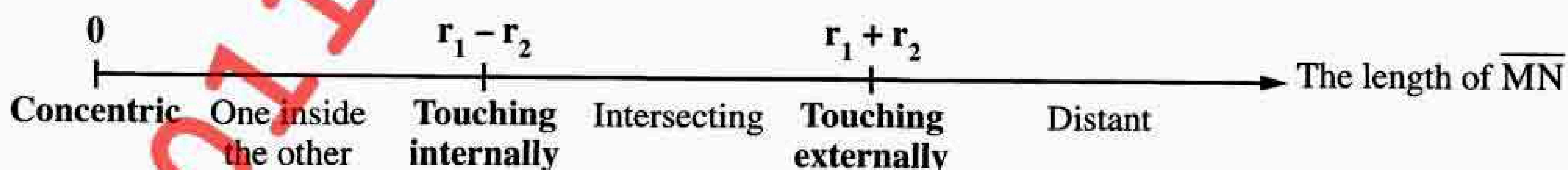
13	**In the opposite figure :** $\overline{AB}$ is a diameter of the circle M , D $\in \overrightarrow{BA}$ If $\overrightarrow{DC}$ is a tangent to the circle at C and m ($\angle$ B) = 25° **Find :** m ($\angle$ D)  *(Beni Suef 2003)* «40°»
14	**In the opposite figure :** $\overline{AB}$ is a diameter in the circle M , $\overrightarrow{AC}$ is a tangent to the circle at A , m ($\angle$ DMB) = 100° **Find by proof :** (1) m ($\angle$ ACB) (2) m ($\angle$ CDM)  *(El-Menia 2011)* «50° , 140°»
15	**In the opposite figure :** A circle of circumference 44 cm. , $\overline{AB}$ is a diameter , $\overleftrightarrow{BC}$ is a tangent at B , and m ($\angle$ C) = 60° **Find :** The length of $\overline{BC}$, $\left(\pi \approx \frac{22}{7}\right)$  *(El-Monofia 2015)* « $\frac{14\sqrt{3}}{3}$ cm.»

Lesson [3] : Positions Of A Circle With Respect To Another Circle

Let M and N be two circles , their radii lengths are r_1 and r_2 respectively , $r_1 > r_2$

If	Then the two circles are	Note that
$MN > r_1 + r_2$	Distant	• The circle M ⋂ the circle N = ∅ • The surface of circle M ⋂ the surface of circle N = ∅
$MN = r_1 + r_2$	Touching externally	• The circle M ⋂ the circle N = {A} • The surface of circle M ⋂ the surface of circle N = {A}
$r_1 - r_2 < MN < r_1 + r_2$	Intersecting	• The circle M ⋂ the circle N = {A , B} • The surface of circle M ⋂ the surface of circle N = the surface of the yellow part.
$MN = r_1 - r_2$	Touching internally	• The circle M ⋂ the circle N = {A} • The surface of circle M ⋂ the surface of circle N = the surface of circle N
$MN < r_1 - r_2$	One inside the other the circle N is inside the circle M	• The circle M ⋂ the circle N = ∅ • The surface of circle M ⋂ the surface of circle N = the surface of circle N
MN = zero	Concentric	• The circle M ⋂ the circle N = ∅ • The surface of circle M ⋂ the surface of circle N = the surface of circle N

Summary



From the previous summary , we notice that :

1. If M and N are two distant circles , then : $MN \in]\, r_1 + r_2 , \infty[$
2. If M and N are two intersecting circles , then : $MN \in]\, r_1 - r_2 , r_1 + r_2[$
3. If M and N (one of them is inside the other) , then : $MN \in]\, 0 , r_1 - r_2[$

Corollary 1

The line of centres of two touching circles passes through the point of tangency and is perpendicular to the common tangent at this point.

In the two opposite figures :

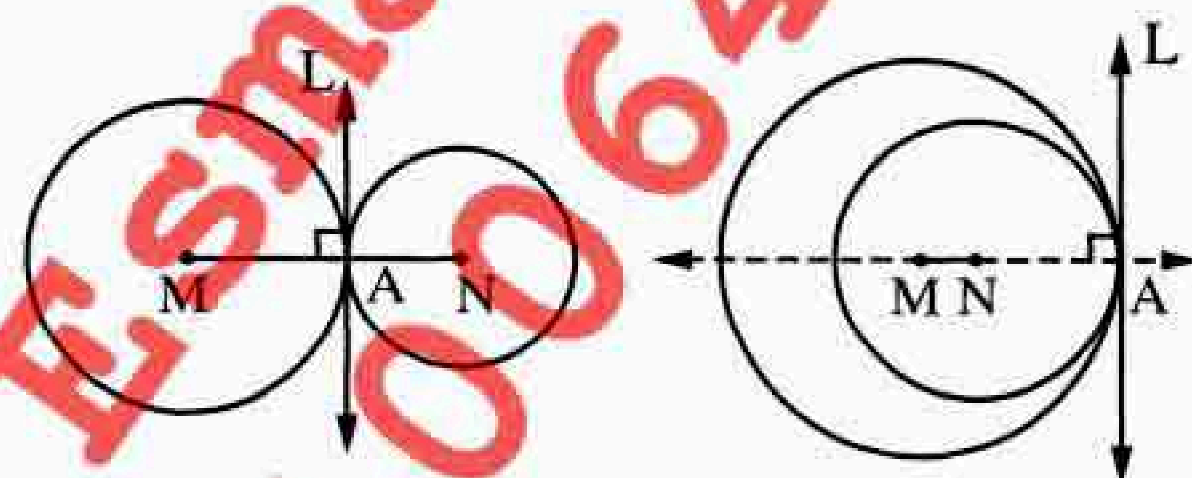
If the two circles

M and N are touching

at A (the point of tangency)

the straight line L is a common tangent to them at A

then $A \in \overleftrightarrow{MN}$ and $\overleftrightarrow{MN} \perp$ the straight line L



Corollary 2

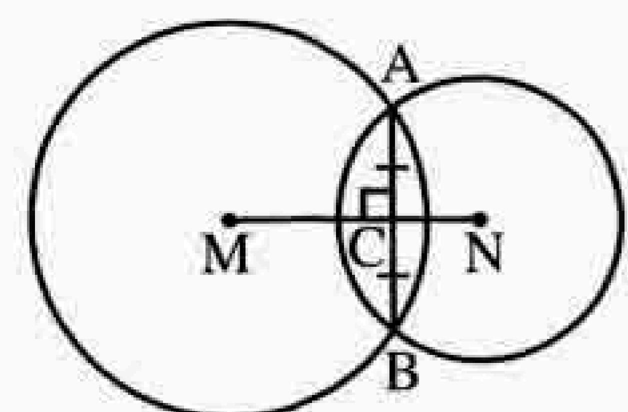
The line of centres of two intersecting circles is perpendicular to the common chord and bisects it.

In the opposite figure :

If M and N are two circles intersecting at A and B ,

then $\overleftrightarrow{MN} \perp \overline{AB}$, $\overleftrightarrow{MN}$ bisects $\overline{AB}$ i.e. AC = BC

This mean that $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$



Lesson [4] : Identifying The Circle

We know that the circle is identified if we know :

1. its centre
2. its radius length

In the following, we will study the possibility of identifying (drawing) the circle under certain conditions.

First Drawing a circle passing through a given point :

i.e. **We can draw an infinite number of circles passing through a given point.**

Second : Drawing a circle passing through two given points :

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$

Remarks

If $\overline{AB}$ is a line segment and the required is drawing a circle passing through the two points A and B , then :

1. If $r > \frac{1}{2} AB$, then we can draw two circles (as shown in the previous example).
2. If $r = \frac{1}{2} AB$, then we can draw one and only one circle (it is the smallest circle) passing through the two points A and B , hence $\overline{AB}$ is a diameter of it and its centre is the midpoint of $\overline{AB}$
3. If $r < \frac{1}{2} AB$, then it is impossible to draw any circle.

- **Any two circles do not intersect at more than two points.**

Third : Drawing a circle passing through three given points :

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

Notice that :

There is a unique circle passing through three points as A , B and C which are not collinear and the centre of this circle is the point of intersection of any two axes of symmetry of the axes of the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$

Corollary 1

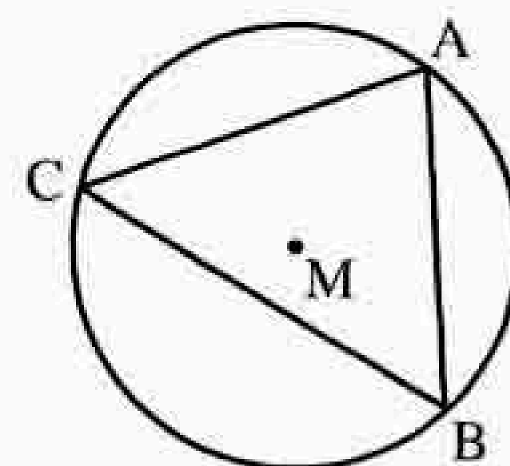
The circle which passes through the vertices of a triangle is called the circumcircle of this triangle.

- The triangle whose vertices lie on a circle is called the inscribed triangle of this circle.

In the opposite figure :

M is the circumcircle of Δ ABC

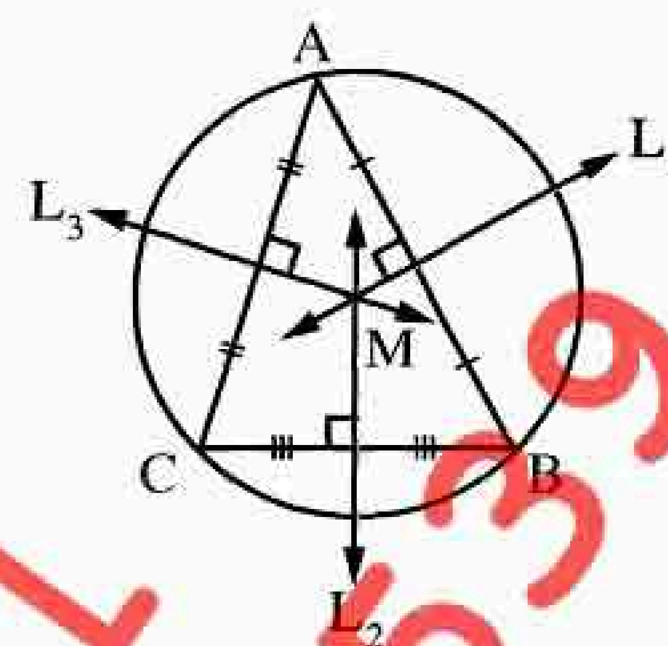
or Δ ABC is the inscribed triangle of the circle M



Corollary 2

The perpendicular bisectors of the sides of a triangle intersect at a point which is the centre of the circumcircle of the triangle.

In the opposite figure :



If the straight lines L_1 , L_2 and L_3 are the axes of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively and $L_1 \cap L_2 \cap L_3 = \{M\}$, then the point M is the centre of the circumcircle of Δ ABC

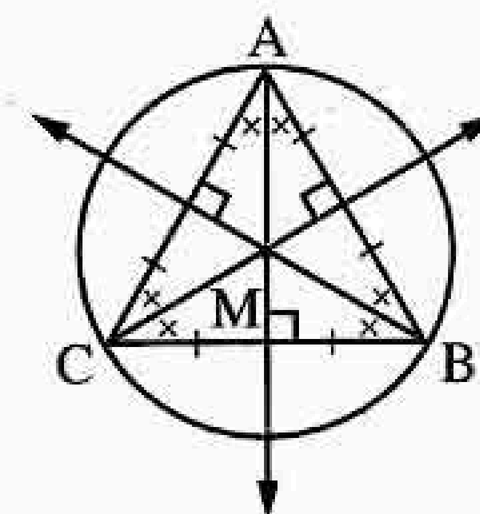
Remarks

The position of the centre of the circumcircle of the triangle as M differs according to the type of the triangle as shown in the following table :

The acute-angled triangle	The right-angled triangle	The obtuse-angled triangle
M is **inside** the triangle	M is the **midpoint of the hypotenuse**	M is **outside** the triangle

A special case :

The centre of the circumcircle of an equilateral triangle is :

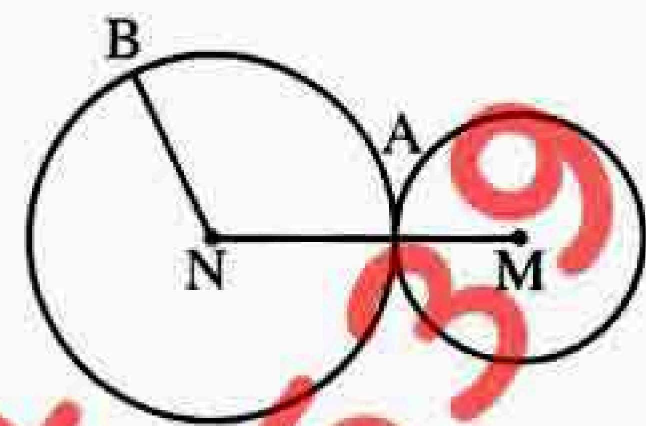
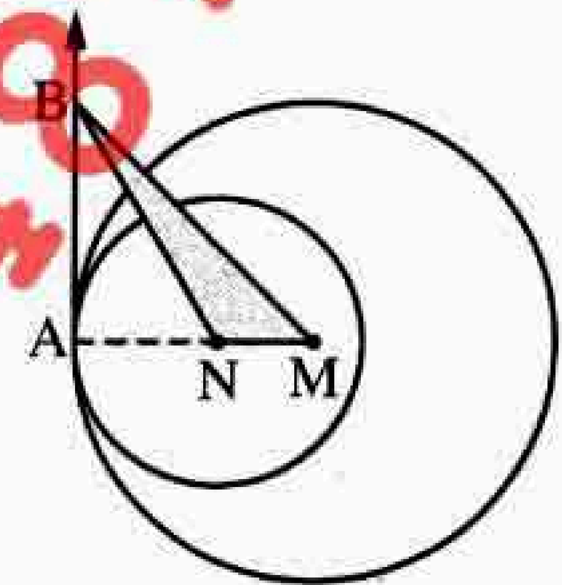
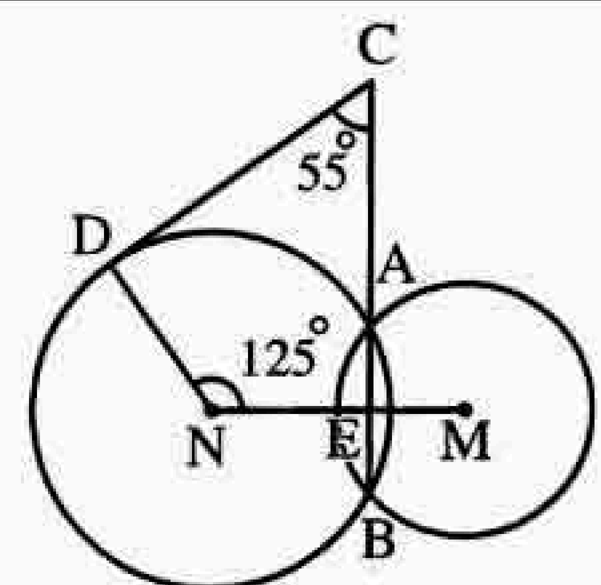
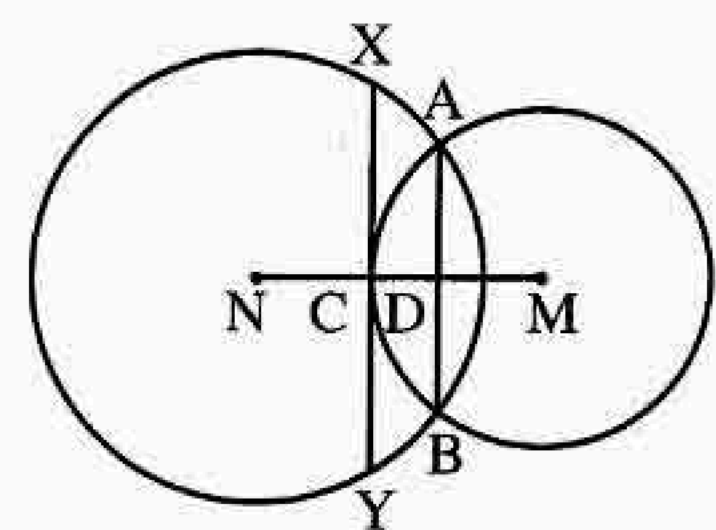


- The point of intersection of its sides axes.
- The point of intersection of its altitudes.
- The point of intersection of its medians.
- The point of intersection of the bisectors of its angles.

Notice that :

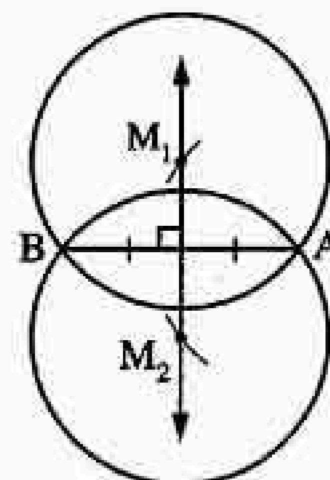
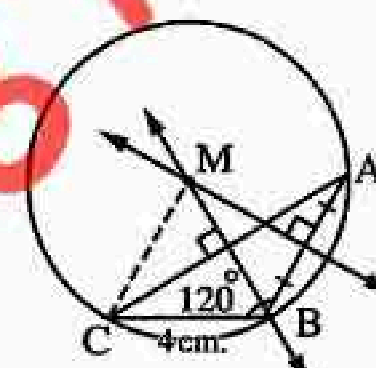
We can draw a circle passing through the vertices of (a rectangle or a square or an isosceles trapezium) while we cannot draw a circle passing through the vertices of (the parallelogram or the rhombus or the trapezium which is not isosceles).

Examples :

1	**In the opposite figure :** M and N are two circles touching at A , the distance between their centres MN = 12 cm. If NB = 7 cm. , **Find :** The length of $\overline{MA}$  *(Kafr El-Sheikh 2006)* «5 cm.»
2	**In the opposite figure :** M and N are two circles with radii lengths of 10 cm. and 6 cm. respectively and they are touching internally at A , $\overleftrightarrow{AB}$ is a common tangent for both. If the area of Δ BMN = 24 cm^2 , **Find :** The length of $\overline{AB}$  *(Qena 2016 , Luxor 2016 , Port Said 2014)* « 12 cm. »
3	**In the opposite figure :** M and N are two intersecting circles at A and B , C ∈ $\overrightarrow{BA}$, D ∈ the circle N , m (∠ MND) = 125° and m (∠ BCD) = 55° **Prove that :** $\overleftrightarrow{CD}$ is a tangent to circle N at D  *(Souhag 2014 , 2015)*
4	**In the opposite figure :** M and N are two intersecting circles , $\overline{AB}$ is the common chord of the two circles M and N $\overline{XY}$ touches the circle M at C **Prove that :** $\overline{AB}$ // $\overline{XY}$  *(Souhag 2008)*
5	Draw $\overline{AB}$ is of length 6 cm. , then draw a circle passing through the two points A and B and its radius length is 4 cm. **How many circles you can draw ?** *(Kafr El-Sheikh 2013 , Luxor 2015)*
6	Using geometrical instruments, draw the isosceles triangle ABC in which m (∠ ABC) = 120° , BC = 4 cm. Determine the centre of the circumcircle of it and find its radius length. *(El-Dakahlia 2011)* « 4 cm. »

Solutions

1	∵ MN = MA + NA , ∵ NA = NB = 7 cm. (lengths of two radii) ∴ 12 = MA + 7 ∴ MA = 5 cm. (The req.)
2	∵ The two circles are touching internally at A ∴ MN = 10 – 6 = 4 cm. , $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$ ∴ The area of Δ BMN = $\frac{1}{2}$ × MN × AB ∴ 24 = $\frac{1}{2}$ × 4 × AB ∴ AB = 12 cm. (The req.)
3	∵ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord ∴ $\overline{AB} \perp \overline{MN}$ ∴ m (∠ AEN) = 90° ∵ The sum of the measures of the interior angles of the quadrilateral CDNE = 360° ∴ m (∠ CDN) = 360° – (55° + 125° + 90°) = 90°
4	∵ $\overleftrightarrow{MN}$ is the line of centres , $\overline{AB}$ is the common chord of the two circles ∴ $\overleftrightarrow{MN} \perp \overline{AB}$ ∴ m (∠ MDA) = 90° ∵ $\overleftrightarrow{XY}$ touches the circle M at C ∴ $\overline{MC} \perp \overleftrightarrow{XY}$ ∴ m (∠ MCX) = 90° ∴ m (∠ MDA) = m (∠ MCX) , but they are corresponding angles ∴ $\overline{AB}$ // $\overleftrightarrow{XY}$ (Q.E.D.)

5	∴ We can draw two circles 
6	 In Δ ABC : ∵ AB = BC ∴ Δ ABC is an isosceles triangle , ∵ $\overleftrightarrow{BM} \perp \overline{AC}$ ∴ $\overleftrightarrow{BM}$ bisects ∠ ABC ∴ m (∠ MBC) = 60° , ∵ MB = MC = r ∴ Δ MBC is an equilateral triangle ∴ MB = MC = BC = r = 4 cm. (The req.)

Exercises

[A] : Choose The Correct Answer : -

1	The number of common tangents of two touching circles externally equals …………… (a) 0 (b) 1 (c) 2 (d) 3
2	The number of common tangents of two distant circles is …………… (a) 1 (b) 2 (c) 3 (d) 4
3	Number of circles passing through a given point …………… (a) one circle. (b) two circles. (c) three circles. (d) infinite number of circles.
4	The number of circles which can be drawn passes through the endpoints of a line segment $\overline{AB}$ equals …………… (a) 1 (b) 2 (c) 3 (d) an infinite number.
5	The number of circles that pass through three collinear points equals …………… (a) zero. (b) one. (c) three. (d) infinite number.
6	Number of the circles that pass through three non-collinear points equals …………… (a) zero (b) one (c) three (d) an infinite number
7	$\overline{AB}$ is a line segment , then the number of the circles passing through the two points A , B is …………… (a) 1 (b) 2 (c) 3 (d) infinite number.
8	If A and B are two points in the plane , if AB = 4 cm. , then the smallest radius length of circle passing through by A and B is …………… cm. (a) 2 (b) 3 (c) 4 (d) 5
9	The centres of all circles passing through the points A and B lie on …………… (a) $\overline{AB}$ (b) midpoint of $\overline{AB}$ (c) the symmetry axis of $\overline{AB}$ (d) the perpendicular to $\overline{AB}$ from B
10	One of the following statments identify one and only one circle , if we have …………… (a) radius length and one of its points. (b) two of its points. (c) only one of its points. (d) its centre and one of its points.
11	The centre of the inscribed circle of any triangle is the intersection point …………… (a) its medians. (b) its heights. (c) the symmetric axes of its sides. (d) bisectors of its interior angles.

12	The centre of the circumcircle of any triangle is the point of intersection of .... (a) the interior bisectors of its angles. (b) the exterior bisectors of its angles. (c) its heights. (d) the symmetric axes of its sides.
13	M and N are two intersecting circles their radii are 5 cm. , 2 cm. , then MN ∈ .... (a)]3 , 7[(b) [3 , 7] (c) [3 , 7[(d)]3 , 7]
14	M and N are two intersecting circles the lengths of their radii are 3 cm. and 5 cm. , then MN ∈ .... (a) [2 , 8] (b) [2 , 8[(c)]2 , 8] (d)]2 , 8[
15	M and N are two intersecting circles , $r_1 = 3$ cm. , $r_2 = 5$ cm. respectively , then MN ∈ .... (a)]0 , 5[(b)]2 , 8[(c)]8 , ∞[(d)]2 , ∞[
16	Two circles M and N with radii lengths 8 cm. and 5 cm. respectively , are touching when MN ∈ .... (a)]13 , 3[(b)]3 , 13[(c) ℝ – [3 , 13] (d) {13 , 3}
17	If the surface of the circle M ∩ the surface of the circle N = {A} , then the two circles M and N are .... (a) distant. (b) concentric. (c) touching externally. (d) intersecting.
18	M and N are two circles their two radii lengths are 5 cm. and 3 cm. respectively. If MN = 8 cm. , then the two circles are .... (a) touching internally. (b) intersecting. (c) touching externally. (d) distant.
19	If the circle M ∩ the circle N = {A , B} , then the two circles M and N are .... (a) intersecting. (b) concentric. (c) touching externally. (d) distant.
20	M and N are two circles of radii lengths 9 cm. , 4 cm. , MN = 5 cm. , then the two circles are .... (a) intersecting. (b) touching internally. (c) touching externally. (d) distant.
21	The surface of the circle M ∩ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. (a) 5 (b) 6 (c) 11 (d) 16

22	The centre of the inscribed circle of any triangle is the point of intersection of its …………… (a) altitudes. (b) medians. (c) axes of symmetry of its sides. (d) bisectors of its interior angles.
23	The centre of the circumcircle of the triangle is the intersection point of its …………… (a) altitudes of triangle. (b) medians of a triangle. (c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.
24	If the two circles M , N are touching externally , the radius length of the circle M is 4 cm. , if MN = 7 cm. then the circumference of the circle N is …………… cm. (a) 4π (b) 6π (c) 7π (d) π
25	If the two circles M and N are touching externally , the radius length of one of them is 5 cm. , and MN = 9 cm. , then the radius length of the other circle equals …………… cm. (a) 4 (b) 5 (c) 9 (d) 14
26	If the two circles M and N are touching externally , their radii lengths are 9 cm. , r cm. , and MN = 14 cm. , then r = …………… cm. (a) 5 (b) 7 (c) 10 (d) 23
27	If M and N are two circles , touching internally , the lengths of their radii are 3 cm. and 5 cm. , then MN = …………… cm. (a) 8 (b) 2 (c) 3 (d) 5
28	If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = …………… cm. (a) 14 (b) 4 (c) 5 (d) 9
29	If the two circles M and N are touching internally , the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = …………… cm. (a) 5 (b) 6 (c) 11 (d) 12
30	If m_1 , m_2 are two slopes of two parallel straight lines , then …………… (a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$
31	If m_1 and m_2 are the slopes of two perpendicular straight lines , then …………… (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$
32	The corresponding angles of the two similar polygons are …………… in measure. (a) equal (b) different (c) proportional (d) alternate
33	If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ……………) (a) X (b) Y (c) Z (d) L

34	The angle whose measure is 50° complements an angle of measure .... (a) 90° (b) 130° (c) 50° (d) 40°
35	The sum of measures of the accumulative angles at a point = ...° (a) 80 (b) 120 (c) 360 (d) 630
36	The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units. (a) – 10 (b) 10 (c) 2 (d) 24
37	If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is .... (a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)
38	If the projection of a line segment on a straight line is a point , then the line segment .... the straight line. (a) // (b) ⊥ (c) ∈ (d) ⊂
39	The image of the point (A , B) by rotation R (0 , 180°) the point .... (a) (– A , B) (b) (– A , – B) (c) (A , – B) (d) (A , B)
40	The image of the point (2 , 3) by rotation R (O , 180°) is the point .... (a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)
41	The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base. (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3
42	The sum of lengths of any two sides of a triangle .... the length of the third side. (a) < (b) > (c) = (d) ≤
43	If $\cos 2x = \frac{1}{2}$ where x is an acute angle , then m (∠ x) = ...° (a) 15 (b) 30 (c) 45 (d) 60
44	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
45	The two angles A and C in the right-angled triangle at B are .... (a) complementary. (b) supplementary. (c) adjacent. (d) vertically opposite angles.
46	ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm. , then its area = ... cm^2. (a) 48 (b) 14 (c) 24 (d) 7

47	Δ XYZ is right-angled triangle at Y , then XZ .... YZ (a) < (b) > (c) = (d) twice
48	ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m ($\angle$ B) = 40° , then m ($\angle$ A) = ... (a) 40° (b) 50° (c) 90° (d) 130°
49	ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and x cm. , then x = ... cm. (a) 5 (b) 8 (c) 10 (d) 12
50	ABC is a triangle in which AB = AC , m ($\angle$ C) = 40° , then m ($\angle$ A) = ... (a) 40° (b) 80° (c) 100° (d) 120°
51	If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is .... (a) 40° (b) 90° (c) 45° (d) 80°
52	The medians of triangle intersect at a same point which divides each in the ratio .... from its base. (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2
53	The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2 (a) 6 (b) 10 (c) 30 (d) 60
54	The numbers 5 , 4 , ... can be side lengths of a triangle. (a) 8 (b) 9 (c) 10 (d) 12
55	If the side length of a rhombus is L cm. , then its perimeter = ... cm. (a) L^2 (b) $2 L^2$ (c) 4 L (d) $2\sqrt{2}$ L
56	The number of the axes of symmetry in the equilateral triangle = ... (a) 1 (b) 2 (c) 3 (d) an infinite number.
57	The diagonals are equal in length and not perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
58	The sum of measures of the interior angles of the quadrilateral = ... (a) 90° (b) 180° (c) 270° (d) 360°
59	The perimeter of the square whose area is 81 cm^2 is .... (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.
60	A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2 (a) 3050 (b) 3500 (c) 2925 (d) 3250

[B] : Essay Problems : -

1

In the opposite figure :

M and N are two circles touching at A ,

the distance between their centres MN = 12 cm.

If NB = 7 cm. ,

Find : The length of $\overline{MA}$

(Kafr El-Sheikh 2006) «5 cm.»

2

In the opposite figure :

M and N are two intersecting circles , $\overline{AB}$ is the common chord of the two circles M and N

$\overline{XY}$ touches the circle M at C

Prove that : $\overline{AB} \parallel \overline{XY}$

(Souhag 2008)

3

Draw $\overline{AB}$ is of length 6 cm. , then draw a circle passing through the two points A and B and its radius length is 4 cm. **How many circles you can draw ?** *(Kafr El-Sheikh 2013 , Luxor 2015)*

4

Draw Δ ABC in which : AB = 5 cm. , BC = 4 cm. , and CA = 3 cm. What is the type of the triangle with respect to the measures of its angles ? then draw a circle whose centre is the point A and touches $\overleftrightarrow{BC}$, another circle whose centre is B and touches $\overleftrightarrow{AC}$ and a third circle whose centre is C and touches $\overleftrightarrow{AB}$ *(Beni Suef 2006)*

5

📖 **In the opposite figure :**

M and N are two circles with radii lengths of 10 cm. and 6 cm. respectively and they are touching internally at A ,

$\overleftrightarrow{AB}$ is a common tangent for both.

If the area of Δ BMN = 24 cm^2 ,

Find : The length of $\overline{AB}$

(Qena 2016 , Luxor 2016 , Port Said 2014) « 12 cm. »

6

📖 M and N are two intersecting circles at A and B , MA = 12 cm. , NA = 9 cm. and MN = 15 cm.

Find : The length of $\overline{AB}$

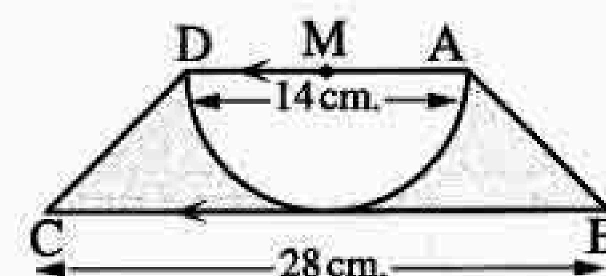
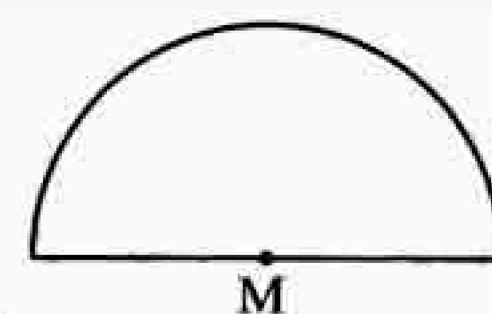
(Port Said 2011) «14.4 cm.»

7

$\overline{AB}$ is a line segment of length 6 cm. Draw the circle that passes through the two points A and B and its radius length is the smallest length. *(Luxor 2005)*

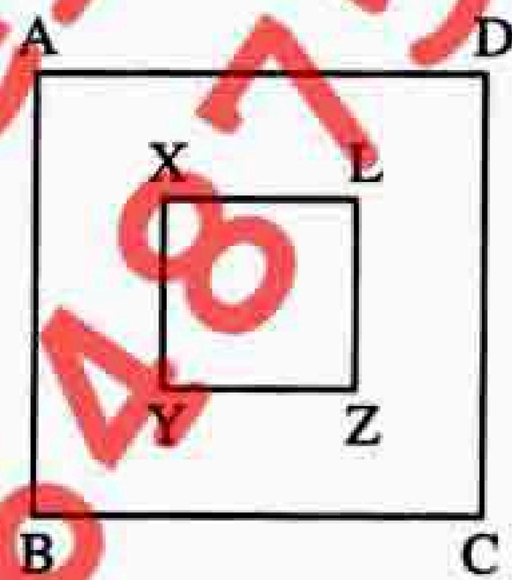
Homework

[A] : Choose The Correct Answer : -

1	$\overline{AB}$ is a line segment , then the number of the circles passing through the two points A , B is .... (a) 1 (b) 2 (c) 3 (d) infinite number.
2	M and N are two intersecting circles their radii are 5 cm. , 2 cm. , then MN $\in$ .... (a) $]3 , 7[$ (b) $[3 , 7]$ (c) $[3 , 7[$ (d) $]3 , 7]$
3	If the circle M $\cap$ the circle N = $\{A , B\}$, then the two circles M and N are .... (a) intersecting. (b) concentric. (c) touching externally. (d) distant.
4	If the two circles M and N are touching externally , the radius length of one of them is 5 cm. , and MN = 9 cm. , then the radius length of the other circle equals .... cm. (a) 4 (b) 5 (c) 9 (d) 14
5	The number of common tangents of two touching circles externally equals .... (a) 0 (b) 1 (c) 2 (d) 3
6	The sum of measures of the interior angles of the quadrilateral = ... (a) 90° (b) 180° (c) 270° (d) 360°
7	The number of the symmetry axes of square is .... (a) 1 (b) 2 (c) 3 (d) 4
8	**In the opposite figure :** ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, $\overline{AD}$ is a diameter of circle M , then the area of the shaded region is ....  (a) 70 cm^2 (b) 147 cm^2 (c) 170 cm^2 (d) 224 cm^2
9	If M is a circle of radius length r cm. , then the length of the semicircle = ... cm. (a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr
10	The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.  (a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$
11	The number of common tangents of two distant circles is .... (a) 1 (b) 2 (c) 3 (d) 4

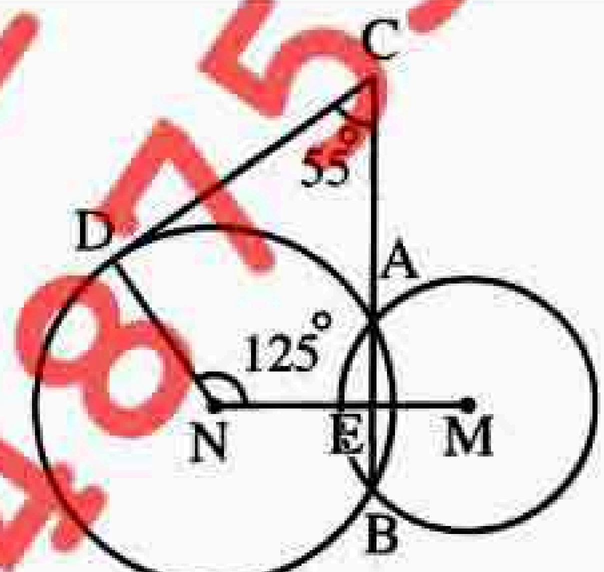
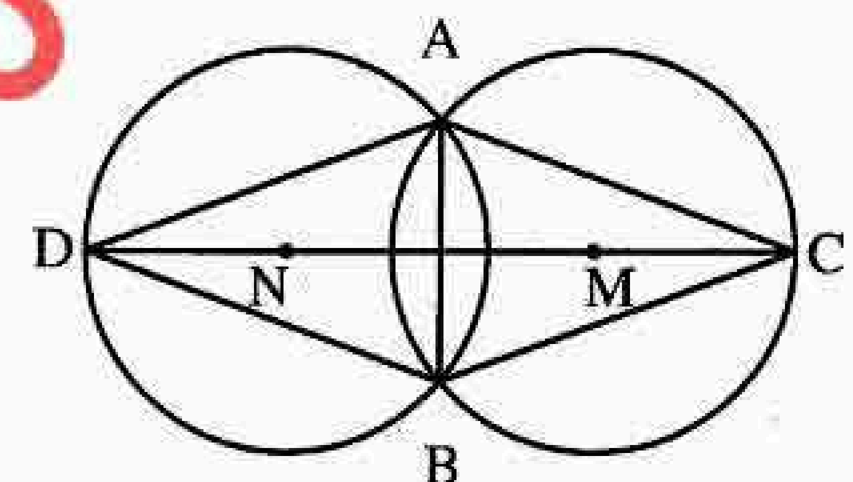
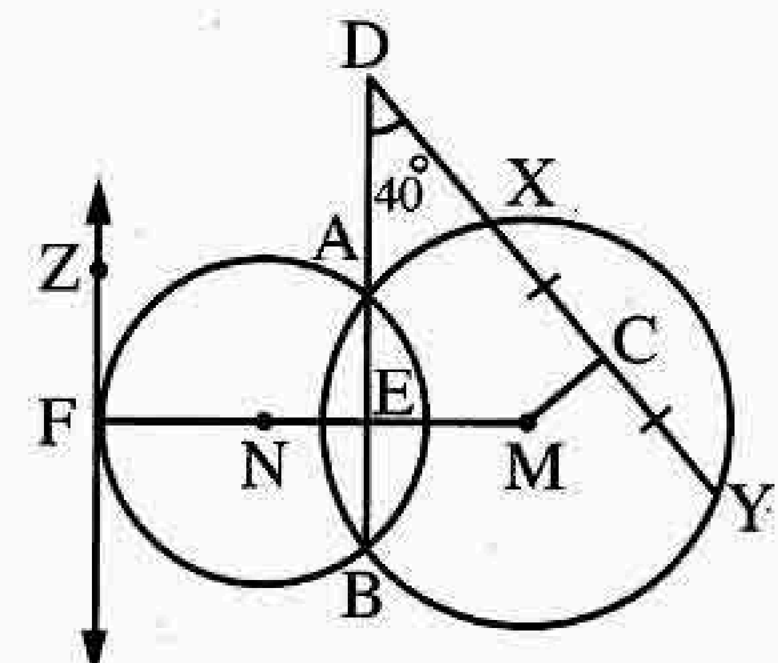
12	If A and B are two points in the plane , if AB = 4 cm. , then the smallest radius length of circle passing through by A and B is .... cm. (a) 2 (b) 3 (c) 4 (d) 5	
13	M and N are two intersecting circles the lengths of their radii are 3 cm. and 5 cm. , then MN $\in$ .... (a) [2 , 8] (b) [2 , 8[(c)]2 , 8] (d)]2 , 8[	
14	M and N are two circles of radii lengths 9 cm. , 4 cm. , MN = 5 cm. , then the two circles are .... (a) intersecting. (b) touching internally. (c) touching externally. (d) distant.	
15	If the two circles M and N are touching externally , their radii lengths are 9 cm. , r cm. , and MN = 14 cm. , then r = ... cm. (a) 5 (b) 7 (c) 10 (d) 23	
16	The perimeter of the square whose area is 81 cm^2 is .... (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.	
17	The area of a square whose diagonal length is 6 cm. equals .... cm^2 (a) 36 (b) 18 (c) 24 (d) 9	
18	The number of symmetric axes of the square is .... (a) 1 (b) 2 (c) 3 (d) 4	
19	A circle whose circumference 20 π cm. its area = ... π cm^2 (a) 10 (b) 100 (c) 200 (d) 400	
20	Number of circles passing through a given point .... (a) one circle. (b) two circles. (c) three circles. (d) infinite number of circles.	
21	The centres of all circles passing through the points A and B lie on .... (a) $\overline{AB}$ (b) midpoint of $\overline{AB}$ (c) the symmetry axis of $\overline{AB}$ (d) the perpendicular to $\overline{AB}$ from B	
22	M and N are two intersecting circles , r_1 = 3 cm. , r_2 = 5 cm. respectively , then MN $\in$ .... (a)]0 , 5[(b)]2 , 8[(c)]8 , ∞[(d)]2 , ∞[	
23	The surface of the circle M $\cap$ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. (a) 5 (b) 6 (c) 11 (d) 16	

24	If M and N are two circles , touching internally , the lengths of their radii are 3 cm. and 5 cm. , then MN = ... cm. (a) 8 (b) 2 (c) 3 (d) 5
25	A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2. (a) 3050 (b) 3500 (c) 2925 (d) 3250
26	A square of perimeter 20 cm. , then its area = ... cm^2. (a) 20 (b) 25 (c) 50 (d) 100
27	The longest chord in the circle is called .... (a) diameter. (b) tangent. (c) secant. (d) radius.
28	Circumference of a circle is 6π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is .... (a) a tangent to the circle. (b) a secant to the circle. (c) outside the circle. (d) the diameter to the circle.
29	The number of circles which can be drawn passes through the endpoints of a line segment $\overline{AB}$ equals .... (a) 1 (b) 2 (c) 3 (d) an infinite number.
30	One of the following statments identify one and only one circle , if we have .... (a) radius length and one of its points. (b) two of its points. (c) only one of its points. (d) its centre and one of its points.
31	Two circles M and N with radii lengths 8 cm. and 5 cm. respectively , are touching when MN $\in$ .... (a) $]13 , 3[$ (b) $]3 , 13[$ (c) $\mathbb{R} - [3 , 13]$ (d) $\{13 , 3\}$
32	The centre of the inscribed circle of any triangle is the point of intersection of its .... (a) altitudes. (b) medians. (c) axes of symmetry of its sides. (d) bisectors of its interior angles.
33	If M , N are two touching circles internally, their radii lengths are 5 cm. , 9 cm. , then MN = ... cm. (a) 14 (b) 4 (c) 5 (d) 9
34	The rhombus in which the lengths of its diagonals are L_1 and L_2 , its area = ... (a) $L_1 L_2$ (b) $L_1 + L_2$ (c) $2 L_1 L_2$ (d) $\frac{1}{2} L_1 L_2$
35	If the two circles M and N are touching internally , the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. (a) 5 (b) 6 (c) 11 (d) 12

36	The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is .... cm^2 (a) 2 (b) 14 (c) 24 (d) 48
37	The number of sides of the regular polygon in which the measure one of its interior angles 135° = ... sides. (a) 4 (b) 6 (c) 8 (d) 10
38	**In the opposite figure :** If the side length of the square ABCD = 7 cm. and the side length of the square XYZL = 3 cm. , then the area of the shaded part = ... cm^2 (a) (7 – 3) (b) 4 (7 – 3) (c) $(7-3)^2$ (d) (7^2-3^2) 
39	If the radius length of the circle M equals 2 cm. , then its circumference equals .... (a) 4 π cm. (b) 5 π cm. (c) 6 π cm. (d) 7 π cm.
40	If the area of the circle is 9 π cm^2 , then its radius length = ... cm. (a) 9 (b) 2 (c) (– 3) (d) 3
41	The number of circles that pass through three collinear points equals .... (a) zero. (b) one. (c) three. (d) infinite number.
42	The centre of the inscribed circle of any triangle is the intersection point .... (a) its medians. (b) its heights. (c) the symmetric axes of its sides. (d) bisectors of its interior angles.
43	If the surface of the circle M ⋂ the surface of the circle N = {A} , then the two circles M and N are .... (a) distant. (b) concentric. (c) touching externally. (d) intersecting.
44	The centre of the circumcircle of the triangle is the intersection point of its .... (a) altitudes of triangle. (b) medians of a triangle. (c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.
45	The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length unit. (a) 3 (b) 4 (c) 5 (d) 6
46	Number of the circles that pass through three non-collinear points equals .... (a) zero (b) one (c) three (d) an infinite number
47	The centre of the circumcircle of any triangle is the point of intersection of .... (a) the interior bisectors of its angles. (b) the exterior bisectors of its angles. (c) its heights. (d) the symmetric axes of its sides.

48	M and N are two circles their two radii lengths are 5 cm. and 3 cm. respectively. If MN = 8 cm. , then the two circles are .... (a) touching internally. (b) intersecting. (c) touching externally. (d) distant.
49	If the two circles M , N are touching externally , the radius length of the circle M is 4 cm. , if MN = 7 cm. then the circumference of the circle N is .... cm. (a) 4π (b) 6π (c) 7π (d) π
50	The diagonals are equal in length and not perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
51	The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm. equals .... cm^2. (a) 2 (b) 18 (c) 40 (d) 80
52	In a regular hexagon , the measure of the angle of its vertex equals .... (a) 60° (b) 108° (c) 120° (d) 135°
53	A circle its radius length is 5 cm. , then its circumference = ... cm. (a) 5π (b) 7π (c) 10π (d) 25π
54	**In the opposite figure :** ABCD is quadrilateral , m (∠ ABD) = 30° , m (∠ DCE) = 120° , then ABCD is .... (a) a rectangle. (b) a rhombus. (c) a cyclic quadrilateral. (d) a parallelogram.
55	If m_1 , m_2 are two slopes of two parallel straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$
56	If m_1 and m_2 are the slopes of two perpendicular straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$
57	The corresponding angles of the two similar polygons are .... in measure. (a) equal (b) different (c) proportional (d) alternate
58	If the area of the circle M = 16π cm^2, A is a point on its plane where MA = 8 cm. , then A is .... (a) outside the circle. (b) inside the circle. (c) on the circle. (d) on the centre of the circle.

[B] : Essay Problems : -

1	If A (2 , 0) and B (– 2 , 3) , draw a circle M of radius length 4 length units and passes through the two points A and B **How many solutions are there for this problem ?** *(North Sinai 2009)*
2	**In the opposite figure :** M and N are two intersecting circles at A and B , $C \in \overrightarrow{BA}$, D ∈ the circle N , m (∠ MND) = 125° and m (∠ BCD) = 55° **Prove that :** $\overleftrightarrow{CD}$ is a tangent to circle N at D *(Souhag 2014 , 2015)* 
3	**In the opposite figure :** M and N are two interacting circles at A , B where C is a point on the circle M , D is a point on the circle N $C \in \overleftrightarrow{MN}$, $D \in \overleftrightarrow{MN}$ **Prove that :** m (∠ CAD) = m (∠ CBD) *(El-Sharkia 2015)* 
4	**In the opposite figure :** M and N are two intersecting circles at A and B , C is the midpoint of $\overline{XY}$, m (∠ D) = 40° , $\overleftrightarrow{FZ}$ is a tangent to the circle N at F where $\overleftrightarrow{MN} \cap \overleftrightarrow{FZ} = \{F\}$ (1) **Find :** m (∠ CME) « 140° » (2) **Prove that :** $\overleftrightarrow{FZ}$ // $\overline{AB}$ *(El-Fayoum 2011)* 
5	**If A ∈ L , draw the circle M passing through A and its radius length = 3 cm. if :** (1) M ∈ the straight line L , how many circles can be drawn ? (2) M ∉ the straight line L , how many circles can be drawn ? *(Assiut 2011)*
6	Using geometrical instruments, draw the isosceles triangle ABC in which m (∠ ABC) = 120° , BC = 4 cm. Determine the centre of the circumcircle of it and find its radius length. *(El-Dakahlia 2011)* « 4 cm. »

Lesson [5] : The Relation Between The Chords Of A Circle With Its Center

Remark [1]

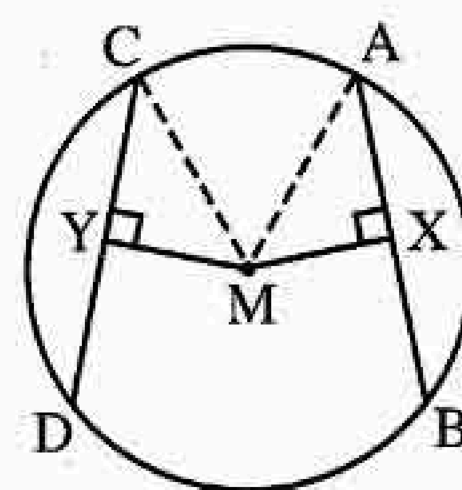
The closer the chord is from the centre of the circle , the longer its length is and vise versa.

i.e. There is a relation between the length of the chord and its distance from the centre of the circle.

The relation between the chords of a circle and its centre :

Theorem

If chords of a circle are equal in length, then they are equidistant from the centre.



Given | $AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{MY} \perp \overline{CD}$

R.T.P. | $MX = MY$

Construction | Draw $\overline{MA}$ and $\overline{MC}$

Proof | $\because \overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$

$\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$

$\because AB = CD$ (given) $\therefore AX = CY$

$\because \Delta\Delta$ AXM and CYM , both have $\begin{cases} AX = CY \text{ (by proof)} \\ MA = MC = r \\ m(\angle AXM) = m(\angle CYM) = 90^\circ \end{cases}$

$\therefore \Delta AXM \equiv \Delta CYM$, then we get : $MX = MY$ (Q.E.D.)

Corollary

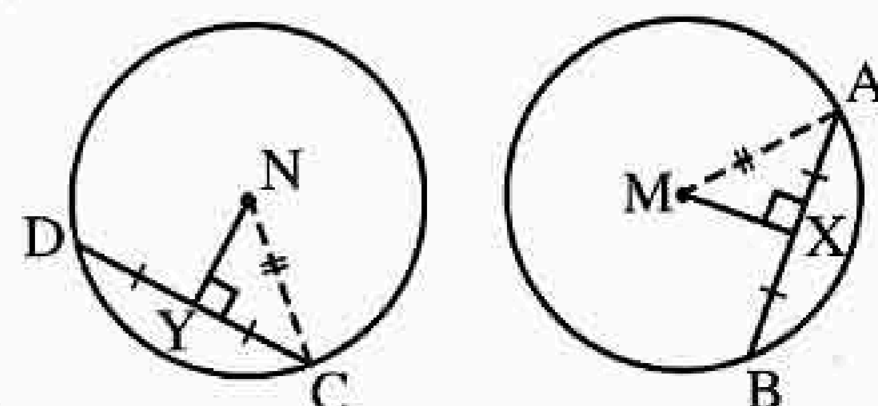
In congruent circles, chords which are equal in length are equidistant from the centres.

In the opposite figure :

If M and N are two congruent circles ,

$AB = CD$, $\overline{MX} \perp \overline{AB}$ and $\overline{NY} \perp \overline{CD}$,

then $MX = NY$



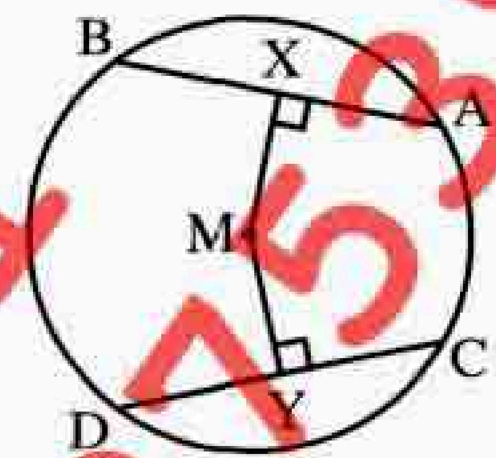
Converse of the theorem (without proof) :

In the same circle (or in congruent circles) ,
chords which are equidistant from the centre (s) are equal in length.

i.e. **In the opposite figure :**

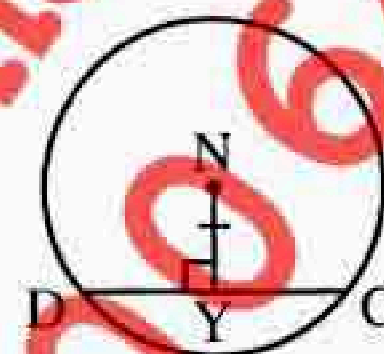
If $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

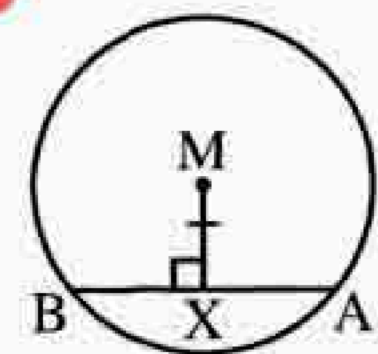
$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$ and MX = MY , then AB = CD



Also in the opposite figure :

If M and N are two congruent circles , $\overline{AB}$ is a chord of circle M and $\overline{CD}$ is a chord of circle N , $\overline{MX} \perp \overline{AB}$, $\overline{NY} \perp \overline{CD}$ and MX = NY , then AB = CD





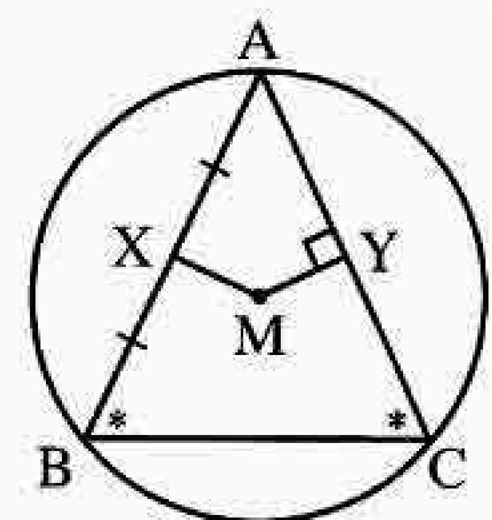
Examples :

1 **In the opposite figure :**

The triangle ABC is an inscribed triangle inside a circle M ,
m (∠ B) = m (∠ C) ,
X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

(Fayoum 2015 , Suez 2014 , Aswan 2011)

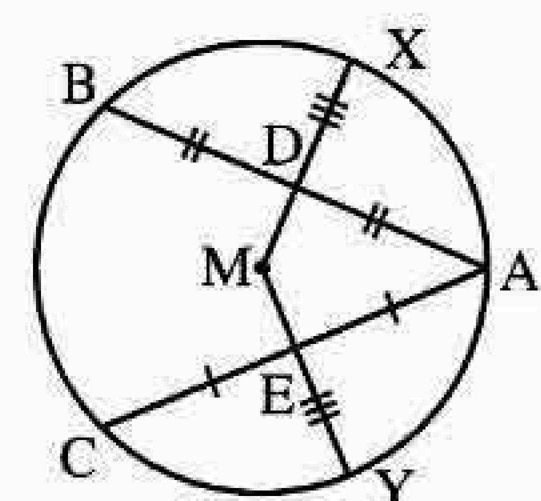


2 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M
, D is the midpoint of $\overline{AB}$
, E is the midpoint of $\overline{AC}$
, DX = EY

Prove that : AB = AC

(El-Kalyoubia 2016)



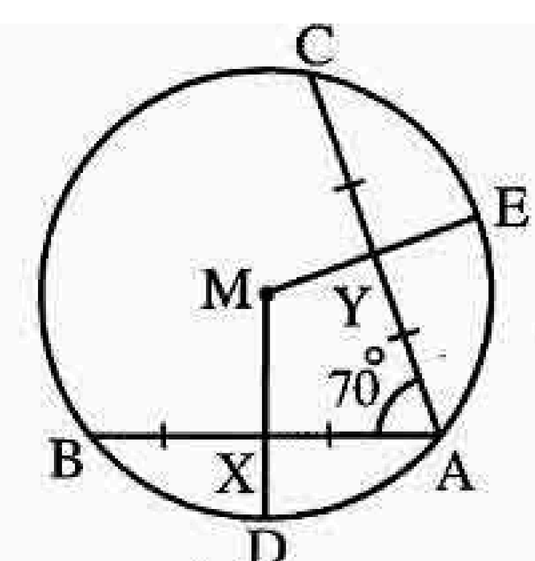
3 📖 **In the opposite figure :**

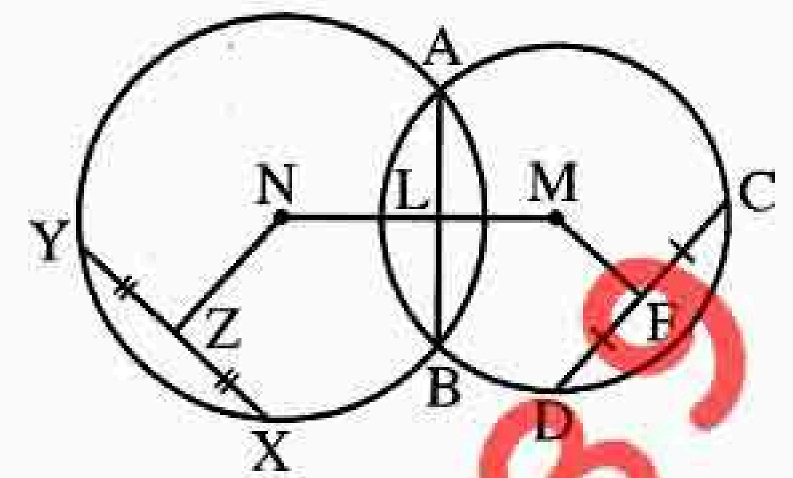
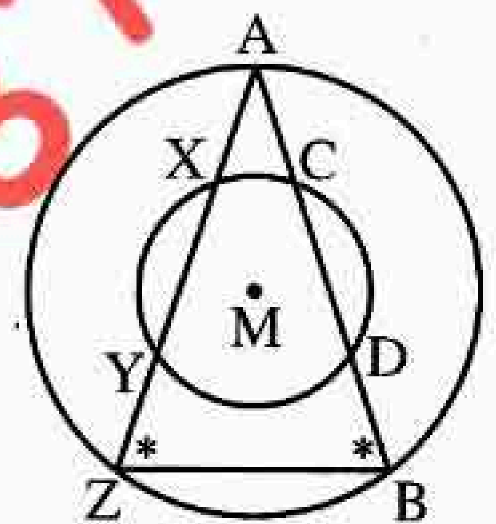
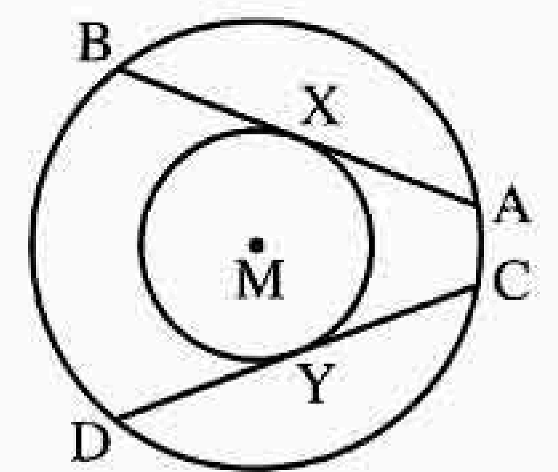
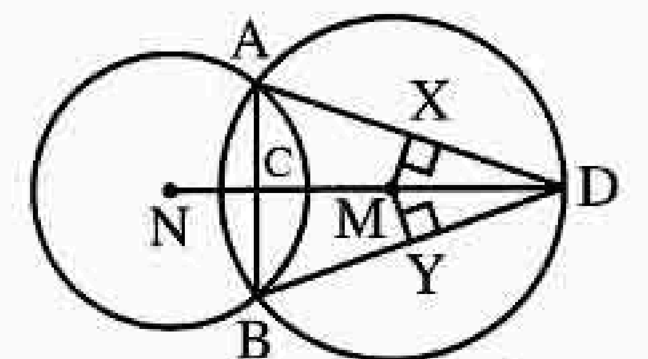
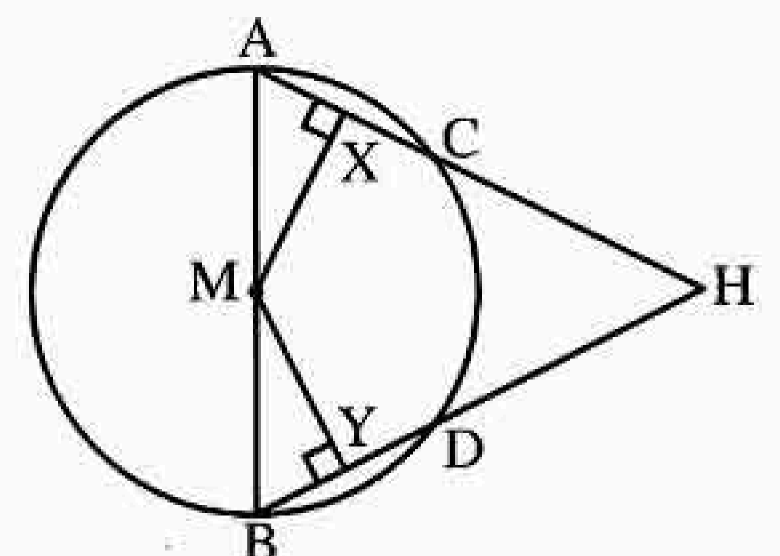
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M
, X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{AC}$ and m (∠ CAB) = 70°

(1) **Calculate :** m (∠ DME) « 110° »

(2) **Prove that :** XD = YE

(Aswan 2016 , Damietta 2013 , New Valley 2012)



4	**In the opposite figure :** M and N are two circles intersecting at A and B , $\overline{MN} \cap \overline{AB} = \{L\}$, F is the midpoint of $\overline{CD}$, Z is the midpoint of $\overline{XY}$, MF = ML and NL = NZ **Prove that :** CD = XY  *(Monofia 2009)*
5	**In the opposite figure :** Two concentric circles at M , $\overline{AB}$ is a chord in the greater circle and cuts the smaller circle at C and D , $\overline{AZ}$ is a chord in the greater circle and cuts the smaller circle at X and Y If m (∠ ABZ) = m (∠ AZB) **Prove that :** CD = XY  *(Souhag 2013)*
6	**In the opposite figure :** Two concentric circles at M , $\overline{AB}$ and $\overline{CD}$ are two chords of the greater circle and touch the smaller circle at X and Y respectively. **Prove that :** AB = CD if the radius length of the greater circle = 5 cm. and the radius length of the smaller circle = 3 cm. , find the length of $\overline{AB}$  *(Gharbia 2004)* « 8 cm. »
7	🕮 **In the opposite figure :** The circle M ∩ the circle N = {A , B} , $\overleftrightarrow{AB} \cap \overleftrightarrow{MN} = \{C\}$, D ∈ $\overleftrightarrow{MN}$, $\overline{MX} \perp \overline{AD}$ and $\overline{MY} \perp \overline{BD}$ **Prove that :** MX = MY  *(El-Sharkia 2011)*
8	**In the opposite figure :** $\overline{AB}$ is a diameter of the circle M , $\overline{AC}$ and $\overline{BD}$ are two chords in it , MX = MY , $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{DB}$ **Prove that :** (1) Δ HAB is isosceles triangle. (2) HC = HD  *(Beni-Suef 2012)*

Solutions

1

In Δ ABC : $\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

, $\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ $\overline{MY} \perp \overline{AC}$, AB = AC $\because$ MX = MY (Q.E.D.)

2

$\because$ MX = MY (lengths of two radii)

, DX = EY $\therefore$ MD = ME

$\because$ D is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MD} \perp \overline{AB}$

$\because$ E is the midpoint of $\overline{AC}$ $\therefore$ $\overline{ME} \perp \overline{AC}$

$\therefore$ AB = AC (Q.E.D.)

3

$\because$ X is the midpoint of AB $\therefore$ MX $\perp$ AB

$\because$ Y is the midpoint of $\overline{AC}$ $\therefore$ $\overline{MY} \perp \overline{AC}$

$\because$ The sum of measures of the interior angles of the quadrilateral AXMY = 360°

$\therefore$ m ($\angle$ XMY) = 360° − (70° + 90° + 90°) = 110° (First req.)

$\because$ AB = AC $\therefore$ MX = MY

$\because$ MD = ME (lengths of two radii)

by subtracting $\therefore$ XD = YE (Second req.)

4

$\because$ F is the midpoint of $\overline{CD}$ $\therefore$ $\overline{MF} \perp \overline{CD}$

$\because$ $\overline{AB}$ is the common chord , $\overleftrightarrow{MN}$ is the line of centres

$\therefore$ $\overline{MN} \perp \overline{AB}$ $\because$ MF = ML

$\therefore$ AB = CD , similarly in the circle N then AB = XY

$\therefore$ CD = XY (Q.E.D.)

5

Constr. :

Draw : $\overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$

Proof : In the great circle :

$\because$ m ($\angle$ ABZ) = m ($\angle$ AZB)

$\therefore$ AB = AZ

$\because$ $\overline{MF} \perp \overline{AB}$, $\overline{ME} \perp \overline{AZ}$ $\therefore$ MF = ME

In the small circle :

$\because$ $\overline{MF} \perp \overline{CD}$, $\overline{ME} \perp \overline{XY}$, MF = ME

$\therefore$ CD = XY (Q.E.D.)

6

Constr. :

Draw : $\overline{MX}$, $\overline{MY}$, $\overline{MA}$

Proof :

$\because$ $\overleftrightarrow{AB}$ is a tangent to the small circle at X

$\therefore$ $\overline{MX} \perp \overline{AB}$ similarly $\overline{MY} \perp \overline{CD}$

$\because$ MX = MY = r (for the small circle)

$\therefore$ AB = CD (First req.)

From Δ AMX which is right-angled at X

$\therefore$ $(AX)^2 = (AM)^2 - (MX)^2 = 25 - 9 = 16$

$\therefore$ AX = 4 cm.

From the great circle : $\because$ $\overline{MX} \perp \overline{AB}$

$\therefore$ X is the midpoint of $\overline{AB}$

$\therefore$ AB = 2 × 4 = 8 cm. (Second req.)

7

$\because$ The circle M $\cap$ the circle N = {A , B}

$\therefore$ $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ In Δ ABD : $\overleftrightarrow{DC}$ is the axis of symmetry of $\overline{AB}$

$\therefore$ AD = BD

$\because$ $\overline{MX} \perp \overline{AD}$, $\overline{MY} \perp \overline{BD}$ $\therefore$ MX = MY (Q.E.D.)

8

$\because$ Δ MXA and Δ MYB which are right-angled triangles

In them $\begin{cases} MA = MB \\ MX = MY \end{cases}$

$\therefore$ The two triangles are congruent , then we deduce that :

m ($\angle$ MAX) = m ($\angle$ MBY)

$\therefore$ Δ HAB is an isosceles triangle. (Q.E.D. 1)

, $\because$ $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{BD}$

, MX = MY

$\therefore$ AC = BD , $\because$ AH = BH

$\therefore$ AH − AC = BH − BD $\therefore$ HC = HD (Q.E.D. 2)

Exercises

[A] : Choose The Correct Answer : -

1	A chord of length 8 cm. in a circle with diameter of length 10 cm. , then the chord at distance from its centre equals .... (a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.
2	$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle M , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively , MX = 3 cm. , then MY = ... cm. (a) 3 (b) 6 (c) $\frac{3}{2}$ (d) 4
3	**In the opposite figure :** AB = CD , $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$, then EX .... YF (a) < (b) = (c) > (d) ≠
4	**In the opposite figure :** D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$, m (∠ A) = 55° , then m (∠ DMH) = ... (a) 120° (b) 130° (c) 135° (d) 125°
5	**In the opposite figure :** MX = MY , m (∠ B) = 50° , then m (∠ A) = ... (a) 50° (b) 60° (c) 70° (d) 80°
6	**In the opposite figure :** m ($\widehat{AC}$) = 50° , $\overline{AB}$ // $\overline{CD}$, then the value of y = ... (a) 5° (b) 10° (c) 15° (d) 20°
7	If m_1 , m_2 are two slopes of two parallel straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$
8	If m_1 and m_2 are the slopes of two perpendicular straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$

9 The corresponding angles of the two similar polygons are .... in measure.

(a) equal (b) different (c) proportional (d) alternate

10 If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

11 The angle whose measure is 50° complements an angle of measure ....

(a) 90° (b) 130° (c) 50° (d) 40°

12 The sum of measures of the accumulative angles at a point = ...°

(a) 80 (b) 120 (c) 360 (d) 630

13 The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units.

(a) – 10 (b) 10 (c) 2 (d) 24

14 If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)

15 If the projection of a line segment on a straight line is a point , then the line segment .... the straight line.

(a) // (b) ⊥ (c) ∈ (d) ⊂

16 The image of the point (A , B) by rotation R (0 , 180°) the point ....

(a) (– A , B) (b) (– A , – B) (c) (A , – B) (d) (A , B)

17 The image of the point (2 , 3) by rotation R (O , 180°) is the point ....

(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

18 The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base.

(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3

19 The sum of lengths of any two sides of a triangle .... the length of the third side.

(a) < (b) > (c) = (d) ≤

20 If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then m (∠ X) = ...°

(a) 15 (b) 30 (c) 45 (d) 60

21 A circle its radius length is 5 cm. , then its circumference = ... cm.

(a) 5 π (b) 7 π (c) 10 π (d) 25 π

22 If M is a circle of radius length r cm. , then the length of the semicircle = ... cm.

(a) 2 π r (b) $\frac{1}{4}$ π r (c) $\frac{1}{2}$ π r (d) π r

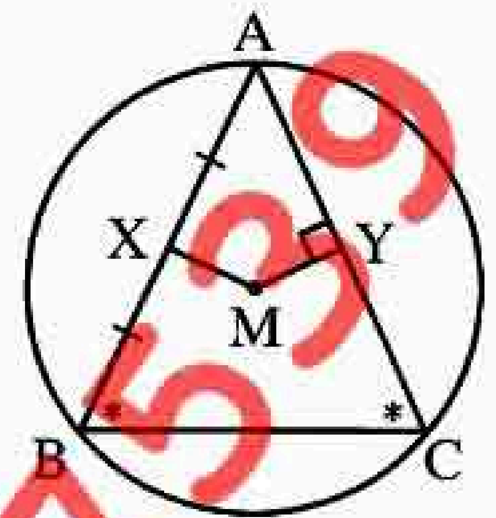
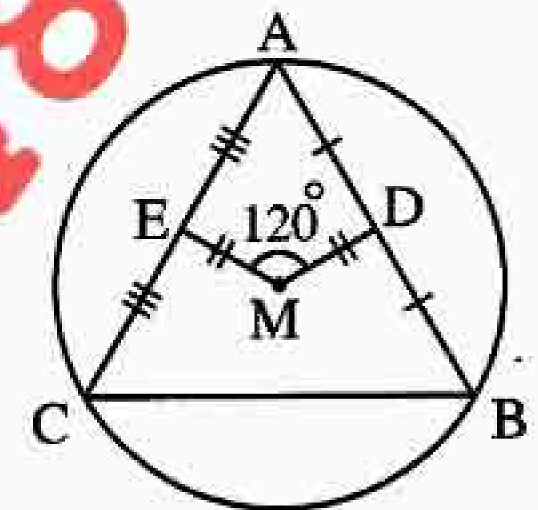
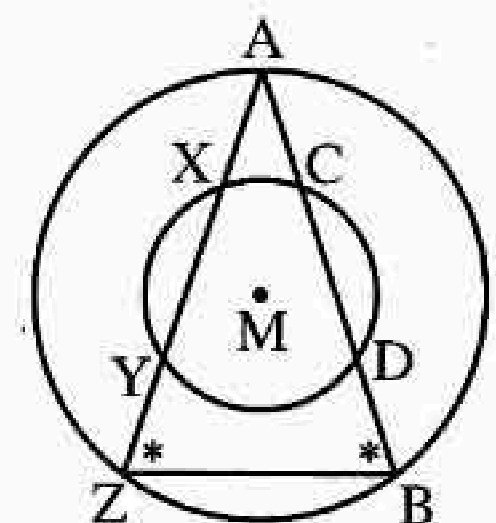
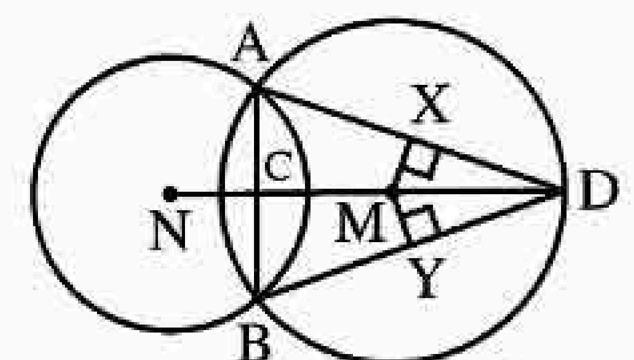
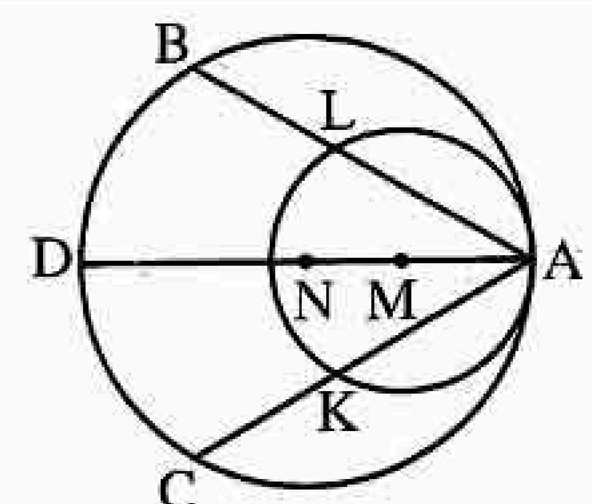
23 A circle whose circumference 20 π cm. its area = ... π cm².

(a) 10 (b) 100 (c) 200 (d) 400

24	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
25	The two angles A and C in the right-angled triangle at B are .... (a) complementary. (b) supplementary. (c) adjacent. (d) vertically opposite angles.
26	ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm. , then its area = ... cm^2. (a) 48 (b) 14 (c) 24 (d) 7
27	Δ XYZ is right-angled triangle at Y , then XZ .... YZ (a) < (b) > (c) = (d) twice
28	ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° , then m (∠ A) = ... (a) 40° (b) 50° (c) 90° (d) 130°
29	ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and x cm. , then x = ... cm. (a) 5 (b) 8 (c) 10 (d) 12
30	ABC is a triangle in which AB = AC , m (∠ C) = 40° , then m (∠ A) = ... (a) 40° (b) 80° (c) 100° (d) 120°
31	If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is .... (a) 40° (b) 90° (c) 45° (d) 80°
32	The medians of triangle intersect at a same point which divides each in the ratio .... from its base. (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2
33	The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2. (a) 6 (b) 10 (c) 30 (d) 60
34	The numbers 5 , 4 , ... can be side lengths of a triangle. (a) 8 (b) 9 (c) 10 (d) 12
35	If the side length of a rhombus is L cm. , then its perimeter = ... cm. (a) L^2 (b) $2 L^2$ (c) 4 L (d) $2\sqrt{2}$ L
36	If the radius length of the circle M equals 2 cm. , then its circumference equals .... (a) 4 π cm. (b) 5 π cm. (c) 6 π cm. (d) 7 π cm.

37	The number of the axes of symmetry in the equilateral triangle = ... (a) 1 (b) 2 (c) 3 (d) an infinite number.
38	**In the opposite figure :** AB = AC , AB = 2 x – 1 and AC = x + 2 , then x = ... (a) 3 (b) 5 (c) 11 (d) 14
39	**In the opposite figure :** ABC is right-angled triangle at B , m (∠ C) = 30° , AB = 3 cm. , then AC = ... cm. (a) 2 (b) 3 (c) $3\sqrt{3}$ (d) 6
40	**In the opposite figure :** CA = CB , $\overline{CX} \perp \overline{AB}$, AB = 2 CX , then m (∠ A) = ... (a) 30° (b) 60° (c) 90° (d) 45°
41	**In the opposite figure :** ABC is a triangle , AB = AC , m (∠ B) = 50° , then m (∠ A) = ... (a) 100° (b) 90° (c) 80° (d) 70°
42	**In the opposite figure :** The length of $\overline{AB}$ = ... cm. (a) $10\sqrt{3}$ (b) 10 (c) 5 (d) $5\sqrt{3}$
43	The diagonals are equal in length and not perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
44	The sum of measures of the interior angles of the quadrilateral = ... (a) 90° (b) 180° (c) 270° (d) 360°
45	The perimeter of the square whose area is 81 cm^2 is .... (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.

[B] : Essay Problems : -

1	**In the opposite figure :** The triangle ABC is an inscribed triangle inside a circle M , $m(\angle B) = m(\angle C)$, X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{AC}$ **Prove that :** MX = MY *(Fayoum 2015 , Suez 2014 , Aswan 2011)* 
2	**In the opposite figure :** Δ ABC is inscribed in the circle M , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ If DM = EM , $m(\angle DME) = 120°$ **Prove that :** Δ ABC is an equilateral triangle. *(Menia 2003)* 
3	**In the opposite figure :** Two concentric circles at M , $\overline{AB}$ is a chord in the greater circle and cuts the smaller circle at C and D , $\overline{AZ}$ is a chord in the greater circle and cuts the smaller circle at X and Y If $m(\angle ABZ) = m(\angle AZB)$ **Prove that :** CD = XY *(Souhag 2013)* 
4	🕮 **In the opposite figure :** The circle M $\cap$ the circle N = {A , B} , $\overleftrightarrow{AB} \cap \overleftrightarrow{MN} = \{C\}$, D $\in \overleftrightarrow{MN}$, $\overline{MX} \perp \overline{AD}$ and $\overline{MY} \perp \overline{BD}$ **Prove that :** MX = MY *(El-Sharkia 2011)* 
5	**In the opposite figure :** M and N are two circles touching internally at A , $\overline{AB}$ and $\overline{AC}$ are two chords drawn in the greater circle N such that they are equal in length to cut the smaller circle M at L and K respectively. **Prove that :** AL = AK *(Dakahlia 2009)* 

6

In the opposite figure :

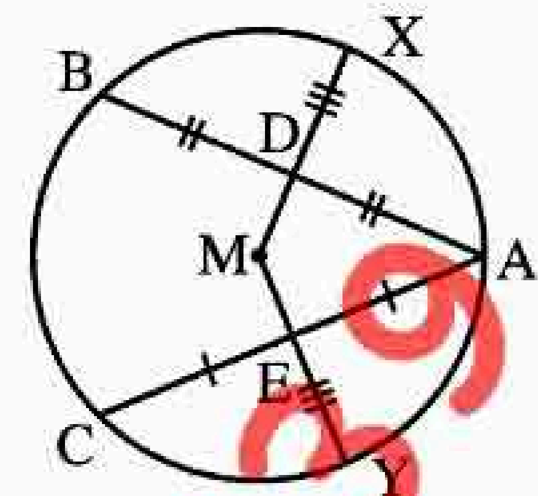
$\overline{AB}$ and $\overline{AC}$ are two chords in the circle M

, D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

, DX = EY

Prove that : AB = AC



(El-Kalyoubia 2016)

7

$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

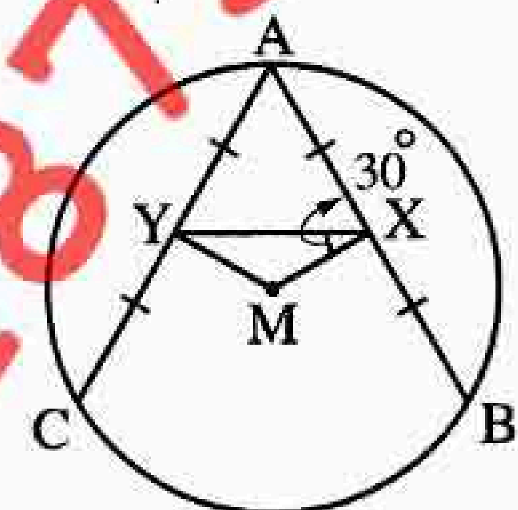
, X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, m (∠ MXY) = 30°

Prove that :

(1) Δ MXY is an isosceles triangle.

(2) Δ AXY is an equilateral triangle.



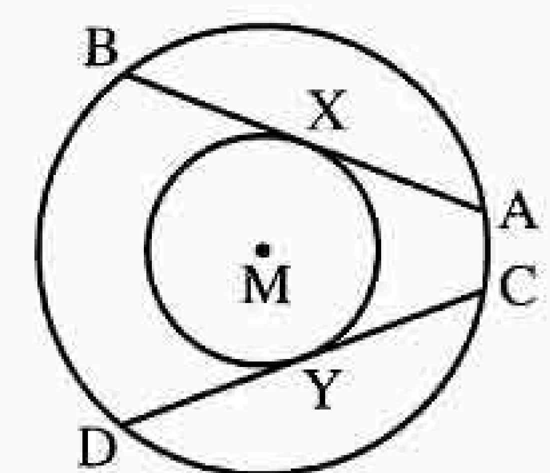
(New Valley 2016)

8

In the opposite figure :

Two concentric circles at M , $\overline{AB}$ and $\overline{CD}$ are two chords of the greater circle and touch the smaller circle at X and Y respectively.

Prove that : AB = CD if the radius length of the greater circle = 5 cm. and the radius length of the smaller circle = 3 cm. , find the length of $\overline{AB}$



(Gharbia 2004) « 8 cm. »

9

In the opposite figure :

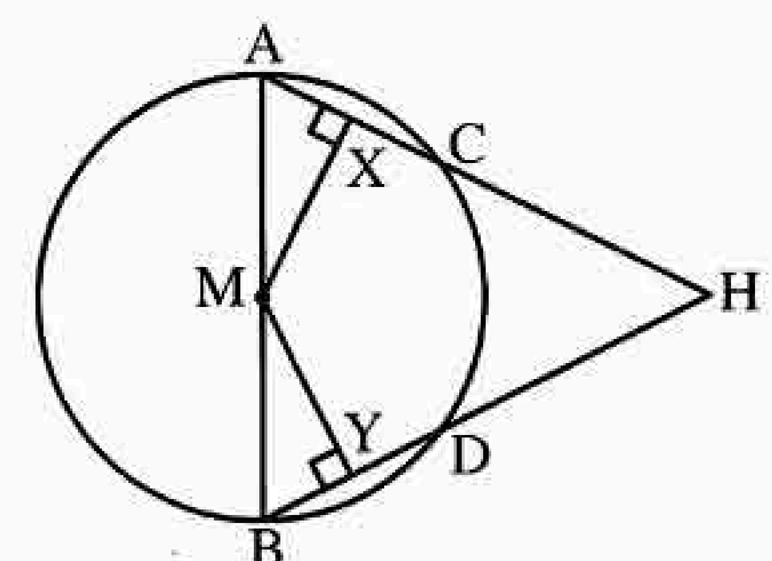
$\overline{AB}$ is a diameter of the circle M , $\overline{AC}$ and $\overline{BD}$ are two chords in it ,

MX = MY , $\overline{MX} \perp \overline{AC}$, $\overline{MY} \perp \overline{DB}$

Prove that :

(1) Δ HAB is isosceles triangle.

(2) HC = HD



(Beni-Suef 2012)

10

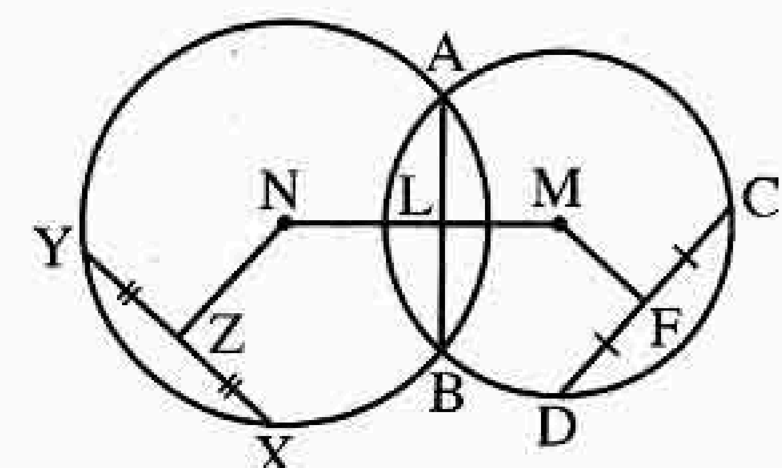
In the opposite figure :

M and N are two circles intersecting at A and B ,

$\overline{MN} \cap \overline{AB} = \{L\}$, F is the midpoint of $\overline{CD}$,

Z is the midpoint of $\overline{XY}$, MF = ML and NL = NZ

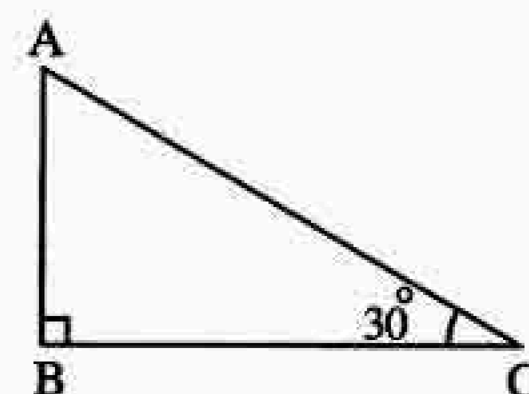
Prove that : CD = XY



(Monofia 2009)

Homework

[A] : Choose The Correct Answer : -

1	A chord of length 8 cm. in a circle with diameter of length 10 cm. , then the chord at distance from its centre equals .... (a) 2 cm. (b) 4 cm. (c) 3 cm. (d) 6 cm.
2	If m_1 , m_2 are two slopes of two parallel straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$
3	If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is .... (a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)
4	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
5	If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is .... (a) 40° (b) 90° (c) 45° (d) 80°
6	**In the opposite figure :** ABC is right-angled triangle at B , m (∠ C) = 30° , AB = 3 cm. , then AC = ... cm.  (a) 2 (b) 3 (c) $3\sqrt{3}$ (d) 6
7	A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2. (a) 3050 (b) 3500 (c) 2925 (d) 3250
8	The perimeter of the square whose area is 81 cm^2 is .... (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.
9	A square of perimeter 20 cm. , then its area = ... cm^2. (a) 20 (b) 25 (c) 50 (d) 100
10	The number of the axes of symmetry in the equilateral triangle = ... (a) 1 (b) 2 (c) 3 (d) an infinite number.
11	The sum of measures of the interior angles of the quadrilateral = ... (a) 90° (b) 180° (c) 270° (d) 360°

12 The area of a square whose diagonal length is 6 cm. equals .... cm^2.

(a) 36 (b) 18 (c) 24 (d) 9

13 If the radius length of the circle M equals 2 cm. , then its circumference equals ....

(a) 4π cm. (b) 5π cm. (c) 6π cm. (d) 7π cm.

14 **In the opposite figure :**

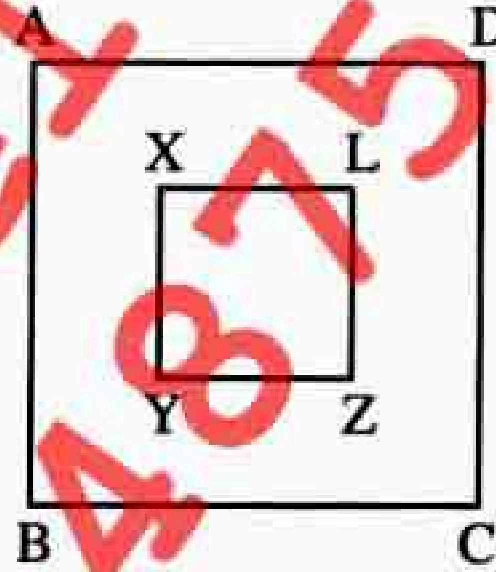
If the side length of the square ABCD = 7 cm.

and the side length of the square XYZL = 3 cm.

, then the area of the shaded part = ... cm^2.

(a) (7 – 3) (b) 4 (7 – 3)

(c) $(7-3)^2$ (d) (7^2-3^2)



15 The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length unit.

(a) 3 (b) 4 (c) 5 (d) 6

16 If the area of the circle M = 16π cm^2. , A is a point on its plane where MA = 8 cm. , then A is ....

(a) outside the circle. (b) inside the circle.

(c) on the circle. (d) on the centre of the circle.

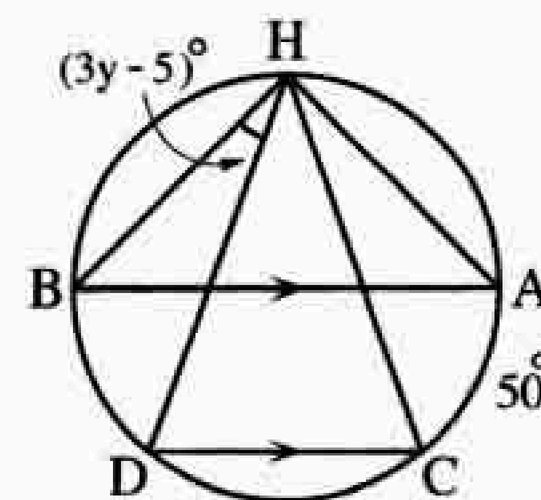
17 **In the opposite figure :**

m $(\overset{\frown}{AC})$ = 50° , $\overline{AB}$ // $\overline{CD}$

, then the value of y = ...

(a) 5° (b) 10°

(c) 15° (d) 20°



18 The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units.

(a) – 10 (b) 10 (c) 2 (d) 24

19 If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then m ($\angle X$) = ...°

(a) 15 (b) 30 (c) 45 (d) 60

20 ABC is a triangle in which AB = AC , m ($\angle$ C) = 40° , then m ($\angle$ A) = ...

(a) 40° (b) 80° (c) 100° (d) 120°

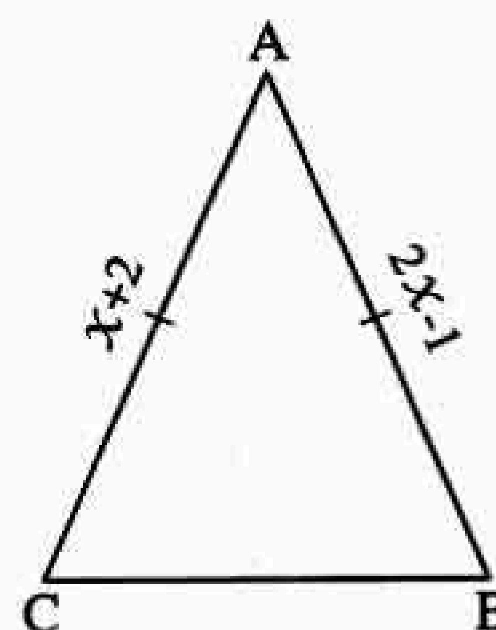
21 **In the opposite figure :**

AB = AC , AB = $2X - 1$ and AC = $X + 2$

, then X = ...

(a) 3 (b) 5

(c) 11 (d) 14



22	**In the opposite figure :** ABCD is quadrilateral , m (∠ ABD) = 30° , m (∠ DCE) = 120° , then ABCD is .... (a) a rectangle. (b) a rhombus. (c) a cyclic quadrilateral. (d) a parallelogram.
23	**In the opposite figure :** MX = MY , m (∠ B) = 50° , then m (∠ A) = ... (a) 50° (b) 60° (c) 70° (d) 80°
24	The sum of measures of the accumulative angles at a point = ...° (a) 80 (b) 120 (c) 360 (d) 630
25	The sum of lengths of any two sides of a triangle .... the length of the third side. (a) < (b) > (c) = (d) ≤
26	ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and X cm. , then X = ... cm. (a) 5 (b) 8 (c) 10 (d) 12
27	The longest chord in the circle is called .... (a) diameter. (b) tangent. (c) secant. (d) radius.
28	If the area of the circle is 9π cm^2 , then its radius length = ... cm. (a) 9 (b) 2 (c) (– 3) (d) 3
29	**In the opposite figure :** D is the midpoint of $\overline{AB}$, H is the midpoint of $\overline{AC}$, m (∠ A) = 55° , then m (∠ DMH) = ... (a) 120° (b) 130° (c) 135° (d) 125°
30	The angle whose measure is 50° complements an angle of measure .... (a) 90° (b) 130° (c) 50° (d) 40°
31	The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base. (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3
32	ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° , then m (∠ A) = ... (a) 40° (b) 50° (c) 90° (d) 130°

33 If the side length of a rhombus is L cm. , then its perimeter = ... cm.

(a) L^2 (b) $2L^2$ (c) 4 L (d) $2\sqrt{2}\,L$

34 The diagonals are equal in length and not perpendicular in ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

35 The number of the symmetry axes of square is ....

(a) 1 (b) 2 (c) 3 (d) 4

36 The number of symmetric axes of the square is ....

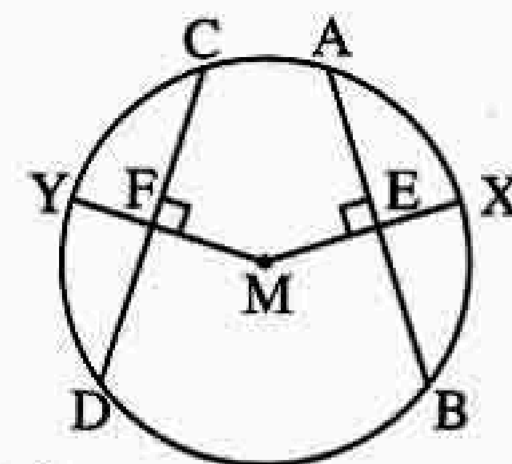
(a) 1 (b) 2 (c) 3 (d) 4

37 Circumference of a circle is 6 π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is ....

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) the diameter to the circle.

38 **In the opposite figure :**



AB = CD , $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$

, then EX .... YF

(a) < (b) = (c) > (d) ≠

39 If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

40 The image of the point (2 , 3) by rotation R (O , 180°) is the point ....

(a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)

41 Δ XYZ is right-angled triangle at Y , then XZ .... YZ

(a) < (b) > (c) = (d) twice

42 The numbers 5 , 4 , ... can be side lengths of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

43 In a regular hexagon , the measure of the angle of its vertex equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

44 If M is a circle of radius length r cm. , then the length of the semicircle = ... cm.

(a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr

45 A circle its radius length is 5 cm. , then its circumference = ... cm.

(a) 5 π (b) 7 π (c) 10 π (d) 25 π

46 If m_1 and m_2 are the slopes of two perpendicular straight lines , then ....

(a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$

[B] : Essay Problems : -

1

📖 $\overline{AB}$ and $\overline{AC}$ are two chords in the circle M , $\overline{MX} \perp \overline{AB}$, Y is the midpoint of $\overline{AC}$,
m (∠ ABC) = 75° , MX = MY

(1) **Find :** m (∠ BAC) « 30° »

(2) **Prove that :** The perimeter of Δ AXY = $\frac{1}{2}$ the perimeter of Δ ABC *(Alexandria 2016)*

2

In the opposite figure :

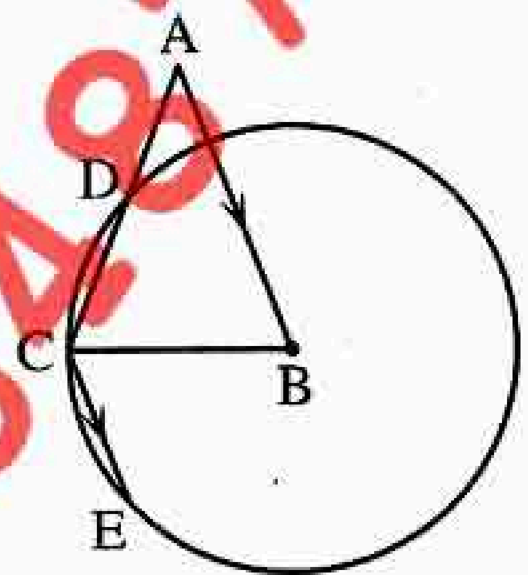
ABC is a triangle in which AB = AC

, a circle of centre B is drawn such that its radius length is $\overline{BC}$,

the circle cuts $\overline{AC}$ at D

Draw $\overline{CE}$ // $\overline{AB}$

Prove that : CE = CD *(Alex. 2009)*



3

📖 **In the opposite figure :**

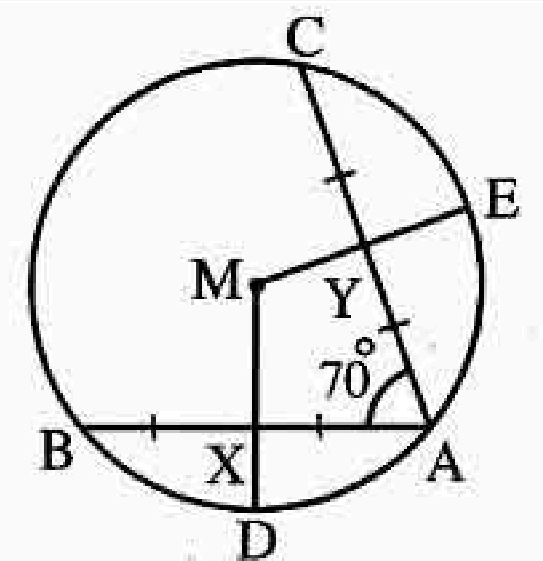
$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$ and m (∠ CAB) = 70°

(1) **Calculate :** m (∠ DME) « 110°»

(2) **Prove that :** XD = YE *(Aswan 2016 , Damietta 2013 , New Valley 2012)*



4

📖 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords of the circle M ,

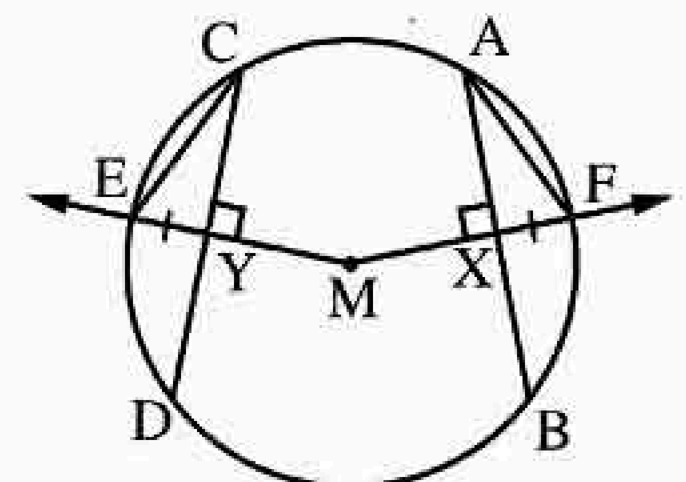
$\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F ,

$\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E ,

FX = EY

Prove that : (1) AB = CD

(2) AF = CE *(El-Gharbia 2016 , Cairo , Kafr El-Sheikh 2011)*



5

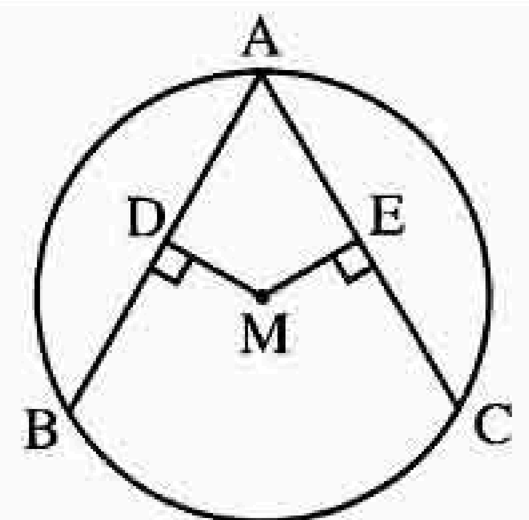
In the opposite figure :

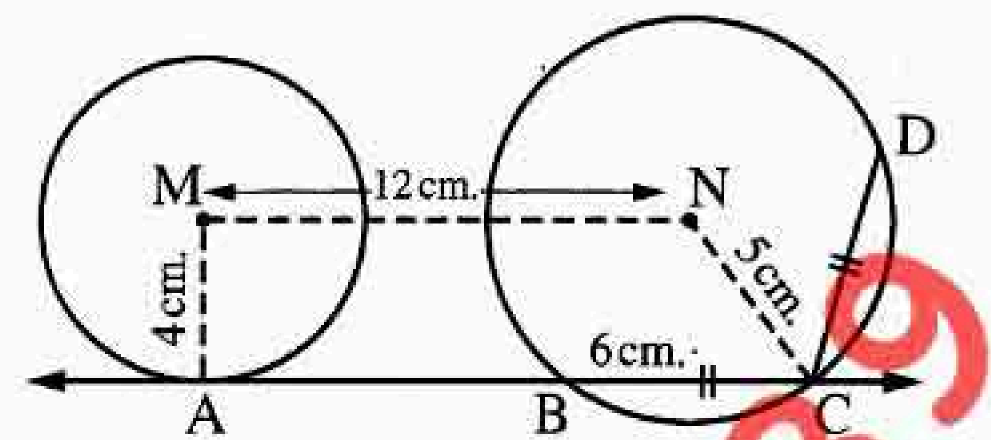
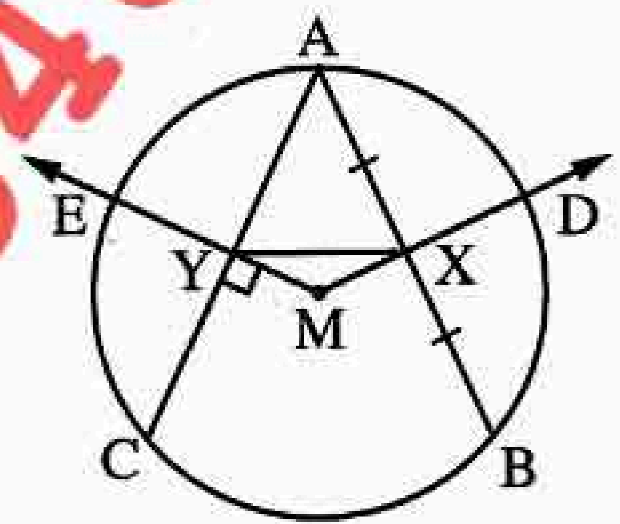
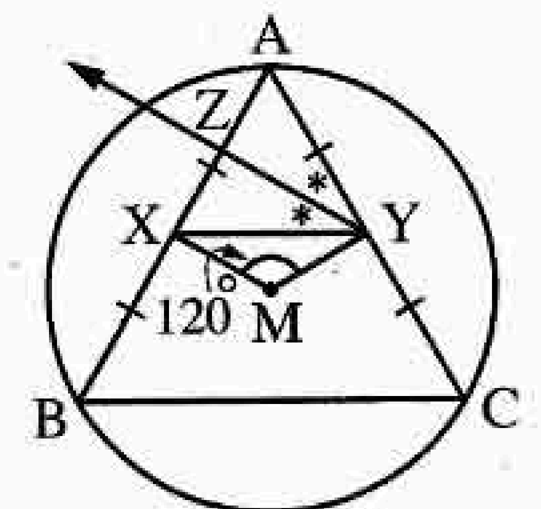
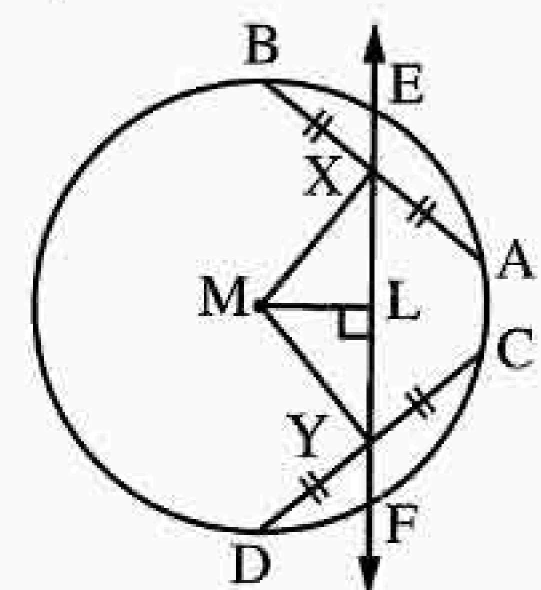
M is a circle , $\overline{MD} \perp \overline{AB}$

, $\overline{ME} \perp \overline{AC}$

, A (2 , 2) , D (1 , 0) and E (3 , 4)

Prove that : ME = MD *(Kafr El-Sheikh 2013)*



6	**In the opposite figure :** M and N are two circles of radii lengths 4 cm. and 5 cm. , $\overline{AC}$ touches the circle M at A and cuts the circle N at B and C , where BC = 6 cm. and MN = 12 cm.  (1) Prove that the quadrilateral MACN is a trapezium then calculate its area. *« 54 cm² »* (2) If CD = CB , find the distance between N and $\overline{CD}$ *(Sharkia 2006) « 4 cm. »*
7	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D , $\overrightarrow{MY} \perp \overline{AC}$ intersects it at Y and intersects the circle at E  **Prove that :** (1) XD = YE (2) m (∠YXB) = m (∠ XYC) *(El-Gharbia 2013)*
8	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two chords of the circle M , equal in length X and Y are their midpoints respectively If m (∠ XMY) = 120° , $\overrightarrow{YZ}$ bisects ∠ AYX  **Prove that :** $\overrightarrow{YZ}$ // $\overleftrightarrow{MX}$ *(Cairo 2008)*
9	**In the opposite figure :** $\overline{AB}$ and $\overline{CD}$ are two chords of the circle M , equal in length , X and Y are the two midpoints of $\overline{AB}$ and $\overline{CD}$ respectively. $\overleftrightarrow{XY}$ is drawn to cut the circle at E and F , $\overline{ML}$ is drawn $\perp$ $\overleftrightarrow{XY}$  **Prove that :** XE = YF *(Cairo 2003)*

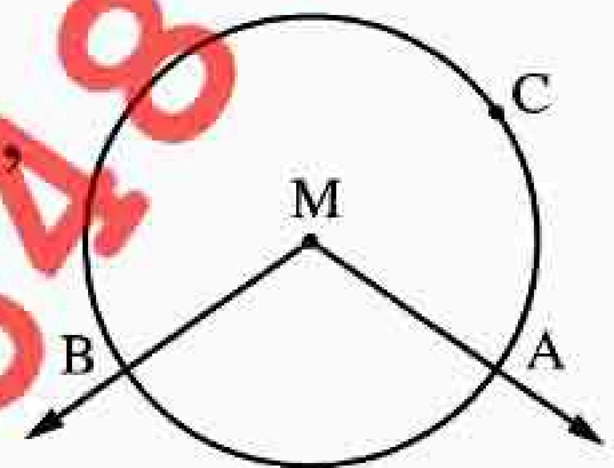
Prep [3] - Second Term - Geometry - Unit [5] - Angles and Arcs in the circle

Lesson [1] : Central Angle And Measuring Arcs

The central angle

It is the angle whose vertex is the centre of the circle and the two sides contain two radii in the circle.

In the opposite figure :



- ∠ AMB is a central angle because its vertex M is the centre of the circle , and its sides $\overrightarrow{MA}$ and $\overrightarrow{MB}$ contain the two radii $\overline{MA}$ and $\overline{MB}$

Notice that :

The two sides of ∠ AMB divide the circle M into two arcs they are :

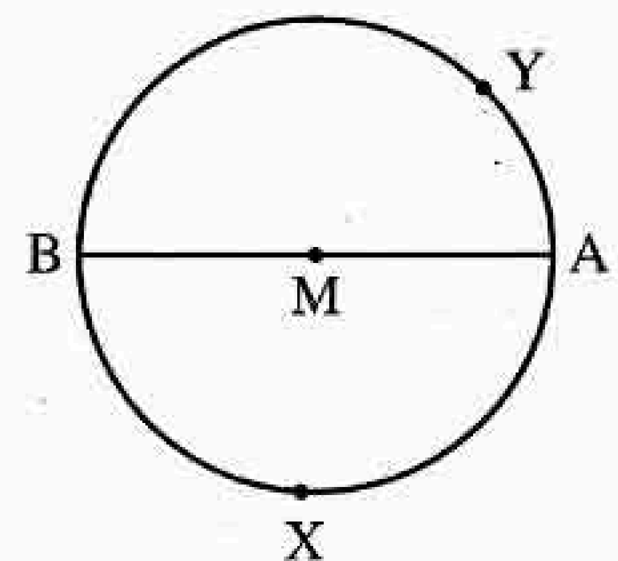
- The minor arc AB and it is denoted by $\overset{\frown}{AB}$
- The major arc AB and it is denoted by $\overset{\frown}{ACB}$ or the major arc $\overset{\frown}{AB}$

Notice that :

The symbol $\overset{\frown}{AB}$ means the minor arc unless there is other stating.

Remark

If $\overline{AB}$ is a diameter of the circle M , then :



∠ AMB is a central straight angle ,

then each of $\overset{\frown}{AXB}$ and $\overset{\frown}{AYB}$

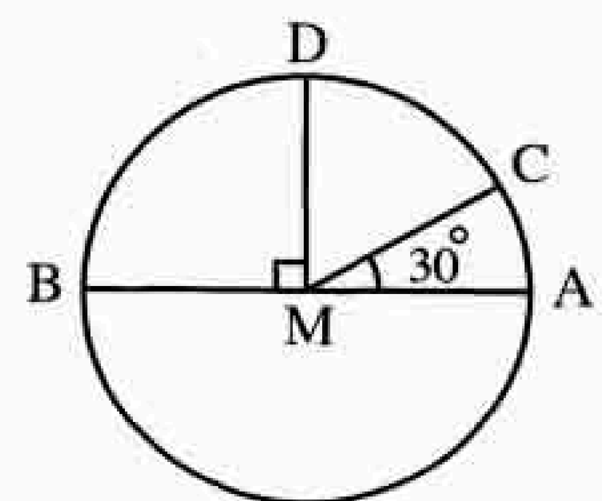
is called a semicircle.

The measure of the arc

It is the measure of the central angle which subtends this arc and it is measured by the measuring units of the angle (degrees , minutes , seconds ...)

For example:

In the opposite figure :



If $\overline{AB}$ is a diameter of the circle M , C and D are two points on the circle M where m (∠ AMC) = 30° , m (∠ AMD) = 90° , then :

1 m ($\overset{\frown}{AC}$) = m (∠ AMC) = 30°

2 m ($\overset{\frown}{CD}$) = m (∠ CMD) = 90° – 30° = 60°

3 m ($\overset{\frown}{DB}$) = m (∠ DMB) = 90°

4 m ($\overset{\frown}{DB}$ the major) = m (∠ DMB the reflex) = 360° – 90° = 270°

5 m ($\overset{\frown}{AB}$) = m (∠ AMB) = 180° , where $\overset{\frown}{AB}$ represents a semicircle.

i.e. The measure of the semicircle = 180° and then the measure of the circle = 2 × 180° = 360°

Remark

The two adjacent arcs are two arcs in the same circle that have only one point in common.

The length of the arc

It is a part of a circle's circumference proportional to its measure and it is measured by length units (centimetre , metre , ...)

- To calculate the length of the arc, you can use the following rule :

$$\text{The length of the arc} = \frac{\text{the measure of the arc}}{\text{the measure of the circle}} \times \text{the circumference of the circle}$$

$$= \frac{\text{the measure of the arc}}{360°} \times 2\pi r$$

Where r is the radius length of the circle and π is the approximated ratio.

Remark

The length of the semicircle = $\frac{1}{2}$ the circumference of the circle = π r length unit

Important corollaries

Corollary 1

In the same circle (or in congruent circles) , if the measures of arcs are equal , then the lengths of the arcs are equal and vice versa.

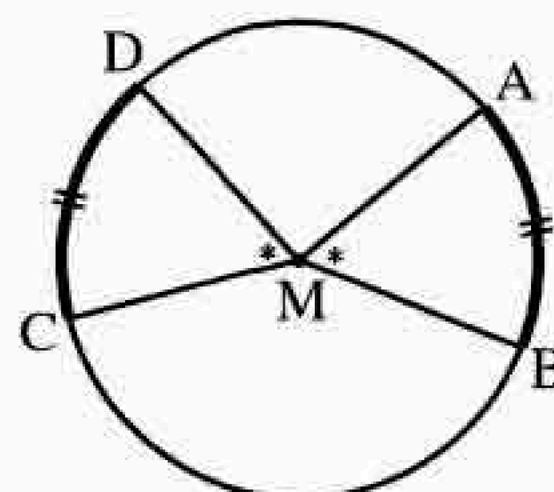
In the opposite figure :

If M is a circle in which m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$)

, then the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

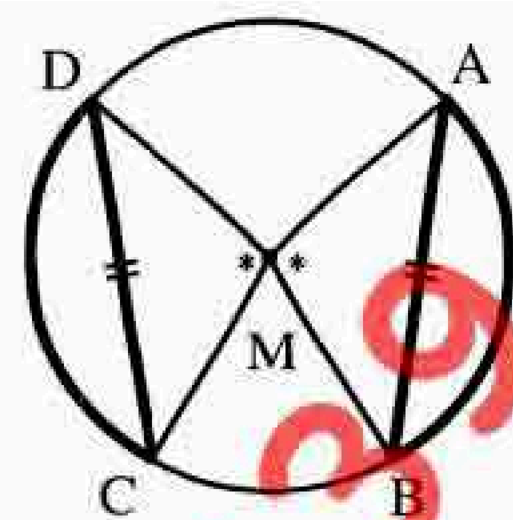
and vice versa if the length of $\overset{\frown}{AB}$

= the length of $\overset{\frown}{CD}$, then m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$)



Corollary 2

In the same circle (or in congruent circles) , if the measures of arcs are equal , then their chords are equal in length , and vice versa.



In the opposite figure :

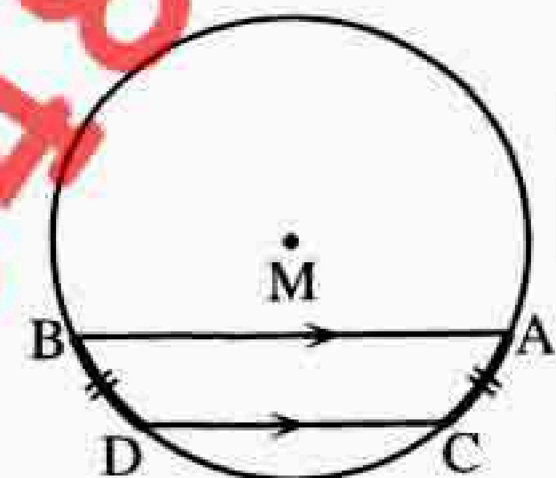
If M is a circle in which

m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$, then AB = CD and vice versa

If AB = CD , then m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$

Corollary 3

If two parallel chords are drawn in a circle , then the measures of the two arcs between them are equal.



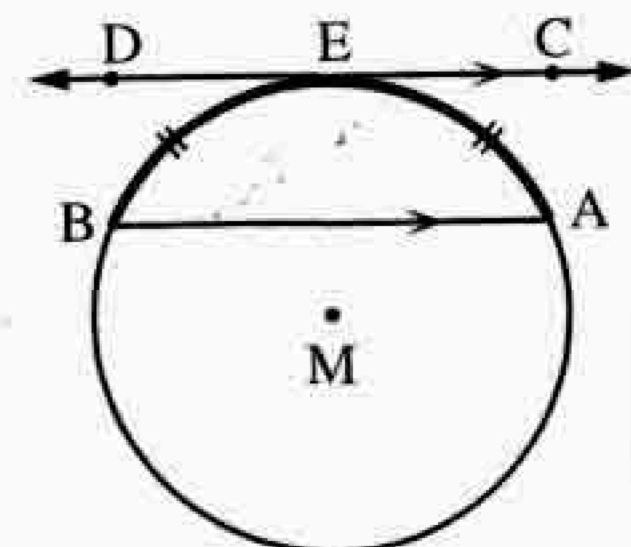
In the opposite figure :

If $\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{AB}$ // $\overline{CD}$, then m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BD})$

Corollary 4

If a chord is parallel to a tangent of a circle , then the measures of the two arcs between them are equal.



In the opposite figure :

If $\overline{AB}$ is a chord in the circle M and

$\overleftrightarrow{CD}$ touches the circle M at E ,

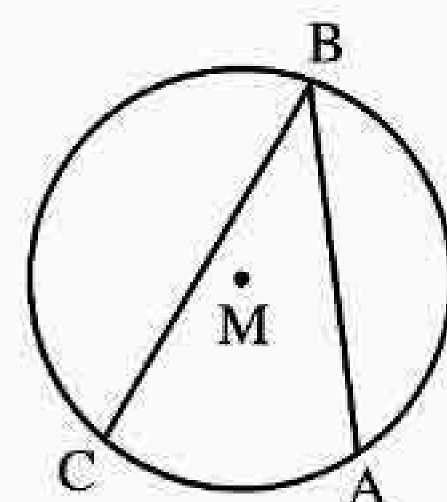
$\overleftrightarrow{CD}$ // $\overline{AB}$, then m $(\overset{\frown}{EA})$ = m $(\overset{\frown}{EB})$

Lesson [2] : The Inscribed And Central Angles

The inscribed angle

It is the angle whose vertex lies on the circle and its sides contain two chords of the circle.

In the opposite figure :



- ∠ ABC is an inscribed angle

  because its vertex B belongs to the circle M

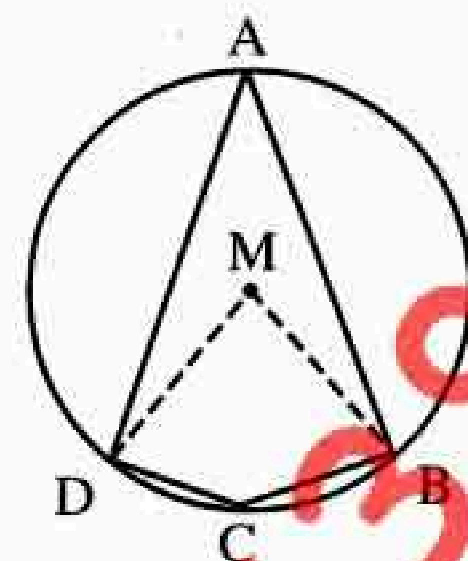
  and its sides $\overrightarrow{BA}$ and $\overrightarrow{BC}$ carry the two chords $\overline{BA}$ and $\overline{BC}$ in the circle M

- The inscribed angle ∠ ABC is subtended by $\overset{\frown}{AC}$

Remark

For each inscribed angle , there is one central angle subtended by the same arc.

In the opposite figure :



- The inscribed angle ∠ BAD is subtended with the central angle ∠ BMD by the arc $\overset{\frown}{BCD}$
- While the inscribed angle ∠ BCD is subtended with the reflex central angle BMD by the arc $\overset{\frown}{BAD}$

Theorem 1

The measure of the inscribed angle is half the measure of the central angle , subtended by the same arc.

Remark

The measure of the central angle equals twice the measure of the inscribed angle subtended by the same arc.

Examples :

1 Find the measure of the arc which represents $\frac{1}{3}$ the measure of the circle, then calculate the length of this arc if the length of the radius is 21 cm. ($\pi \approx \frac{22}{7}$) (Show steps)

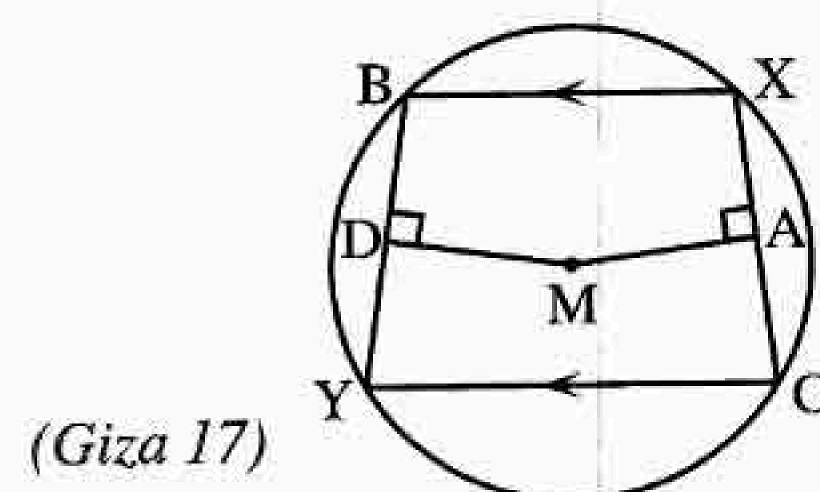
(El-Monofia 16) « 120° , 44 cm. »

2 **In the opposite figure :**

$\overline{XB} // \overline{CY}$, $\overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

Prove that : MA = MD

(Giza 17)



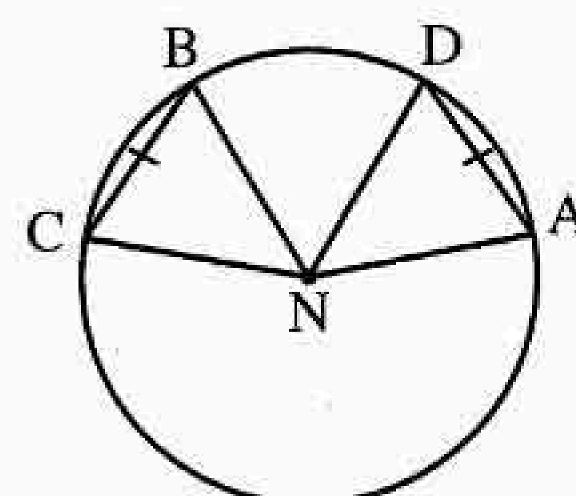
3 **In the opposite figure :**

A and B are two points belonging to the circle N

, D ∈ $\overset{\frown}{AB}$, C ∈ the major arc $\overset{\frown}{AB}$

such that AD = BC

Prove that : m (∠ ANB) = m (∠ CND)



(Souhag 2005)

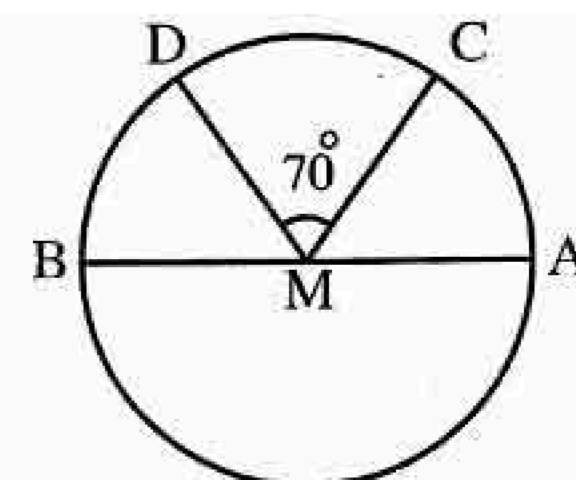
4 **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M

, m (∠ CMD) = 70° ,

m ($\overset{\frown}{AC}$) : m ($\overset{\frown}{DB}$) = 5 : 6

Find : m ($\overset{\frown}{ACD}$)



(Assiut 2012) « 120° »

5 ABCD is a quadrilateral inscribed in a circle. If $\overline{AB} \parallel \overline{DC}$, E is the midpoint of $\overset{\frown}{AB}$

Prove that : CE = DE *(Damietta 2016 , Giza 2005)*

6 🕮 **In the opposite figure :**

$\overline{AB}$ is a diameter in a circle M

, $\overleftrightarrow{DE}$ is a tangent to it at C

, $\overline{AB} \parallel \overline{DE}$, X is the midpoint of $\overset{\frown}{AC}$

, m $(\overset{\frown}{BY})$ = 2 m $(\overset{\frown}{CY})$

Find : The measures of the angles of Δ MDE *(El-Monofia , Damietta 2013)* « 45° , 75° , 60° »

7 🕮 **In the opposite figure :**

$\overline{AB}$ is a diameter in a circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30°

, m $(\overset{\frown}{AC})$ = 80° **Find :** m $(\overset{\frown}{CD})$ *(El-Sharkia 17 , Aswan 17)* « 80° »

8 **In the opposite figure :**

A circle M , BC = 6 cm.

, BM = $2\sqrt{3}$ cm.

Find : m (∠ BAC)

(Hint : Draw $\overline{MD} \perp \overline{BC}$) *(New Valley 13)* « 60° »

9 **In the opposite figure :**

M is a circle , m (∠ EMC) = 120°

and BE = AB

Find with proof : m (∠ EAC)

(South Sinai 16) « 30° »

10 **Using the opposite figure :**

Write the given data then find :

(1) m (∠ ABC) (2) m $(\overset{\frown}{BC}$ the major)

(New Valley 06) « 70° , 280° »

11

In the opposite figure :

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : Δ CAB is equilateral. *(Alexandria 16 , Ismailia 13)*

12

ABC is a triangle inscribed in the circle M such that m (∠ AMB) = 90° ,

m (∠ BMC) = 130°

Find : The measures of the angles of Δ ABC *(El-Menia 13)* « 65° , 45° , 70° »

13

In the opposite figure :

$\overline{AB}$ is a chord in the circle M ,

$\overline{CM}$ // $\overline{AB}$, $\overline{BC}$ ∩ $\overline{AM}$ = {E}

Prove that : BE > AE

(El-Gharbia 17 , Beni Suef 16 , Port Said 15 , El-Gharbia 14 , El-Dakahlia 13)

Solutions

1

The measure of the arc = $\frac{1}{3}$ the measure of the circle

= $\frac{1}{3} \times 360° = 120°$

The length of the arc = $\frac{1}{3}$ the circumference of the circle

= $\frac{1}{3} \times 2\pi r$

= $\frac{1}{3} \times 2 \times \frac{22}{7} \times 21 = 44$ cm.

2

∵ $\overline{XB}$ // $\overline{CY}$ ∴ m ($\overset{\frown}{XC}$) = m ($\overset{\frown}{BY}$)

∴ XC = BY , ∵ $\overline{MA} \perp \overline{XC}$, $\overline{MD} \perp \overline{BY}$

∴ MA = MD (Q.E.D.)

3

∵ AD = BC ∴ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{BC}$)

∴ m (∠ AND) = m (∠ CNB)

Adding m (∠ DNB) to both sides

∴ m (∠ ANB) = m (∠ CND) (Q.E.D.)

4

∵ m (∠ CMD) = 70° ∴ m ($\overset{\frown}{CD}$) = 70°

∵ m ($\overset{\frown}{AC}$) + m ($\overset{\frown}{CD}$) + m ($\overset{\frown}{DB}$) = 180°

∴ m ($\overset{\frown}{AC}$) + m ($\overset{\frown}{DB}$) = 180° − 70° = 110°

Let m ($\overset{\frown}{AC}$) be $5x$, m ($\overset{\frown}{DB}$) = $6x$

∴ $5x + 6x = 110$ ∴ $11x = 110$

∴ $x = 10°$

∴ m ($\overset{\frown}{AC}$) = 5 × 10° = 50°

∴ m ($\overset{\frown}{ACD}$) = 50° + 70° = 120° (The req.)

5

∵ $\overline{AB}$ // $\overline{DC}$

∴ m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{AD}$)

∵ E is the midpoint of $\overset{\frown}{AB}$

∴ m ($\overset{\frown}{EB}$) = m ($\overset{\frown}{AE}$) adding

∴ m ($\overset{\frown}{CE}$) = m ($\overset{\frown}{DE}$)

∴ CE = DE (Q.E.D.)

6

∵ $\overline{AB}$ is a diameter in the circle M

∴ m ($\overset{\frown}{AB}$) = 180° , ∵ $\overline{AB}$ // $\overline{DE}$

∴ m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{CB}$) = $\frac{180°}{2}$ = 90°

, ∵ X is the midpoint of $\overset{\frown}{AC}$

∴ m ($\overset{\frown}{AX}$) = 45° ∴ m (∠ AMX) = m ($\overset{\frown}{AX}$) = 45°

, ∵ $\overline{DE}$ // $\overline{AB}$, $\overleftrightarrow{DM}$ is a transversal

∴ m (∠ EDM) = m (∠ AMD) = 45° (alternate angles)

, ∵ m ($\overset{\frown}{BY}$) = 2 m ($\overset{\frown}{CY}$) ∴ m ($\overset{\frown}{BY}$) = 60°

∴ m (∠ YMB) = m ($\overset{\frown}{BY}$) = 60°

, ∵ $\overline{DE}$ // $\overline{AB}$, $\overleftrightarrow{ME}$ is a transversal

∴ m (∠ DEM) = m (∠ EMB) = 60° (alternate angles)

In Δ MDE : m (∠ DME) = 180° − (45° + 60°) = 75°

(The req.)

7

Construction :
Draw $\overline{AC}$, $\overline{MC}$, $\overline{MD}$
Proof : $\because$ m $(\widehat{AC}) = 80°$
$\therefore$ m $(\angle AMC) = 80°$
$\therefore$ m $(\angle CME) = 180° - 80° = 100°$
In Δ CME : $\therefore$ m $(\angle MCE) = 180° - (100° + 30°) = 50°$
In Δ MCD : $\because$ MC = MD = r
$\therefore$ m $(\angle CMD) = 180° - (50° + 50°) = 80°$
$\therefore$ m $(\widehat{CD}) = 80°$ (The req.)

8

$\because \overline{MD} \perp \overline{BC}$
$\therefore$ D is the midpoint of $\overline{BC}$
$\therefore$ BD = 3 cm.
$\therefore \cos(\angle MBD) = \frac{BD}{BM} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$
$\therefore$ m $(\angle MBD) = 30°$
In Δ MBC : $\because$ MB = MC = r
$\therefore$ m $(\angle MCB) = 30°$
$\therefore$ m $(\angle BMC) = 180° - 2 \times 30° = 120°$
$\therefore$ m $(\angle BAC) = \frac{1}{2}$ m $(\angle BMC)$
(inscribed and central angles subtended by the same arc $\widehat{BC}$)
$\therefore$ m $(\angle BAC) = \frac{1}{2} \times 120° = 60°$ (The req.)

9

$\because$ m $(\angle EBC) = \frac{1}{2}$ m $(\angle M)$
(inscribed and central angles subtended the same arc $\widehat{EC}$)
$\therefore$ m $(\angle EBC) = \frac{1}{2} \times 120° = 60°$
, $\because \angle EBC$ is an exterior angle of Δ ABE
$\therefore$ m $(\angle BEA)$ + m $(\angle A) = 60°$
, $\because$ BE = BA $\therefore$ m $(\angle A) = \frac{60°}{2} = 30°$ (The req.)

10

$\because$ m $(\angle BAC) = \frac{1}{2}$ m $(\angle BNC)$
(inscribed and central angles subtended the same arc $\widehat{BC}$)
$\therefore$ m $(\angle BAC) = \frac{1}{2} \times 80° = 40°$ $\because$ AB = AC
$\therefore$ m $(\angle ABC)$ = m $(\angle ACB)$
$= \frac{180° - 40°}{2} = 70°$ (First req.)
$\because$ m $(\widehat{BC})$ = m $(\angle N) = 80°$
$\therefore$ m $(\widehat{BC}$ the major$) = 360° - 80° = 280°$ (Second req.)

11

$\because$ m $(\angle ACB) = \frac{1}{2}$ m $(\angle AMB)$
(inscribed and central angles subtended the same arc $\widehat{AB}$)
$\therefore$ m $(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)
$\because \overleftrightarrow{CD}$ // $\overline{AB}$ $\therefore$ m $(\widehat{AC})$ = m $(\widehat{BC})$
$\therefore$ AC = BC (2)
From (1) and (2) : $\therefore \Delta$ CAB is equilateral (Q.E.D.)

12

$\because$ m $(\angle A) = \frac{1}{2}$ m $(\angle BMC)$
(inscribed and central angles subtended by the same arc $\widehat{BC}$)
$\therefore$ m $(\angle A) = \frac{1}{2} \times 130° = 65°$
, $\because$ m $(\angle C) = \frac{1}{2}$ m $(\angle AMB)$
(inscribed and central angles subtended by the same arc $\widehat{AB}$)
$\therefore$ m $(\angle C) = \frac{1}{2} \times 90° = 45°$
$\therefore$ m $(\angle B) = 180° - (65° + 45°) = 70°$ (The req.)

13

$\because$ m $(\angle AMC)$ = 2 m $(\angle ABC)$
(inscribed and central angles subtended the same arc $\widehat{AC}$)
$\because \overline{CM}$ // $\overline{AB}$, $\overleftrightarrow{MA}$ is a transversal to them
$\therefore$ m $(\angle MAB)$ = m $(\angle AMC)$ (alternate angles)
In Δ AEB : $\therefore$ m $(\angle EAB)$ = 2 m $(\angle EBA)$
$\therefore$ m $(\angle EAB)$ > m $(\angle EBA)$
$\therefore$ BE > AE (Q.E.D.)

Exercises

[A] : Choose The Correct Answer : -

1 The measure of the arc which equals half the measure of the circle equals ……………

(a) 360° (b) 180° (c) 120° (d) 90°

2 The length of the arc which represents $\frac{1}{4}$ of the perimeter of the circle = ……………

(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

3 **In the opposite figure :**

If M is centre of the circle , then m (∠ CMB) = ……………°

(a) 36 (b) 72

(c) 144 (d) 180

4 **In the opposite figure :**

$\overline{AB}$ is a diameter of a circle M

, m (∠ B) = 30° , AC = 6 cm. , then AB = …………… cm.

(a) 3 (b) 6 (c) 9 (d) 12

5 **In the opposite figure :**

m (∠ ABC) = 60°

, m (∠ AMC) = (y + 20)°

, then y = ……………°

(a) 30 (b) 40 (c) 80 (d) 100

6 **In the opposite figure :**

Circle of centre M

If m $(\widehat{AB})$ = 50° , then m (∠ ADB) = ……………

(a) 25° (b) 50° (c) 100° (d) 150°

7 The inscribed angle which opposite to the minor arc in a circle is ……………

(a) reflex. (b) right. (c) obtuse. (d) acute.

8 **In the opposite figure :**

Two concentric circles.

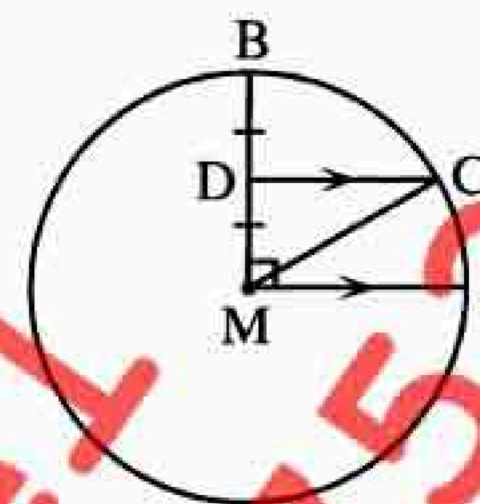
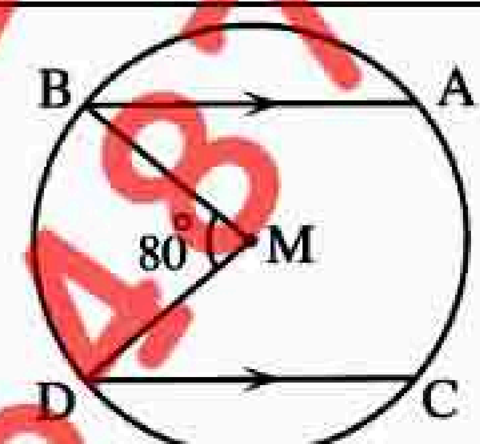
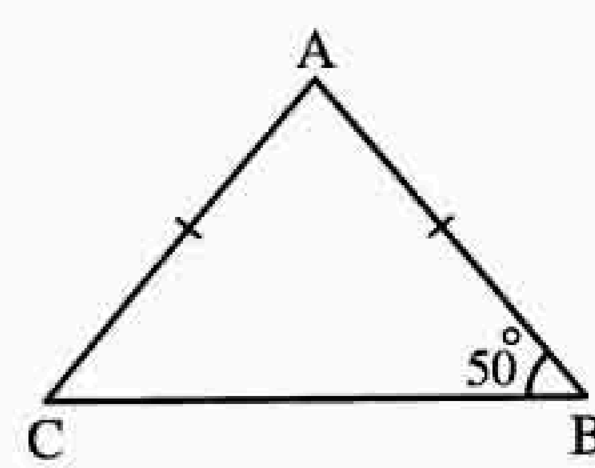
If the lengths of their radii are 2 cm. and 5 cm.

, then $\frac{m(\widehat{AB})}{m(\widehat{CD})}$ = ……………

(a) $\frac{2}{5}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{5}$

9	The corresponding angles of the two similar polygons are .... in measure. (a) equal (b) different (c) proportional (d) alternate
10	The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units. (a) – 10 (b) 10 (c) 2 (d) 24
11	The image of the point (2 , 3) by rotation R (O , 180°) is the point .... (a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)
12	The two angles A and C in the right-angled triangle at B are .... (a) complementary. (b) supplementary. (c) adjacent. (d) vertically opposite angles.
13	ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and X cm. , then X = ... cm. (a) 5 (b) 8 (c) 10 (d) 12
14	The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2. (a) 6 (b) 10 (c) 30 (d) 60
15	**In the opposite figure :** AB = AC , AB = 2 X – 1 and AC = X + 2 , then X = ... (a) 3 (b) 5 (c) 11 (d) 14 Figure labels: A, B, C, x+2, 2x-1
16	**In the opposite figure :** The length of $\overline{AB}$ = ... cm. (a) $10\sqrt{3}$ (b) 10 (c) 5 (d) $5\sqrt{3}$ Figure labels: A, B, C, D, 60°, 60°, 8cm., 6cm.
17	A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2. (a) 3050 (b) 3500 (c) 2925 (d) 3250
18	The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm. equals .... cm^2. (a) 2 (b) 18 (c) 40 (d) 80
19	In a regular hexagon , the measure of the angle of its vertex equals .... (a) 60° (b) 108° (c) 120° (d) 135°
20	If the radius length of the circle M equals 2 cm. , then its circumference equals .... (a) 4 π cm. (b) 5 π cm. (c) 6 π cm. (d) 7 π cm.

21	**In the opposite figure :** ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$, $\overline{AD}$ is a diameter of circle M , then the area of the shaded region is .... (a) 70 cm² (b) 147 cm² (c) 170 cm² (d) 224 cm²
22	The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length unit. (a) 3 (b) 4 (c) 5 (d) 6
23	Circumference of a circle is 6 π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is .... (a) a tangent to the circle. (b) a secant to the circle. (c) outside the circle. (d) the diameter to the circle.
24	The measure of the arc that is opposite the inscribed angle of measure 60° = ...° (a) 60 (b) 30 (c) 120 (d) 90
25	The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc. (a) half (b) third (c) quarter (d) double
26	**In the opposite figure :** $\overline{AB}$ is a diameter of circle M , m (∠ ABC) = 50° , then m ($\overset{\frown}{BC}$) = ...° (a) 40 (b) 50 (c) 80 (d) 100
27	**In the opposite figure :** M is a circle , m (∠ BAC) = 55° , then m (∠ MCB) = ...° (a) 110 (b) 55 (c) 35 (d) 25
28	**In the opposite figure :** ABC is an equilateral triangle inscribed in circle M , then m (∠ BMC) = ... (a) 50° (b) 120° (c) 60° (d) 100°
29	A circle whose circumference 20 π cm. its area = ... π cm² (a) 10 (b) 100 (c) 200 (d) 400

30	The perimeter of the square whose area is 81 cm^2 is …………… (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.
31	**In the opposite figure :** $\overline{AM} // \overline{CD}$, MD = DB , m (∠ AMB) = 90° , then m $(\overset{\frown}{AC})$ = …………… (a) 45° (b) 60° (c) 30° (d) 90° 
32	**In the opposite figure :** In a circle M , $\overline{AB} // \overline{CD}$, m (∠ BMD) = 80° , then m $(\overset{\frown}{AC})$ = …………… (a) 20° (b) 40° (c) 80° (d) 160° 
33	If m_1 and m_2 are the slopes of two perpendicular straight lines , then …………… (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$
34	The sum of measures of the accumulative angles at a point = ……………° (a) 80 (b) 120 (c) 360 (d) 630
35	The image of the point (A , B) by rotation R (0 , 180°) the point …………… (a) (– A , B) (b) (– A , – B) (c) (A , – B) (d) (A , B)
36	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals …………… the length of the hypotenuse. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2
37	ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° , then m (∠ A) = …………… (a) 40° (b) 50° (c) 90° (d) 130°
38	The medians of triangle intersect at a same point which divides each in the ratio …………… from its base. (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2
39	The number of the axes of symmetry in the equilateral triangle = …………… (a) 1 (b) 2 (c) 3 (d) an infinite number.
40	**In the opposite figure :** ABC is a triangle , AB = AC , m (∠ B) = 50° , then m (∠ A) = …………… (a) 100° (b) 90° (c) 80° (d) 70° 

[B] : Essay Problems : -

1

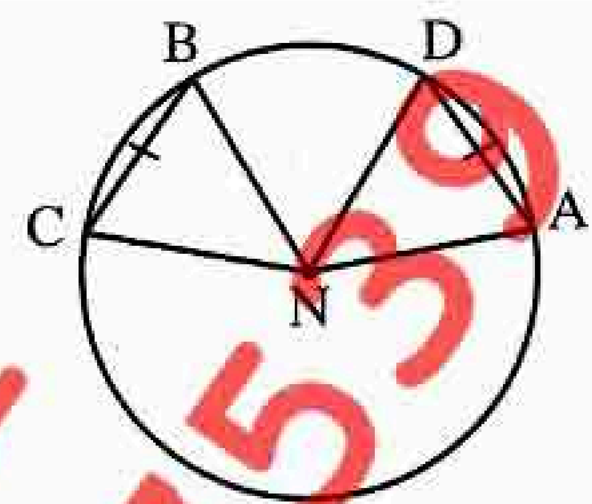
In the opposite figure :

A and B are two points belonging to the circle N

, D ∈ $\overset{\frown}{AB}$, C ∈ the major arc $\overset{\frown}{AB}$

such that AD = BC

Prove that : m (∠ ANB) = m (∠ CND)



(Souhag 2005)

2

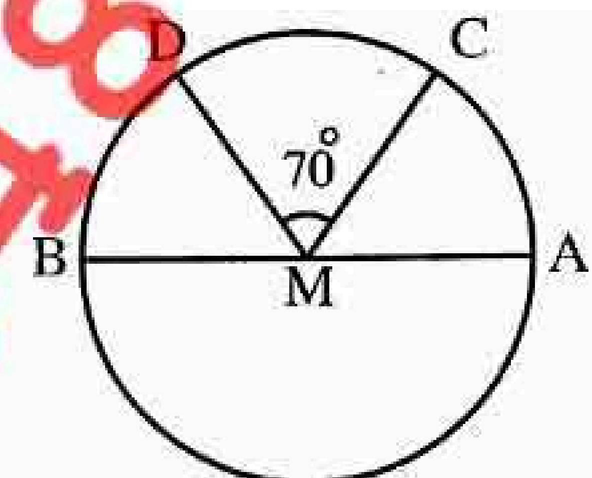
In the opposite figure :

$\overline{AB}$ is a diameter of the circle M

, m (∠ CMD) = 70° ,

m ($\overset{\frown}{AC}$) : m ($\overset{\frown}{DB}$) = 5 : 6

Find : m ($\overset{\frown}{ACD}$)



(Assiut 2012) « 120° »

3

ABCD is a quadrilateral inscribed in a circle. If $\overline{AB}$ // $\overline{DC}$, E is the midpoint of $\overset{\frown}{AB}$

Prove that : CE = DE

(Damietta 2016 , Giza 2005)

4

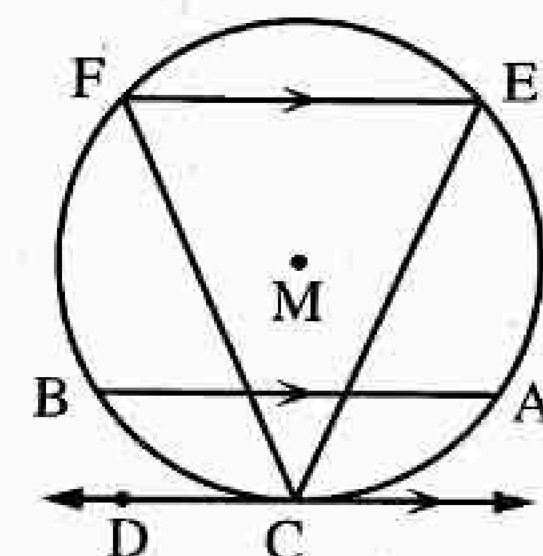
In the opposite figure :

M is a circle , $\overleftrightarrow{CD}$ is a tangent to the circle at C ,

$\overline{AB}$ and $\overline{EF}$ are two chords in the circle

, where $\overline{AB}$ // $\overline{EF}$ // $\overleftrightarrow{CD}$

Prove that : CE = CF



(El-Beheira 2014 , Alex. 2011)

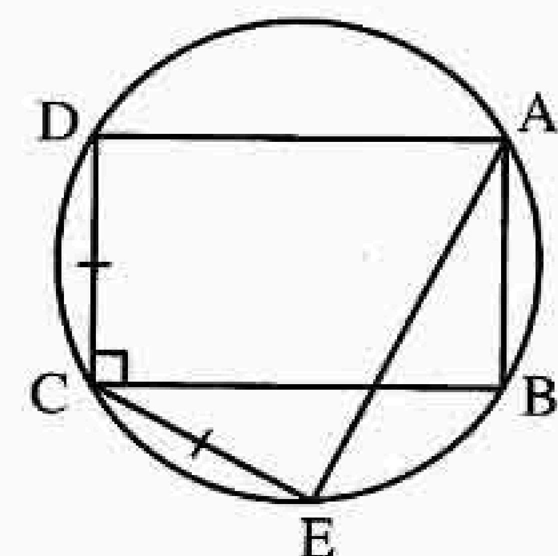
5

In the opposite figure :

ABCD is a rectangle inscribed in a circle. Draw the chord $\overline{CE}$

, where CE = CD

Prove that : AE = BC



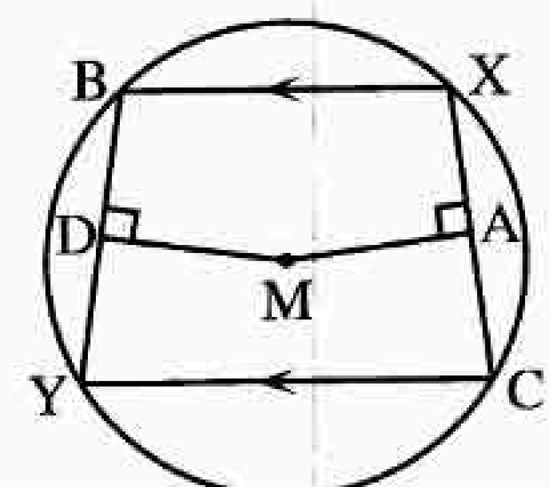
(El-Monofia 2016 , El-Fayoum 2011 , Souhag 2015)

6

In the opposite figure :

$\overline{XB}$ // $\overline{CY}$, $\overline{MA}$ ⊥ $\overline{XC}$, $\overline{MD}$ ⊥ $\overline{BY}$

Prove that : MA = MD

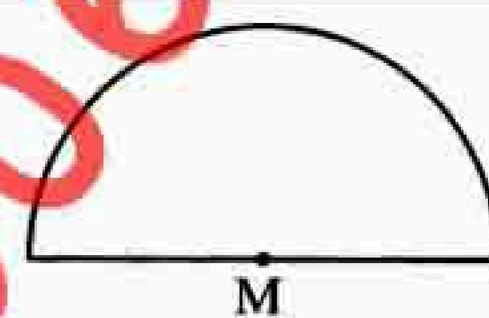
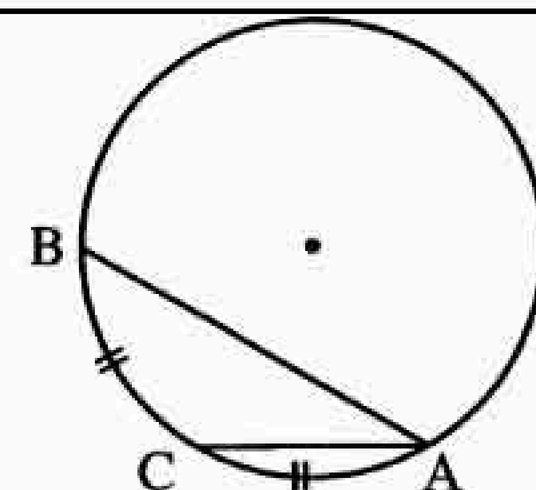
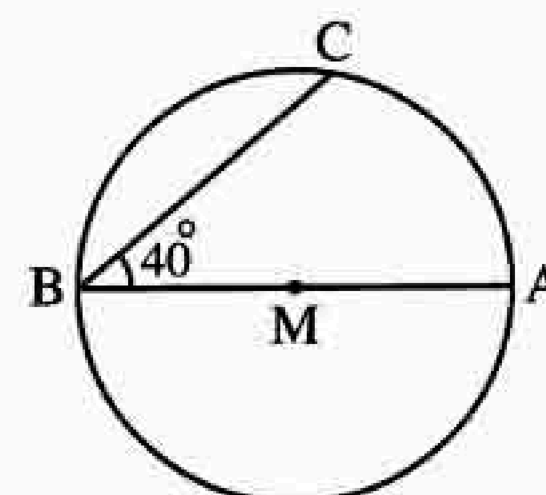
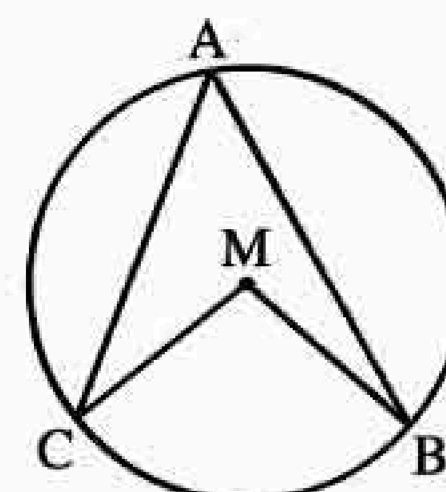


(Giza 17)

7	**In the opposite figure :** $\overline{AB}$ is a diameter in a circle M , $\overleftrightarrow{DE}$ is a tangent to it at C , $\overline{AB}$ // $\overline{DE}$, X is the midpoint of $\overset{\frown}{AC}$, m $(\overset{\frown}{BY})$ = 2 m $(\overset{\frown}{CY})$ **Find :** The measures of the angles of Δ MDE *(El-Monofia , Damietta 2013)* « 45° , 75° , 60° »
8	Find the measure of the arc which represents $\frac{1}{3}$ the measure of the circle, then calculate the length of this arc if the length of the radius is 21 cm. $(\pi \approx \frac{22}{7})$ (Show steps) *(El-Monofia 16)* « 120° , 44 cm. »
9	**In the opposite figure :** M is a circle of radius length 7 cm. , m (∠ AMB) = 120° Find the length of $\overset{\frown}{AB}$ $(\pi \approx \frac{22}{7})$ *(Suez 17)* « $14\frac{2}{3}$ cm. »
10	**In the opposite figure :** $\overline{AB}$ is a diameter in a circle M , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ AEC) = 30° , m $(\overset{\frown}{AC})$ = 80° **Find :** m $(\overset{\frown}{CD})$ *(El-Sharkia 17 , Aswan 17)* « 80° »
11	**In the opposite figure :** m (∠ A) = 30° , BC = 7 cm. **Find :** The area of the circle M $(\pi = \frac{22}{7})$ *(Gharbia 09)* « 154 cm^2 »
12	**In the opposite figure :** A circle M , BC = 6 cm. , BM = $2\sqrt{3}$ cm. **Find :** m (∠ BAC) (Hint : Draw $\overline{MD} \perp \overline{BC}$) *(New Valley 13)* « 60° »

Homework

[A] : Choose The Correct Answer :

1	The sum of measures of the interior angles of the quadrilateral = ... (a) 90° (b) 180° (c) 270° (d) 360°
2	The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is .... cm^2 (a) 2 (b) 14 (c) 24 (d) 48
3	A square of perimeter 20 cm. , then its area = ... cm^2 (a) 20 (b) 25 (c) 50 (d) 100
4	The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.  (a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$
5	The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals .... (a) 60° (b) 120° (c) 90° (d) 240°
6	The ratio between the measure of the inscribed angle to the measure of the central angle subtended by the same arc equals .... (a) 2 : 1 (b) 1 : 2 (c) 2 : 2 (d) 2 : 3
7	**In the opposite figure :** If C is the midpoint of $\overset{\frown}{AB}$, then AB .... 2 AC  (a) < (b) > (c) ≥ (d) =
8	**In the opposite figure :** $\overline{AB}$ is a diameter in the circle M , m (∠ ABC) = 40° , then m ($\overset{\frown}{BC}$) = ...  (a) 40° (b) 50° (c) 90° (d) 100°
9	**In the opposite figure :** In the circle M , if m (∠ M) – m (∠ A) = 50° , then m (∠ A) = ...  (a) 40° (b) 50° (c) 100° (d) 130°

10	**In the opposite figure :** If m (∠ ABC) = 40° , then m (∠ AMC) = ...° (a) 20 (b) 40 (c) 80 (d) 140
11	The measure of the inscribed angle drawn in a semicircle equals .... (a) 45° (b) 90° (c) 120° (d) 80°
12	If m_1 , m_2 are two slopes of two parallel straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$
13	The angle whose measure is 50° complements an angle of measure .... (a) 90° (b) 130° (c) 50° (d) 40°
14	If the projection of a line segment on a straight line is a point , then the line segment .... the straight line. (a) // (b) ⊥ (c) ∈ (d) ⊂
15	If $\cos 2X = \frac{1}{2}$ where X is an acute angle , then m (∠ X) = ...° (a) 15 (b) 30 (c) 45 (d) 60
16	Δ XYZ is right-angled triangle at Y , then XZ .... YZ (a) < (b) > (c) = (d) twice
17	If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is .... (a) 40° (b) 90° (c) 45° (d) 80°
18	The sum of lengths of any two sides of a triangle .... the length of the third side. (a) < (b) > (c) = (d) ≤
19	**In the opposite figure :** CA = CB , $\overline{CX} \perp \overline{AB}$, AB = 2 CX , then m (∠ A) = ... (a) 30° (b) 60° (c) 90° (d) 45°
20	The diagonals are equal in length and not perpendicular in .... (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
21	If the side length of a rhombus is L cm. , then its perimeter = ... cm. (a) L^2 (b) $2L^2$ (c) 4 L (d) $2\sqrt{2}$ L

22	The area of a square whose diagonal length is 6 cm. equals .... cm^2. (a) 36 (b) 18 (c) 24 (d) 9
23	The number of sides of the regular polygon in which the measure one of its interior angles 135° = ... sides. (a) 4 (b) 6 (c) 8 (d) 10
24	The longest chord in the circle is called .... (a) diameter. (b) tangent. (c) secant. (d) radius.
25	If M is a circle of radius length r cm. , then the length of the semicircle = ... cm. (a) $2\pi r$ (b) $\frac{1}{4}\pi r$ (c) $\frac{1}{2}\pi r$ (d) πr
26	If the area of the circle M = 16π cm^2. , A is a point on its plane where MA = 8 cm. , then A is .... (a) outside the circle. (b) inside the circle. (c) on the circle. (d) on the centre of the circle.
27	The measure of the central angle is .... the measure of the arc which is opposite to it. (a) twice (b) half (c) equals (d) more than
28	**In the opposite figure :** If m (∠ AMB) = 52° , then m $(\widehat{ADB})$ = ...° (a) 52 (b) 104 (c) 128 (d) 308
29	**In the opposite figure :** $\overline{AB}$ is a diameter in a circle M , m (∠ CAB) = 45° , then m (∠ ABC) = ...° (a) 40 (b) 45 (c) 50 (d) 90
30	**In the opposite figure :** M is a circle , m (∠ MBC) = 32° , then m $(\widehat{BC}$ the minor) = ... (a) 116° (b) 23° (c) 58° (d) 64°
31	**In the opposite figure :** m (∠ ACB) = ... (a) 40° (b) 80° (c) 90° (d) 180°

32 The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

33 **In the opposite figure :**

Two concentric circle M , m $(\widehat{BC}) = 80°$

, if the radius length of the smaller circle is 7 cm.

and the redius length of the larger circle is 14 cm. , $\left(\pi = \frac{22}{7}\right)$, then :

First : The perimeter of the smaller circle = ... cm.

(a) 44 (b) 22 (c) 154 (d) 88

Second : m $(\widehat{AD})$ = ...°

(a) 80 (b) 40 (c) 20 (d) 160

34 If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ....)

(a) X (b) Y (c) Z (d) L

35 If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)

36 The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base.

(a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3

37 ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm.

, then its area = ... cm^2

(a) 48 (b) 14 (c) 24 (d) 7

38 ABC is a triangle in which AB = AC , m (∠ C) = 40° , then m (∠ A) = ...

(a) 40° (b) 80° (c) 100° (d) 120°

39 The numbers 5 , 4 , ... can be side lengths of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

40 **In the opposite figure :**

ABC is right-angled triangle at B

, m (∠ C) = 30° , AB = 3 cm.

, then AC = ... cm.

(a) 2 (b) 3 (c) $3\sqrt{3}$ (d) 6

41 A circle its radius length is 5 cm. , then its circumference = ... cm.

(a) 5 π (b) 7 π (c) 10 π (d) 25 π

42 If the area of the circle is 9 π cm^2 , then its radius length = ... cm.

(a) 9 (b) 2 (c) (– 3) (d) 3

[B] : Essay Problems : -

1

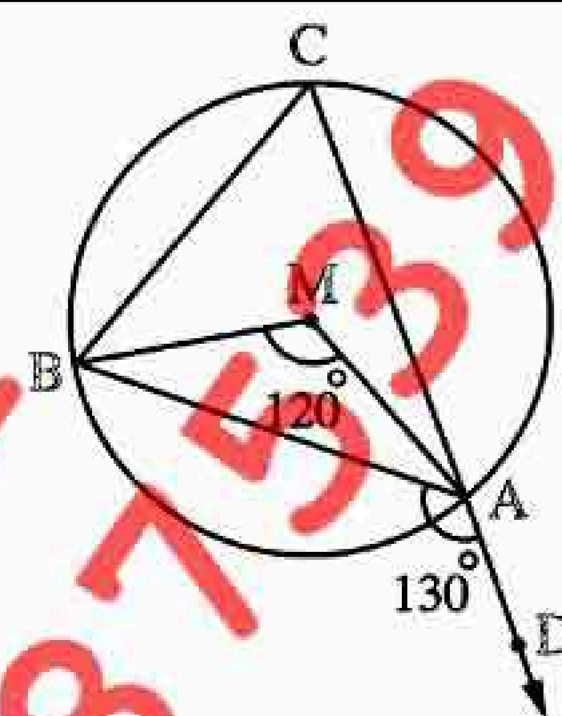
In the opposite figure :

Δ ABC is inscribed in the circle M

, D ∈ $\overrightarrow{CA}$, m (∠ BAD) = 130°

, m (∠ AMB) = 120°

Find : m (∠ MBC)



(Monofia 09) « 40° »

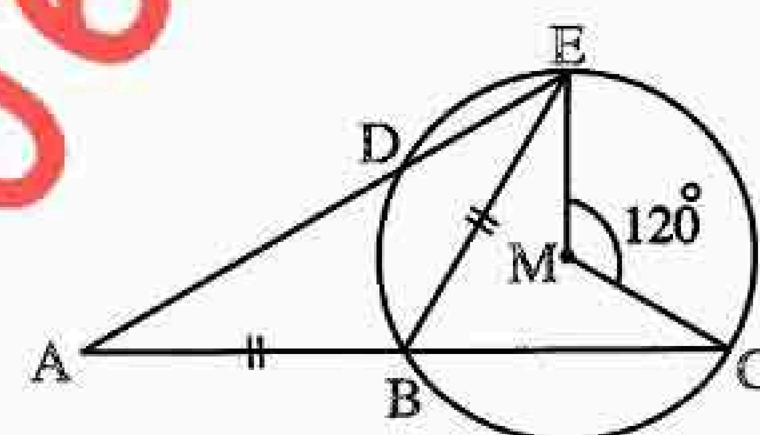
2

In the opposite figure :

M is a circle , m (∠ EMC) = 120°

and BE = AB

Find with proof : m (∠ EAC)



(South Sinai 16) « 30° »

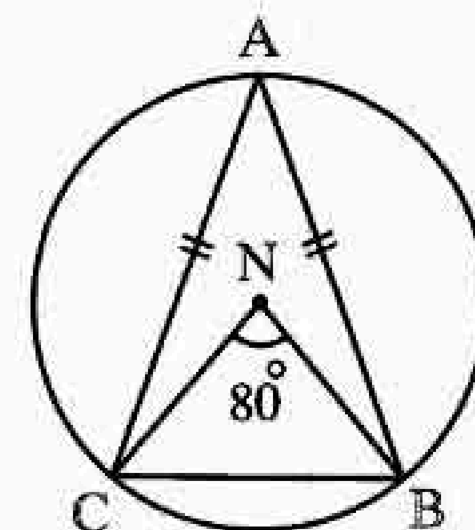
3

Using the opposite figure :

Write the given data then find :

(1) m (∠ ABC) (2) m ($\overset{\frown}{BC}$ the major)

(New Valley 06) « 70° , 280° »



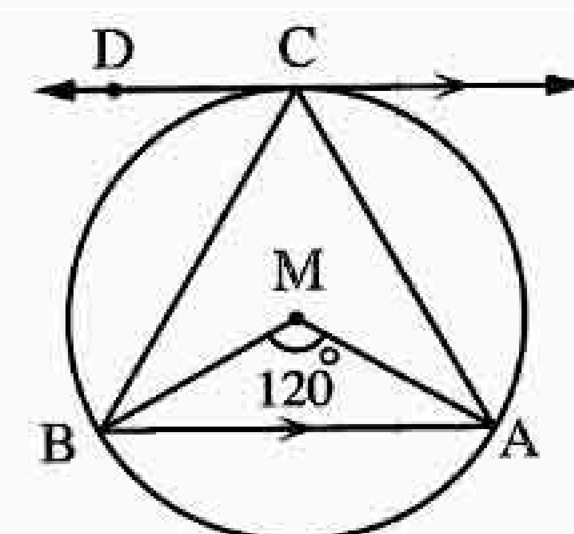
4

In the opposite figure :

$\overleftrightarrow{CD}$ is a tangent to the circle at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

Prove that : Δ CAB is equilateral. *(Alexandria 16 , Ismailia 13)*



5

ABC is a triangle inscribed in the circle M such that m (∠ AMB) = 90° ,

m (∠ BMC) = 130°

Find : The measures of the angles of Δ ABC *(El-Menia 13)* « 65° , 45° , 70° »

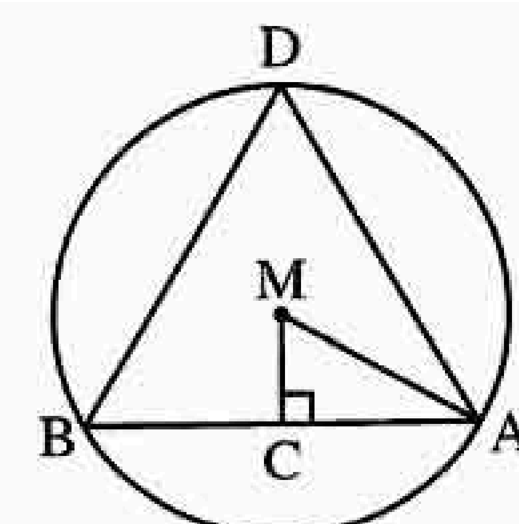
6

In the opposite figure :

$\overline{AB}$ is a chord in the circle M ,

$\overline{MC}$ ⊥ $\overline{AB}$

Prove that : m (∠ AMC) = m (∠ ADB)



(Port Said 14 , El-Beheira 13)

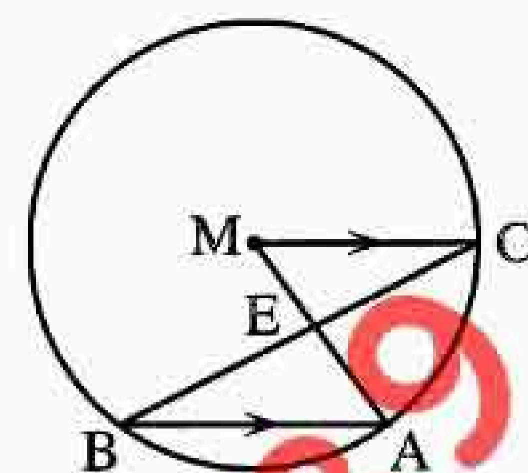
7

In the opposite figure :

$\overline{AB}$ is a chord in the circle M ,

$\overline{CM}$ // $\overline{AB}$, $\overline{BC} \cap \overline{AM} = \{E\}$

Prove that : BE > AE



(El-Gharbia 17 , Beni Suef 16 , Port Said 15 , El-Gharbia 14 , El-Dakahlia 13)

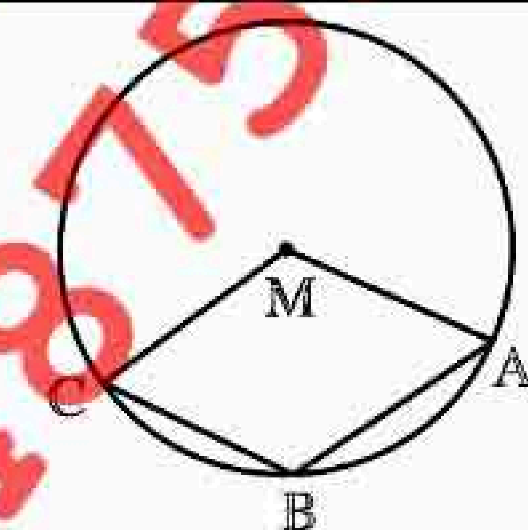
8

In the opposite figure :

If M is the centre of the circle

, m (∠ AMC) = m (∠ B)

Find : m (∠ B)



(Monofia 06) « 120° »

9

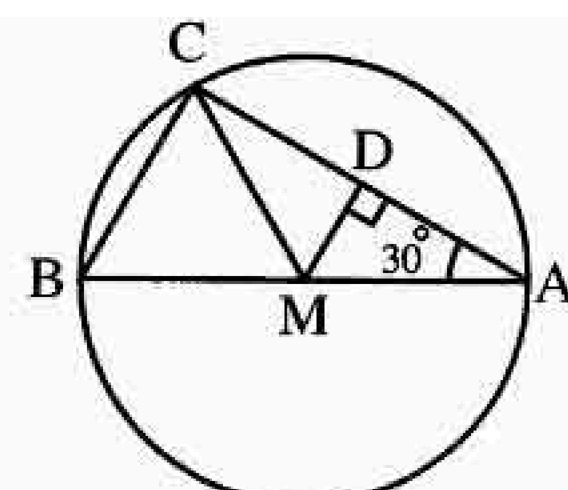
In the opposite figure :

$\overline{AB}$ is a diameter of a circle M , $\overline{AC}$ is a chord ,

$\overline{MD} \perp \overline{AC}$, m (∠ A) = 30°

Prove that : (1) $\overline{MD}$ // $\overline{BC}$

(2) Δ MBC is an equilateral triangle.



(Fayoum 2012)

10

In the opposite figure :

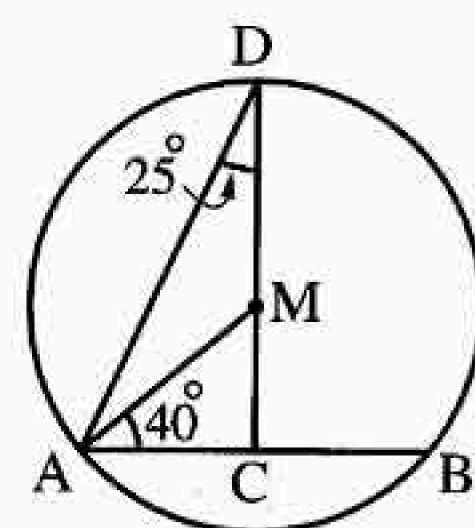
$\overline{AB}$ is a chord of the circle M ,

m (∠ D) = 25°

and m (∠ MAC) = 40°

Prove that :

C is the midpoint of $\overline{AB}$



(Kafr El-Sheikh 2009)

11

In the opposite figure :

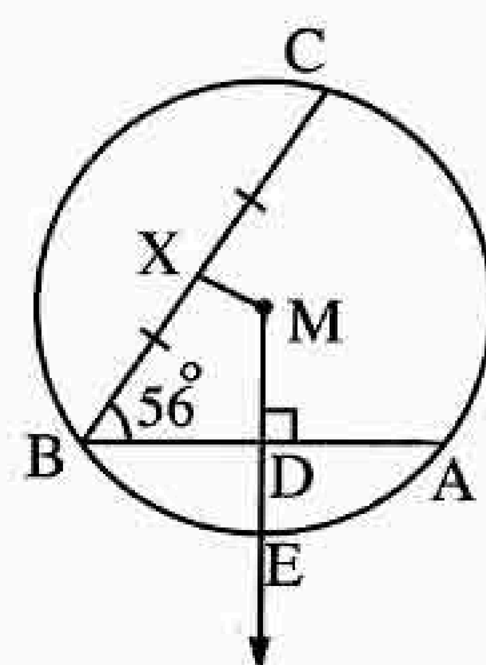
$\overline{AB}$ and $\overline{BC}$ are two chords in circle M ,

which has radius length of 5 cm. ,

$\overrightarrow{MD} \perp \overline{AB}$ intersects $\overline{AB}$ at D and inersects the circle M at E ,

X is the midpoint of $\overline{BC}$, AB = 8 cm. , m (∠ ABC) = 56°

Find : (1) m (∠ DMX)   (2) The length of $\overline{DE}$



(Souhag 2015 , Alexandria 2011) « 124° , 2 cm.»

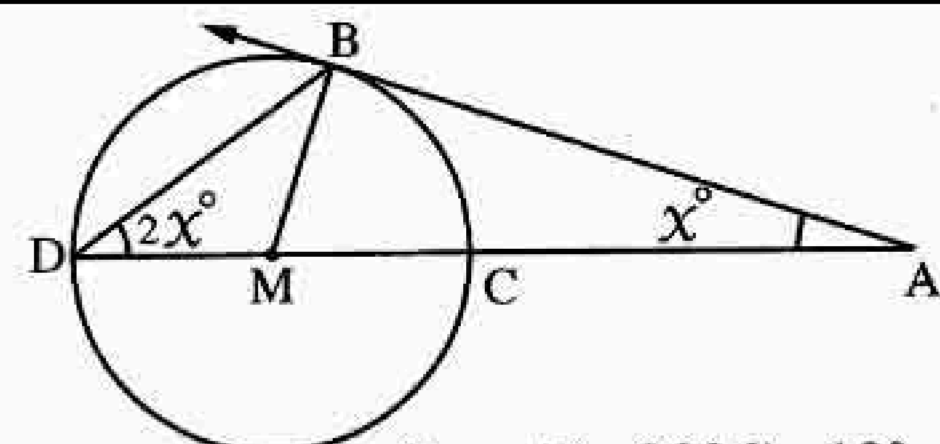
12

In the opposite figure :

$\overrightarrow{AB}$ touches the circle M at B , $\overline{CD}$ is a diameter of it ,

m (∠ BAM) = x° and m (∠ MDB) = 2 x°

Find : The value of x in degrees.



(Ismailia 2006) «18°»

Lesson [3] : Inscribed Angles Subtended By The Same Arc

Corollary 1

The measure of an inscribed angle is half the measure of the subtended arc.

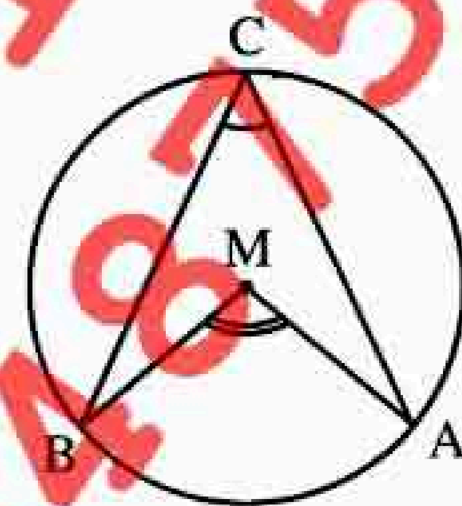
In the opposite figure :

$m(\angle C) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles with common arc $\overset{\frown}{AB}$),

$m(\angle AMB) = m(\overset{\frown}{AB})$

$\therefore m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$



Remark

The measure of the arc equals twice the measure of the inscribed angle subtended by this arc.

Corollary 2

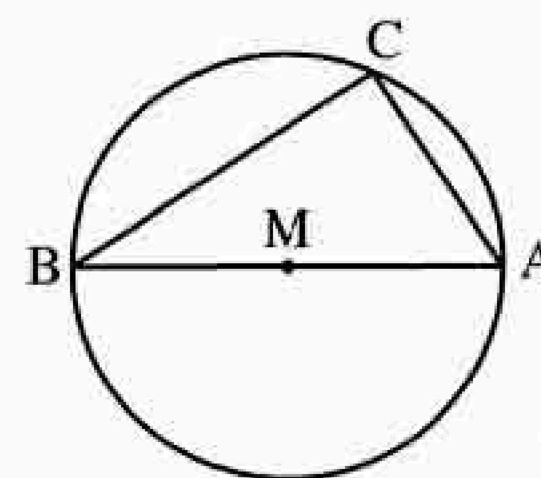
The inscribed angle in a semicircle is a right angle.

In the opposite figure :

$\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$ (corollary 1),

$\therefore m(\overset{\frown}{AB}) = 180°$

$\therefore m(\angle C) = 90°$



Remarks

1. The inscribed angle which is right angle is drawn in a semicircle.
2. The inscribed angle which is subtended by an arc of measure less than the measure of a semicircle is an acute angle.
3. The inscribed angle which is subtended by an arc of measure greater than the measure of a semicircle is an obtuse angle.

Well known problem (1)

If two chords intersect at a point inside a circle , then the measure of the included angle equals half of the sum of the two measures of the two opposite arcs.

$\overline{AB}$, $\overline{CD}$ are two chords in a circle intersecting at the point E

1. $m(\angle AEC) = \frac{1}{2}\left[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})\right]$
2. $m(\angle CEB) = \frac{1}{2}\left[m(\overset{\frown}{BC}) + m(\overset{\frown}{AD})\right]$

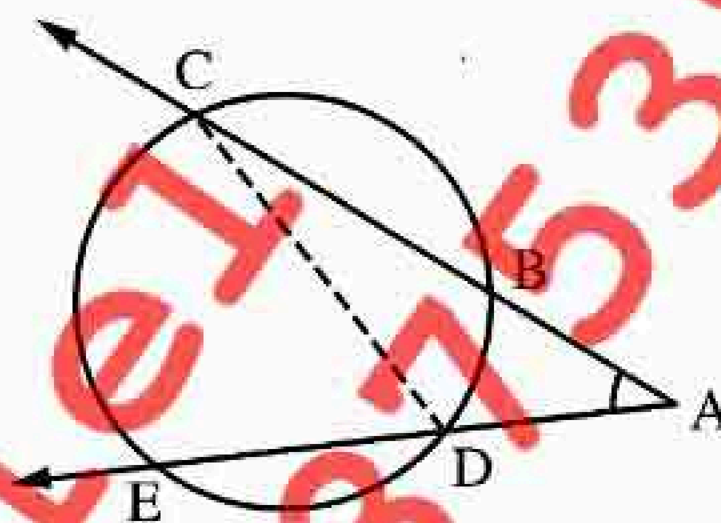
Well known problem (2)

If two rays carrying two chords in a circle are intersecting outside it, then the measure of their intersecting angle equals half of the measure of the major arc subtracted from it half of the measure of the minor arc in which both are included by the two sides of this angle.

$\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$

$m(\angle A) = \frac{1}{2}[m(\overset{\frown}{CE}) - m(\overset{\frown}{BD})]$

Draw $\overline{CD}$



Theorem 2

In the same circle , the measures of all inscribed angles subtended by the same arc are equal.

Given $\angle C$, $\angle D$ and $\angle E$ are inscribed angles subtended by $\overset{\frown}{AB}$

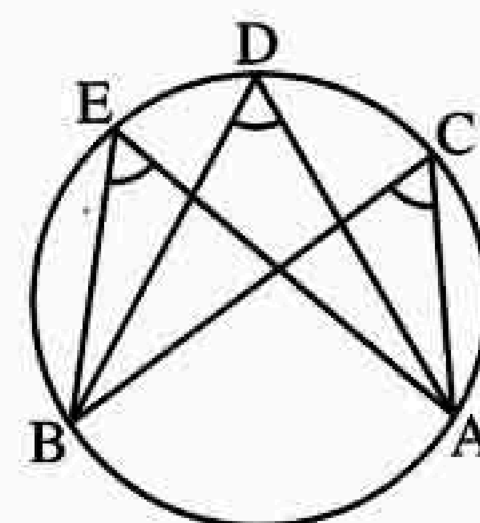
R.T.P. $m(\angle C) = m(\angle D) = m(\angle E)$

Proof $\because m(\angle C) = \frac{1}{2} m(\overset{\frown}{AB})$

, $m(\angle D) = \frac{1}{2} m(\overset{\frown}{AB})$

, $m(\angle E) = \frac{1}{2} m(\overset{\frown}{AB})$

$\therefore m(\angle C) = m(\angle D) = m(\angle E)$ (Q.E.D.)



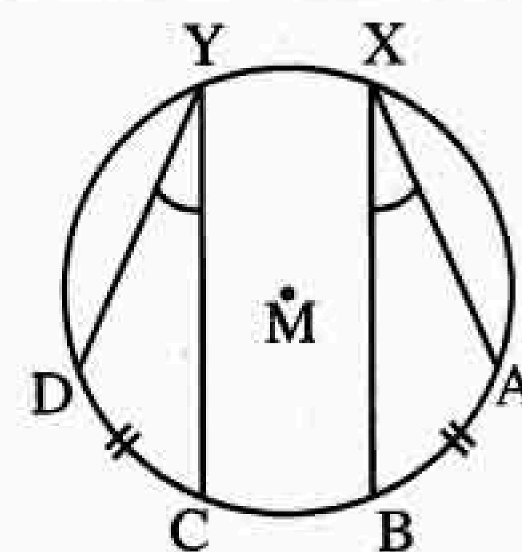
Corollary

In the same circle (or in any number of circles) the measures of the inscribed angles subtended by arcs of equal measures are equal.

i.e. In the circle M

If $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

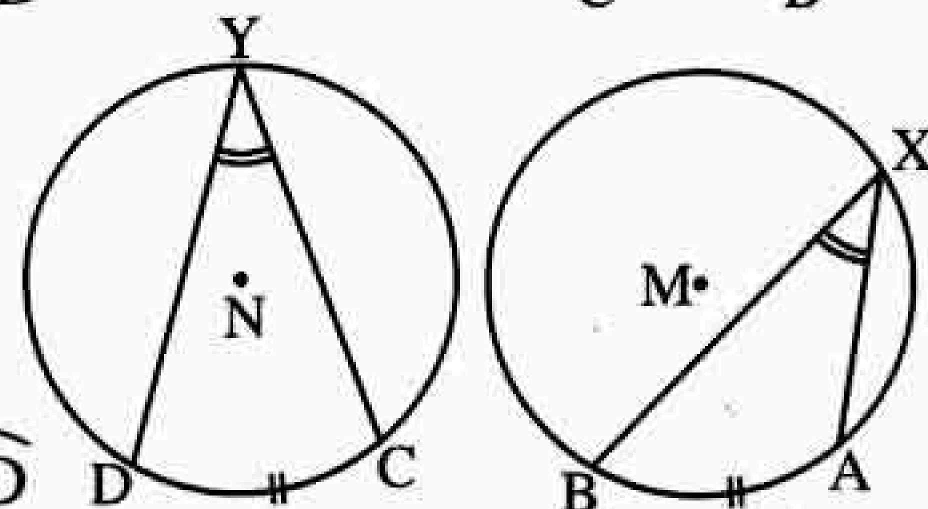


Notice that: In this case , the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

Also : If M and N are two congruent circles

and $m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$,

then $m(\angle X) = m(\angle Y)$

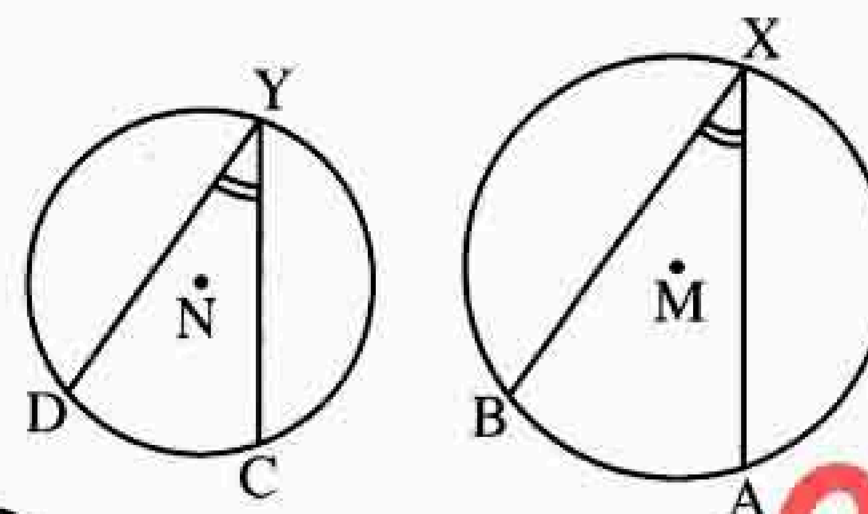


Notice that: In this case , the length of $\overset{\frown}{AB}$ = the length of $\overset{\frown}{CD}$

Similarly : In any two circles M and N

If m $(\overset{\frown}{AB})$ = m $(\overset{\frown}{CD})$,

then m (∠ X) = m (∠ Y)



Notice that: In this case , the length of $\overset{\frown}{AB}$ ≠ the length of $\overset{\frown}{CD}$

The converse of the previous corollary is true also

i.e. **In the same circle (or in any number of circles) the inscribed angles of equal measures subtend arcs of equal measures.**

Examples :

1 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

$\overline{AB}$ // $\overline{DC}$, m $(\overset{\frown}{DC})$ = 80° ,

m $(\overset{\frown}{AH})$ = 100°

Find by proof : m (∠ DHB) , m (∠ AOH)

(El-Menia 17) « 25° , 75° »

2 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{BD}$,

m (∠ CAB) = 35°

Find by proof : m (∠ CBD)

(El-Menia 11) « 100° »

3 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $(\overset{\frown}{AD})$ = the length of $(\overset{\frown}{DC})$,

m (∠ ABC) = 70°

Find each of : m (∠ DCA) , m (∠ CAB)

(El-Ismailia 05) « 35° , 20° »

4 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ touches the circle at A

If AC = 9 cm. , BM = 6 cm.

Find the length of each of : $\overline{BC}$, $\overline{AD}$

(Souhag 17 , Kafr El-Sheikh 04) « 15 cm. , 7.2 cm. »

5

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

$m(\angle B) = 40°$, $\overleftrightarrow{DH}$ is a tangent to the circle M at D ,

$\overleftrightarrow{DH} \parallel \overline{BC}$

Find : $m(\overset{\frown}{DC})$

(El-Monofia 17) « 50° »

6

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

$\overline{DC} \parallel \overline{AB}$, $m(\angle AMD) = 70°$

Find by proof : $m(\angle ACD)$, $m(\angle ABC)$

(El-Menia 17) « 35° , 55° »

7

$\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord such that $m(\angle BAC) = 30°$, draw $\overline{BC}$ and draw $\overline{MD} \perp \overline{AC}$ and to intersect it at D

(1) **Prove that :** $\overline{MD} \parallel \overline{BC}$

(2) **Prove that :** length of $\overline{BC}$ = length of the radius of this circle.

(El-Monofia 17)

8

In the opposite figure :

$m(\angle A) = 40°$, $m(\overset{\frown}{BD}) = 60°$

, $m(\overset{\frown}{BC}) = m(\overset{\frown}{DE})$

Find : (1) $m(\overset{\frown}{EC})$ (2) $m(\overset{\frown}{BC})$

(Port Said 17 , North Sinai 17) « 140° , 80° »

9

In the opposite figure :

M is a circle , $\overline{BC}$ is a diameter in it

, $m(\angle A) = 70°$, $\overline{DE} \parallel \overline{BC}$

Find : $m(\overset{\frown}{BD})$

(El-Dakahlia 17 , El-Sharkia 13) « 70° »

10

M , N are two touching externally circles at A , $\overleftrightarrow{BA}$, $\overleftrightarrow{CA}$ are two secants cut the circle M at B , C and the circle N at D , E respectively, $m(\angle BMC) = 140°$

Find : $m(\overset{\frown}{ED})$

(El-Dakahlia 2016) « 140° »

11	**In the opposite figure :** $\overrightarrow{CB} \cap \overrightarrow{HD} = \{A\}$, m (∠ A) = 40° $\overline{DC} \cap \overline{BH} = \{X\}$ and m (∠ DCB) = 26° **Find :** (1) m $(\overset{\frown}{CH})$ (2) m (∠ HXC) *(El-Gharbia 17 , Ismailia 16)* « 132° , 92° »
12	**In the opposite figure :** ABC is an inscribed triangle in circle M , m $(\overset{\frown}{AB})$: m $(\overset{\frown}{BC})$: m $(\overset{\frown}{AC})$ = 4 : 5 : 3 **Find :** m (∠ ACB) *(Alexandria 16)* « 60° »
13	🕮 **In the opposite figure :** AB = AC , E ∈ $\overset{\frown}{BC}$ **Prove that :** m (∠ AEB) = m (∠ AEC) *(Souhag 2015)*
14	🕮 **In the opposite figure :** ABC is a triangle inscribed in a circle , $\overline{DE}$ // $\overline{BC}$ **Prove that :** m (∠ DAC) = m (∠ BAE) *(El-Gharbia 2016 , Fayoum 2015 , Qena 2013)*
15	🕮 **In the opposite figure :** $\overline{AB} \cap \overline{CD} = \{E\}$, EA = ED **Prove that :** EB = EC *(El-Sharkia 2016 , Suez 2015 , El-Beheira 2014 , S. Sinai 2013)*
16	**In the opposite figure :** $\overline{AB}$ is a diameter in the circle M , C ∈ the circle M , m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$, $\overline{DB} \cap \overline{AC} = \{H\}$ (1) **Find :** m (∠ BDC) and m $(\overset{\frown}{AD})$ (2) **Prove that :** $\overline{AB}$ // $\overline{DC}$ *(Cairo 17)* « 30° , 60° »

Solutions

1

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\overset{\frown}{AB}) = 180°$, $\because m(\overset{\frown}{CD}) = 80°$

$\therefore m(\overset{\frown}{AC}) + m(\overset{\frown}{BD}) = 180° - 80° = 100°$

, $\because \overline{AB} \parallel \overline{CD}$

$\therefore m(\overset{\frown}{AC}) = m(\overset{\frown}{BD}) = \frac{100°}{2} = 50°$

, $\because m(\angle DHB) = \frac{1}{2} m(\overset{\frown}{BD})$

$\therefore m(\angle DHB) = \frac{1}{2} \times 50° = 25°$

, $\because m(\angle AOH) = \frac{1}{2}\left[m(\overset{\frown}{AH}) + m(\overset{\frown}{BD})\right]$

$\therefore m(\angle AOH) = \frac{1}{2}[100° + 50°] = 75°$ (The req.)

2

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90°$

$\because m(\angle ABC) = 180° - (90° + 35°) = 55°$

similarly $m(\angle ADB) = 90°$

, $\because$ the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{DB}$

$\therefore AD = DB$, from ΔABD :

$\therefore m(\angle DBA) = m(\angle DAB) = \frac{180° - 90°}{2} = 45°$

$\therefore m(\angle CBD) = 55° + 45° = 100°$ (The req.)

3

$\because m(\overset{\frown}{AC}) = 2\, m(\angle ABC) = 140°$

$\therefore$ D is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AD}) = \frac{140°}{2} = 70°$

$\therefore m(\angle DCA) = 35°$ (First req.)

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90°$

$\therefore m(\angle CAB) = 180° - (90° + 70°) = 20°$ (Second req.)

4

$\because \overleftrightarrow{AC}$ touches the circle at A $\therefore \overline{MA} \perp \overline{AC}$

In ΔABC : $(CB)^2 = (AB)^2 + (AC)^2 = (12)^2 + (9)^2 = 225$

$\therefore CB = 15$ cm. (First req.)

$\because \overline{AB}$ is a diameter $\therefore m(\angle ADB) = 90°$

$\therefore AD = \frac{AC \times AB}{BC} = \frac{9 \times 12}{15} = 7.2$ cm. (Second req.)

5

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\overset{\frown}{AB}) = 180°$

, $\because m(\overset{\frown}{AC}) = 2\, m(\angle ABC) = 2 \times 40° = 80°$

$\therefore m(\overset{\frown}{BDC}) = 180° - 80° = 100°$

, $\because \overleftrightarrow{DH} \parallel \overline{BC}$

$\therefore m(\overset{\frown}{CD}) = m(\overset{\frown}{BD}) = \frac{100°}{2} = 50°$ (The req.)

6

$\because m(\angle ACD) = \frac{1}{2} m(\angle AMD)$

(inscribed and central angles subtended by $\overset{\frown}{AD}$)

$\therefore m(\angle ACD) = \frac{1}{2} \times 70° = 35°$

, $\because \overline{DC} \parallel \overline{AB}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle A) = m(\angle ACD) = 35°$ (alternate angles)

, $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90°$

$\therefore$ From ΔABC : $m(\angle ABC) = 180° - (90° + 35°) = 55°$

(The req.)

7

$\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle C) = 90°$

, $\because \overline{MD} \perp \overline{AC}$

$\therefore m(\angle ADM) = 90°$

$\therefore m(\angle C) = m(\angle ADM) = 90°$

and they are corresponding angles

$\therefore \overline{DM} \parallel \overline{BC}$ (Q.E.D.1)

, $\because \Delta ABC$ is right-angled at C

, $m(\angle A) = 30°$

$\therefore BC = \frac{1}{2} AB$, $\because \overline{AB}$ is a diameter in the circle M

$\therefore$ BC = the radius length of the circle (Q.E.D.2)

8

$\because \frac{1}{2}\left[m(\overset{\frown}{EC}) - 60°\right] = 40°$ $\therefore m(\overset{\frown}{EC}) - 60° = 80°$

$\therefore m(\overset{\frown}{EC}) = 140°$ (First req.)

$\because m(\overset{\frown}{BD}) + m(\overset{\frown}{BC}) + m(\overset{\frown}{CE}) + m(\overset{\frown}{DE}) = 360°$

, $\because m(\overset{\frown}{BC}) = m(\overset{\frown}{DE})$

$\therefore 60° + 2\, m(\overset{\frown}{BC}) + 140° = 360°$

$\therefore m(\overset{\frown}{BC}) = 80°$ (Second req.)

9

$\because m(\angle A) = \frac{1}{2}\left[m(\overset{\frown}{BC}) - m(\overset{\frown}{DE})\right]$

$\therefore 70° = \frac{1}{2}\left[180° - m(\overset{\frown}{DE})\right]$

$\therefore 140° = 180° - m(\overset{\frown}{DE})$

$\therefore m(\overset{\frown}{DE}) = 180° - 140° = 40°$

, $\because \overline{DE} \parallel \overline{BC}$ $\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{CE})$

, $\because m(\overset{\frown}{BD}) + m(\overset{\frown}{DE}) + m(\overset{\frown}{CE}) = 180°$

$\therefore m(\overset{\frown}{BD}) + 40° + m(\overset{\frown}{BD}) = 180°$

$\therefore 2\, m(\overset{\frown}{BD}) = 180° - 40° = 140°$

$\therefore m(\overset{\frown}{BD}) = \frac{140°}{2} = 70°$ (The req.)

10

$\because$ m (∠ BAC) = $\frac{1}{2}$ m (∠ M)
(inscribed and central angles subtended by the same arc)
$\therefore$ m (∠ BAC) = 70°
, $\because$ $\overline{BD} \cap \overline{CE} = \{A\}$
$\therefore$ m (∠ EAD) = m (∠ BAC) = 70° (V.O.A.)
$\therefore$ m ($\overset{\frown}{ED}$) = 2 m (∠ EAD) = 2 × 70° = 140°
(The req.)

11

$\because$ m ($\overset{\frown}{BD}$) = 2 m (∠ BCD)
$\therefore$ m ($\overset{\frown}{BD}$) = 2 × 26° = 52°
, $\because$ m (∠ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) – m ($\overset{\frown}{BD}$)]
$\therefore$ 40° = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) – 52°]
$\therefore$ m ($\overset{\frown}{CE}$) – 52° = 80°
$\therefore$ m ($\overset{\frown}{CE}$) = 132° (First req.)
, $\because$ m (∠ HXC) = $\frac{1}{2}$ [m ($\overset{\frown}{CH}$) + m ($\overset{\frown}{BD}$)]
$\therefore$ m (∠ HXC) = $\frac{1}{2}$ [132° + 52°] = 92°
(Second req.)

12

$\because$ m ($\overset{\frown}{AB}$) : m ($\overset{\frown}{BC}$) : m ($\overset{\frown}{AC}$) = 4 : 5 : 3
$\therefore$ m ($\overset{\frown}{AB}$) = 4 x , m ($\overset{\frown}{BC}$) = 5 x , m ($\overset{\frown}{AC}$) = 3 x
$\therefore$ 4 x + 5 x + 3 x = 360° $\therefore$ 12 x = 360°
$\therefore$ $x = \frac{360°}{12}$ = 30° $\therefore$ m ($\overset{\frown}{AB}$) = 4 × 30° = 120°
$\therefore$ m (∠ ACB) = $\frac{1}{2}$ × 120° = 60° (The req.)

13

$\because$ AB = AC $\therefore$ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{AC}$)
$\therefore$ m (∠ AEB) = m (∠ AEC) (Q.E.D.)

14

$\because$ $\overline{DE}$ // $\overline{BC}$ $\therefore$ m ($\overset{\frown}{DB}$) = m ($\overset{\frown}{EC}$)
$\therefore$ m (∠ DAB) = m (∠ EAC)
Adding m (∠ BAC) to both sides
$\therefore$ m (∠ DAC) = m (∠ BAE) (Q.E.D.)

15

$\because$ m (∠ A) = m (∠ C)
(two inscribed angles subtended by $\overset{\frown}{BD}$)
, m (∠ B) = m (∠ D)
(two inscribed angles subtended by $\overset{\frown}{AC}$)
$\because$ EA = ED $\therefore$ m (∠ A) = m (∠ D)
$\therefore$ m (∠ B) = m (∠ C) $\therefore$ EB = EC (Q.E.D.)

16

$\because$ m (∠ BDC) = m (∠ BAC)
(two inscribed angles subtended by $\overset{\frown}{BC}$)
$\therefore$ m (∠ BDC) = 30°
, $\because$ m ($\overset{\frown}{BC}$) = 2 m (∠ BDC) = 2 × 30° = 60°
, $\because$ $\overline{AB}$ is a diameter of the circle M
$\therefore$ m ($\overset{\frown}{AB}$) = 180°
$\therefore$ m ($\overset{\frown}{AC}$) = 180° – 60° = 120°
, $\because$ D is the midpoint of $\overset{\frown}{AC}$
$\therefore$ m ($\overset{\frown}{AD}$) = $\frac{120°}{2}$ = 60° (First req.)
, $\because$ m (∠ ACD) = $\frac{1}{2}$ m ($\overset{\frown}{AD}$)
$\therefore$ m (∠ ACD) = $\frac{1}{2}$ × 60° = 30°
$\therefore$ m (∠ ACD) = m (∠ BAC) = 30°
and they are alternate angles
$\therefore$ $\overline{AB}$ // $\overline{CD}$ (Second req.)

Exercises

[A] : Choose The Correct Answer : -

1	If two chords intersect at a point inside a circle then the measure of the included angle equals .... of the two opposite arcs. (a) half of the difference (b) half of the sum (c) twice the sum (d) twice the difference
2	**In the opposite figure :** M is a circle in which $\overline{AB} \perp \overline{CD}$, then m $(\widehat{AC})$ + m $(\widehat{BD})$ = ...° (a) 45 (b) 90 (c) 180 (d) 270
3	**In the opposite figure :** If m $(\widehat{AC}) = 50°$, m $(\widehat{BD}) = 110°$, then m (∠ E) = ...° (a) 60 (b) 50 (c) 40 (d) 30
4	**In the opposite figure :** If $\overline{AB}$ is a diameter in circle , m $(\widehat{AC})$ = m $(\widehat{CD})$ = m $(\widehat{DE})$ = m $(\widehat{EF})$ = m $(\widehat{FB})$, then m (∠ DXE) = ... (a) 72° (b) 54° (c) 36° (d) 18°
5	The length of the arc which represents $\frac{1}{4}$ of the perimeter of the circle = ... (a) 2 π r (b) π r (c) $\frac{1}{2}$ π r (d) 4 π r
6	**In the opposite figure :** If M is centre of the circle , then m (∠ CMB) = ...° (a) 36 (b) 72 (c) 144 (d) 180
7	**In the opposite figure :** $\overline{AB}$ is a diameter of a circle M , m (∠ B) = 30° , AC = 6 cm. , then AB = ... cm. (a) 3 (b) 6 (c) 9 (d) 12

8	**In the opposite figure :** $m(\angle ABC) = 60°$, $m(\angle AMC) = (y + 20)°$, then y = ...° (a) 30 (b) 40 (c) 80 (d) 100
9	**In the opposite figure :** Circle of centre M. If $m(\widehat{AB}) = 50°$, then $m(\angle ADB) =$ .... (a) 25° (b) 50° (c) 100° (d) 150°
10	The inscribed angle which opposite to the minor arc in a circle is .... (a) reflex. (b) right. (c) obtuse. (d) acute.
11	**In the opposite figure :** Two concentric circles. If the lengths of their radii are 2 cm. and 5 cm. , then $\frac{m(\widehat{AB})}{m(\widehat{CD})} =$ .... (a) $\frac{2}{5}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{3}{5}$
12	**In the opposite figure :** $m(\angle AHC) =$ .... (a) 25° (b) 50° (c) 75° (d) 100°
13	**In the opposite figure :** If $m(\widehat{XZ}) = 60°$, $m(\widehat{YN}) = 40°$, then $m(\angle E) =$ ....° (a) 10 (b) 30 (c) 20 (d) 40
14	**In the opposite figure :** Two concentric circles with centre M , the radii lengths of them are 6 cm. and 3 cm. , if $m(\widehat{AB}) = 60°$, then $m(\widehat{DC}) =$ .... (a) 60° (b) 30° (c) 120° (d) 40°
15	The measure of the arc that is opposite the inscribed angle of measure 60° = ...° (a) 60 (b) 30 (c) 120 (d) 90

16	The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc. (a) half (b) third (c) quarter (d) double
17	**In the opposite figure :** $\overline{AB}$ is a diameter of circle M , m (∠ ABC) = 50° , then m ($\widehat{BC}$) = ...° (a) 40 (b) 50 (c) 80 (d) 100
18	**In the opposite figure :** M is a circle , m (∠ BAC) = 55° , then m (∠ MCB) = ...° (a) 110 (b) 55 (c) 35 (d) 25
19	**In the opposite figure :** ABC is an equilateral triangle inscribed in circle M , then m (∠ BMC) = ... (a) 50° (b) 120° (c) 60° (d) 100°
20	**In the opposite figure :** $\overline{AM}$ // $\overline{CD}$, MD = DB , m (∠ AMB) = 90° , then m ($\widehat{AC}$) = ... (a) 45° (b) 60° (c) 30° (d) 90°
21	**In the opposite figure :** In a circle M , $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 80° , then m ($\widehat{AC}$) = ... (a) 20° (b) 40° (c) 80° (d) 160°
22	**In the opposite figure :** If m ($\widehat{AC}$) = 30° , $\overline{AB}$ // $\overline{CD}$, then m (∠ BED) = ... (a) 10° (b) 15° (c) 30° (d) 60°
23	The measure of the inscribed angle drawn in a semicircle equals .... (a) 45° (b) 90° (c) 120° (d) 80°

24 **In the opposite figure :**

m ($\overset{\frown}{CE}$) = 100°

, m ($\overset{\frown}{BD}$) = 30°

, then m (∠ A) = ...°

(a) 70 (b) 65 (c) 50 (d) 35

25 **In the opposite figure :**

$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40°

(c) 30° (d) 45°

26 The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 120° (c) 90° (d) 240°

27 The ratio between the measure of the inscribed angle to the measure of the central angle subtended by the same arc equals ....

(a) 2 : 1 (b) 1 : 2 (c) 2 : 2 (d) 2 : 3

28 **In the opposite figure :**

If C is the midpoint of $\overset{\frown}{AB}$

, then AB .... 2 AC

(a) < (b) > (c) ≥ (d) =

29 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABC) = 40° , then m ($\overset{\frown}{BC}$) = ...

(a) 40° (b) 50°

(c) 90° (d) 100°

30 **In the opposite figure :**

In the circle M ,

if m (∠ M) – m (∠ A) = 50°

, then m (∠ A) = ...

(a) 40° (b) 50° (c) 100° (d) 130°

31 **In the opposite figure :**

If m (∠ ABC) = 40° , then m (∠ AMC) = ...°

(a) 20 (b) 40

(c) 80 (d) 140

[B] : Essay Problems : -

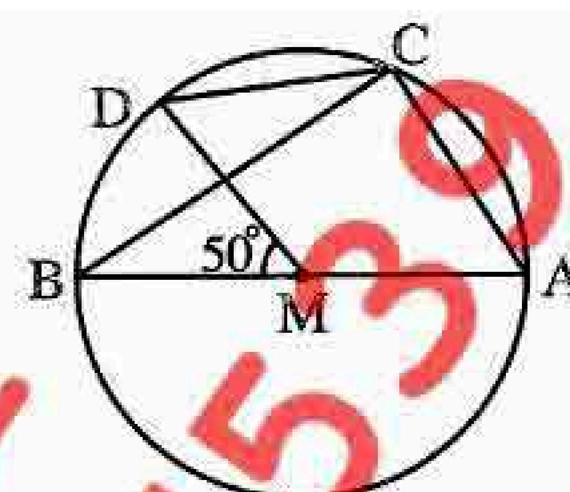
1 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

m (∠ BMD) = 50°

Find with proof :

m (∠ ACD)



(Damietta 14) « 115° »

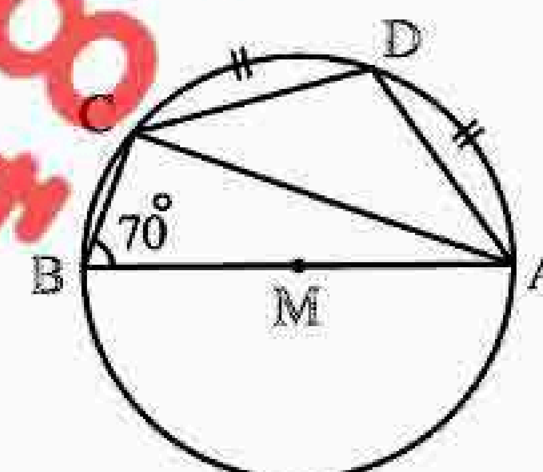
2 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

the length of $(\overset{\frown}{AD})$ = the length of $(\overset{\frown}{DC})$,

m (∠ ABC) = 70°

Find each of : m (∠ DCA) , m (∠ CAB)



(El-Ismailia 05) « 35° , 20° »

3 $\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord such that m (∠ BAC) = 30° , draw $\overline{BC}$ and draw $\overline{MD} \perp \overline{AC}$ and to intersect it at D

(1) **Prove that :** $\overline{MD}$ // $\overline{BC}$

(2) **Prove that :** length of $\overline{BC}$ = length of the radius of this circle. *(El-Monofia 17)*

4 M , N are two touching externally circles at A , $\overleftrightarrow{BA}$, $\overleftrightarrow{CA}$ are two secants cut the circle M at B , C and the circle N at D , E respectively, m (∠ BMC) = 140°

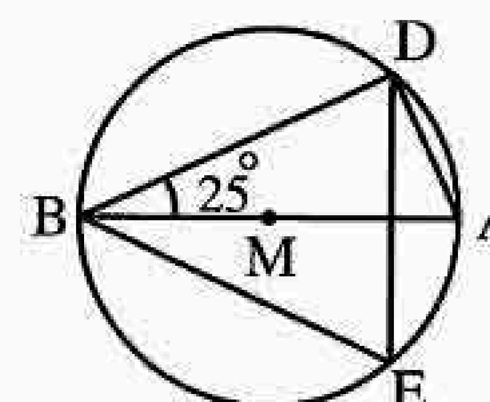
Find : m $(\overset{\frown}{ED})$ *(El-Dakahlia 2016)* « 140° »

5 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

Find : m (∠ DEB) in degrees.



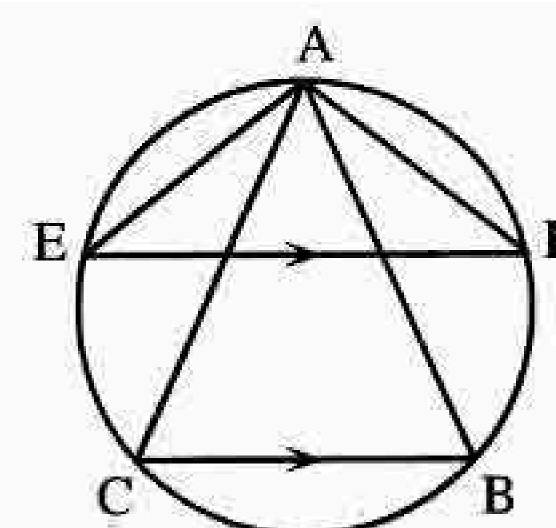
(Suez 2011) « 65° »

6 🕮 **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



(El-Gharbia 2016 , Fayoum 2015 , Qena 2013)

7

In the opposite figure :

$\overline{AD}$ and $\overline{BE}$ are two equal chords in length in the circle

, $\overrightarrow{AD} \cap \overrightarrow{BE} = \{C\}$

Prove that : CD = CE

(Beni Suef 2014 , El-Kalyoubia 2013)

8

In the opposite figure :

ABC is an equilateral triangle inscribed in a circle

, $D \in \widehat{AB}$, $E \in \overline{DC}$, where AD = DE

Prove that : The triangle ADE is equilateral.

(Matrouh 2016 , Fayoum 2015 , Alex. 2011)

9

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

the length of $\widehat{AD}$ = the length of $\widehat{BD}$,

m (∠ CAB) = 35°

Find by proof : m (∠ CBD)

(El-Menia 11) « 100° »

10

In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

$\overline{DC}$ // $\overline{AB}$, m (∠ AMD) = 70°

Find by proof : m (∠ ACD) , m (∠ ABC)

(El-Menia 17) « 35° , 55° »

11

In the opposite figure :

M is a circle , $\overline{BC}$ is a diameter in it

, m (∠ A) = 70° , $\overline{DE}$ // $\overline{BC}$

Find : m $(\widehat{BD})$

(El-Dakahlia 17 , El-Sharkia 13) « 70° »

12

ABC is an equilateral triangle inscribed in the circle M

Draw the diameter $\overline{CD}$

Prove that : m (∠ ABD) = m (∠ CBM) = m (∠ ACD)

(Beni Suef 2008)

Homework

[A] : Choose The Correct Answer : -

1 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$

, m $(\overset{\frown}{AC}) = 60°$

, m $(\overset{\frown}{BD}) = 100°$, then m (∠ DEB) = ...°

(a) 160 (b) 60 (c) 80 (d) 100

2 **In the opposite figure :**

m (∠ E) = 35°

, m (∠ C) = 20°

, then m $(\overset{\frown}{AC})$ = ...°

(a) 135 (b) 110 (c) 65 (d) 55

3 The measure of the arc which equals half the measure of the circle equals ....

(a) 360° (b) 180° (c) 120° (d) 90°

4 The measure of the central angle is .... the measure of the arc which is opposite to it.

(a) twice (b) half (c) equals (d) more than

5 **In the opposite figure :**

If m (∠ AMB) = 52°

, then m $(\overset{\frown}{ADB})$ = ...°

(a) 52 (b) 104 (c) 128 (d) 308

6 **In the opposite figure :**

$\overline{AB}$ is a diameter in a circle M

, m (∠ CAB) = 45°

, then m (∠ ABC) = ...°

(a) 40 (b) 45 (c) 50 (d) 90

7 **In the opposite figure :**

M is a circle , m (∠ MBC) = 32°

, then m ($\overset{\frown}{BC}$ the minor) = ...

(a) 116° (b) 23° (c) 58° (d) 64°

8

In the opposite figure :

m (∠ ACB) = ...

(a) 40° (b) 80°

(c) 90° (d) 180°

9

The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

10

In the opposite figure :

Two concentric circle M , m $(\widehat{BC}) = 80°$

, if the radius length of the smaller circle is 7 cm.

and the redius length of the larger circle is 14 cm. , $\left(\pi = \frac{22}{7}\right)$, then :

First : The perimeter of the smaller circle = ... cm.

(a) 44 (b) 22 (c) 154 (d) 88

Second : m $(\widehat{AD})$ = ...°

(a) 80 (b) 40 (c) 20 (d) 160

11

If two chords intersect at a point inside a circle then the measure of the included angle equals .... of the two opposite arcs.

(a) half of the difference (b) half of the sum

(c) twice the sum (d) twice the difference

12

The inscribed angle which opposite to the minor arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

13

In the opposite figure :

$\overline{AM} \parallel \overline{CD}$, MD = DB

, m (∠ AMB) = 90° , then m $(\widehat{AC})$ = ...

(a) 45° (b) 60°

(c) 30° (d) 90°

14

In the opposite figure :

In the circle M ,

if m (∠ M) – m (∠ A) = 50°

, then m (∠ A) = ...

(a) 40° (b) 50° (c) 100° (d) 130°

15

In the opposite figure :

Circle of centre M

If m $(\widehat{AB})$ = 50° , then m (∠ ADB) = ...

(a) 25° (b) 50° (c) 100° (d) 150°

16	**In the opposite figure :** ABC is an equilateral triangle inscribed in circle M , then m (∠ BMC) = ... (a) 50° (b) 120° (c) 60° (d) 100°
17	**In the opposite figure :** $\overline{AB}$ is a diameter in the circle M , m (∠ ABC) = 40° , then m $(\widehat{BC})$ = ... (a) 40° (b) 50° (c) 90° (d) 100°
18	**In the opposite figure :** m (∠ ABC) = 60° , m (∠ AMC) = (y + 20)° , then y = ...° (a) 30 (b) 40 (c) 80 (d) 100
19	**In the opposite figure :** M is a circle , m (∠ BAC) = 55° , then m (∠ MCB) = ...° (a) 110 (b) 55 (c) 35 (d) 25
20	**In the opposite figure :** If C is the midpoint of $\widehat{AB}$, then AB .... 2 AC (a) < (b) > (c) ≥ (d) =
21	**In the opposite figure :** $\overline{AB}$ is a diameter of a circle M , m (∠ B) = 30° , AC = 6 cm. , then AB = ... cm. (a) 3 (b) 6 (c) 9 (d) 12
22	**In the opposite figure :** $\overline{AB}$ is a diameter of circle M , m (∠ ABC) = 50° , then m $(\widehat{BC})$ = ...° (a) 40 (b) 50 (c) 80 (d) 100

23 The ratio between the measure of the inscribed angle to the measure of the central angle subtended by the same arc equals ....

(a) 2 : 1 (b) 1 : 2 (c) 2 : 2 (d) 2 : 3

24 **In the opposite figure :**

If M is centre of the circle , then m (∠ CMB) = ...°

(a) 36 (b) 72

(c) 144 (d) 180

25 The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc.

(a) half (b) third (c) quarter (d) double

26 The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 120° (c) 90° (d) 240°

27 The length of the arc which represents $\frac{1}{4}$ of the perimeter of the circle = ...

(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

28 The measure of the arc that is opposite the inscribed angle of measure 60° = ...°

(a) 60 (b) 30 (c) 120 (d) 90

29 **In the opposite figure :**

$\overline{AB}$ // $\overline{CD}$, m (∠ AWC) = 40° ,

then m (∠ DEB) = ...

(a) 50° (b) 40°

(c) 30° (d) 45°

30 **In the opposite figure :**

If $\overline{AB}$ is a diameter in circle

, m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{EF}$) = m ($\overset{\frown}{FB}$)

, then m (∠ DXE) = ...

(a) 72° (b) 54° (c) 36° (d) 18°

31 **In the opposite figure :**

Two concentric circles with centre M

, the radii lengths of them are 6 cm. and 3 cm.

, if m ($\overset{\frown}{AB}$) = 60° , then m ($\overset{\frown}{DC}$) = ...

(a) 60° (b) 30° (c) 120° (d) 40°

32 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 120° (d) 80°

[B] : Essay Problems : -

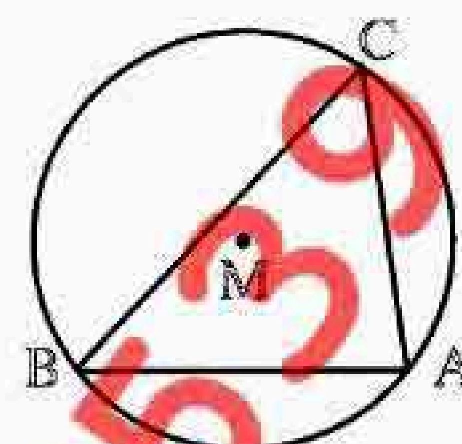
1

In the opposite figure :

ABC is an inscribed triangle in circle M

, m $(\overset{\frown}{AB})$: m $(\overset{\frown}{BC})$: m $(\overset{\frown}{AC})$ = 4 : 5 : 3

Find : m (∠ ACB)



(Alexandria 16) « 60° »

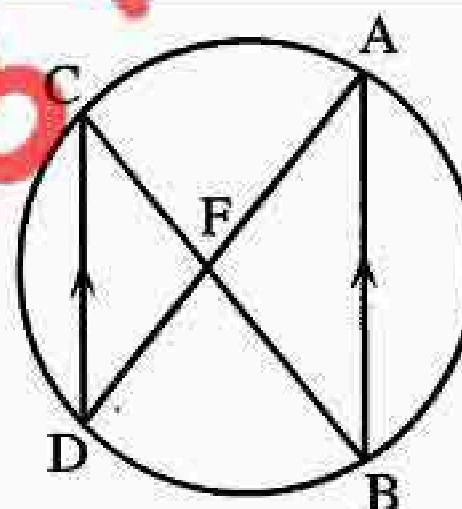
2

In the opposite figure :

$\overline{AB}$ and $\overline{CD}$ are two parallel chords in the circle

, $\overline{AD} \cap \overline{CB} = \{F\}$

Prove that : AF = FB



(Kafr El-Sheikh 2008)

3

In the opposite figure :

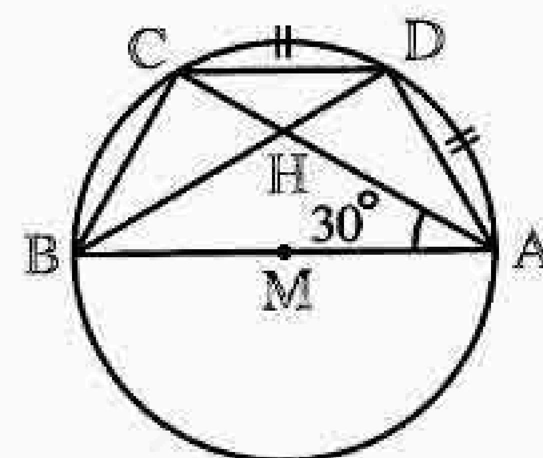
$\overline{AB}$ is a diameter in the circle M , C ∈ the circle M ,

m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$,

$\overline{DB} \cap \overline{AC} = \{H\}$

(1) **Find :** m (∠ BDC) and m $(\overset{\frown}{AD})$

(2) **Prove that :** $\overline{AB}$ // $\overline{DC}$



(Cairo 17) « 30° , 60° »

4

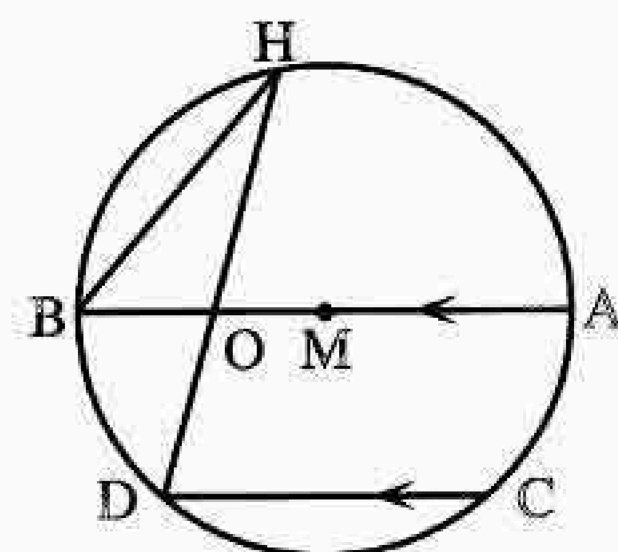
In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

$\overline{AB}$ // $\overline{DC}$, m $(\overset{\frown}{DC})$ = 80° ,

m $(\overset{\frown}{AH})$ = 100°

Find by proof : m (∠ DHB) , m (∠ AOH)



(El-Menia 17) « 25° , 75° »

5

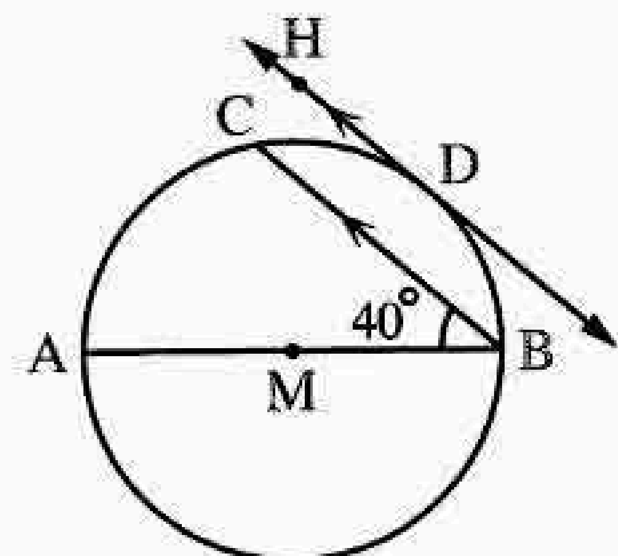
In the opposite figure :

$\overline{AB}$ is a diameter in the circle M ,

M (∠ B) = 40° , $\overleftrightarrow{DH}$ is a tangent to the circle M at D ,

$\overleftrightarrow{DH}$ // $\overline{BC}$

Find : m $(\overset{\frown}{DC})$



(El-Monofia 17) « 50° »

6 **In the opposite figure :**

$m(\angle A) = 40^\circ$, $m(\overset{\frown}{BD}) = 60^\circ$

, $m(\overset{\frown}{BC}) = m(\overset{\frown}{DE})$

Find : (1) $m(\overset{\frown}{EC})$ (2) $m(\overset{\frown}{BC})$

(Port Said 17 , North Sinai 17) « 140° , 80° »

7 **In the opposite figure :**

D is the midpoint of $\overset{\frown}{AB}$

Prove that :

$m(\angle ACD) = \frac{1}{4} m(\angle ANB)$

(Beni Suef 04)

8 🕮 **In the opposite figure :**

$AB = AC$, $E \in \overset{\frown}{BC}$

Prove that :

$m(\angle AEB) = m(\angle AEC)$

(Souhag 2015)

9 🕮 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two equal chords in length in the circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

Prove that : The triangle ACE is an isosceles triangle.

(El-Kalyoubia 2011)

10 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{AC}$ touches the circle at A

If AC = 9 cm. , BM = 6 cm.

Find the length of each of : $\overline{BC}$, $\overline{AD}$

(Souhag 17 , Kafr El-Sheikh 04) « 15 cm. , 7.2 cm. »

11 **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle ,

$\overline{AB} \cap \overline{CD} = \{E\}$, if $m(\overset{\frown}{BD}) = 60^\circ$, $m(\overset{\frown}{AD}) = 100^\circ$,

$m(\overset{\frown}{AC}) = 120^\circ$

Calculate : (1) $m(\overset{\frown}{CB})$ (2) $m(\angle CEB)$

(Alex. 05) « 80° , 90° »

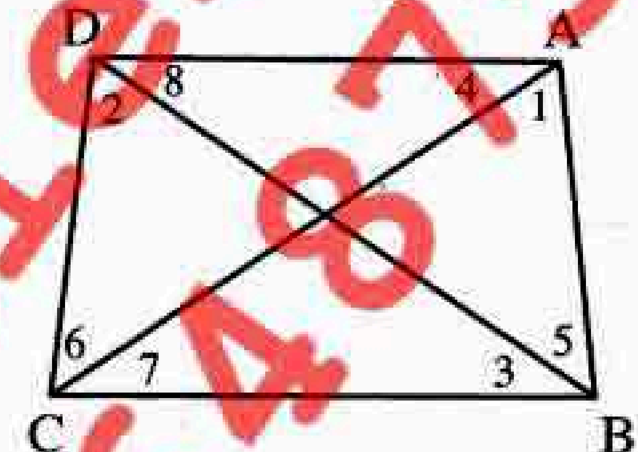
Lesson [4] : Cyclic Quadrilaterals

The cyclic quadrilateral

It is a quadrilateral figure whose four vertices belong to one circle.

Generally

If the figure ABCD is a cyclic quadrilateral , then we can deduce the following :

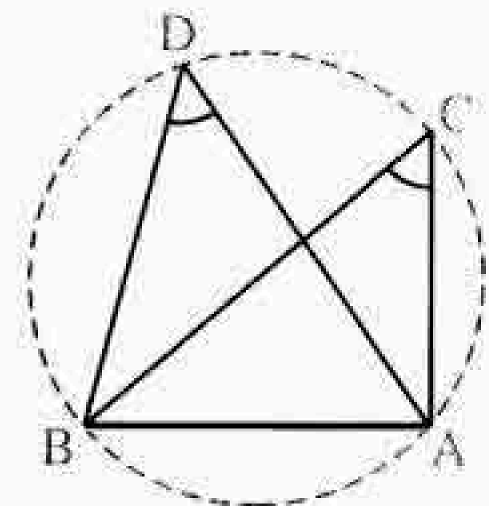


- m (∠ 1) = m (∠ 2) , m (∠ 3) = m (∠ 4) , m (∠ 5) = m (∠ 6) , m (∠ 7) = m (∠ 8)
- The line segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$ and $\overline{BD}$ are chords in this circle.
- $\overline{AC}$ is a diameter of the circle which passes through the points A , B , C and D if the measure of each of the two angles ABC and ADC equals 90°
- Similarly , $\overline{BD}$ is a diameter of the circle which passes through the points A , B , C and D if the measure of each of the two angles BAD and BCD equals 90°

The converse of theorem 2 (without proof)

If two angles subtended by the same base and on the same side of it have the same measure , then their vertices are on an arc of a circle and the base is a chord of it.

In the opposite figure :



If ∠ C and ∠ D are drawn on the base $\overline{AB}$ and on the same side of it , m (∠ C) = m (∠ D) , then the points A , B , C and D lie on a unique circle , then $\overline{AB}$ is a chord of it.

From the previous, we deduce that :

The quadrilateral in which there are two angles drawn on one of its sides as a base and their vertices are two vertices of the quadrilateral and equal in measure and on one side of this side , is a cyclic quadrilateral.

Remarks

1. If there are two angles drawn on one of the sides of a quadrilateral , they are on the same side of it and they are not equal in measure , then the quadrilateral is not cyclic.
2. Each of the rectangle , the square and the isosceles trapezium are cyclic quadrilaterals while each of the parallelogram, the rhombus and the trapezium that is not isosceles are not cyclic quadrilaterals.

Lesson [5] : Properties Of Cyclic Quadrilaterals

Theorem 3

In a cyclic quadrilateral , each two opposite angles are supplementary.

Corollary

The measure of the exterior angle at a vertex of a cyclic quadrilateral is equal to the measure of the interior angle at the opposite vertex.

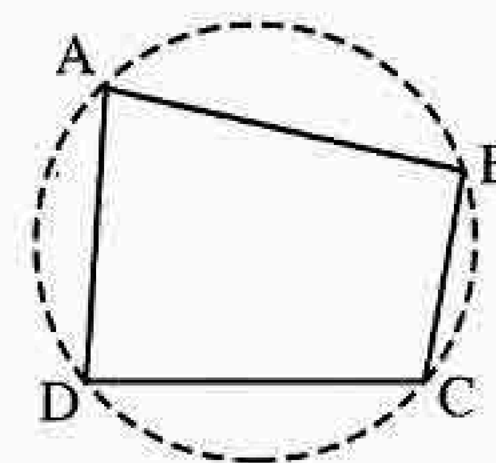
Very Important Notes :

If two opposite angles of a quadrilateral are supplementary , then the quadrilateral is cyclic.

In the opposite figure :

If $m(\angle B) + m(\angle D) = 180°$ or $m(\angle A) + m(\angle C) = 180°$

, then the figure ABCD is a cyclic quadrilateral



An important remark

If one of a cyclic quadrilateral's angles is right , then the diagonal opposite to this angle is a diameter of the circumcircle of this cyclic quadrilateral and the midpoint of this diagonal is the centre of this circle.

Corollary

If the measure of the exterior angle at a vertex of a quadrilateral figure is equal to the measure of the interior angle at the opposite vertex , then the figure is a cyclic quadrilateral.

A summary of the cases in which the quadrilateral is cyclic :

The quadrilateral is cyclic if one of the following conditions is verified :

1. If there is a point in the plane of the figure such that it is equidistant from its vertices.
2. If there are two equal angles in measure and drawn on one of its sides as a base and on one side of this side.
3. If there are two opposite supplementary angles «their sum = 180°»
4. If there is an exterior angle at any of its vertices equal in measure to the measure of the interior angle at the opposite vertex.

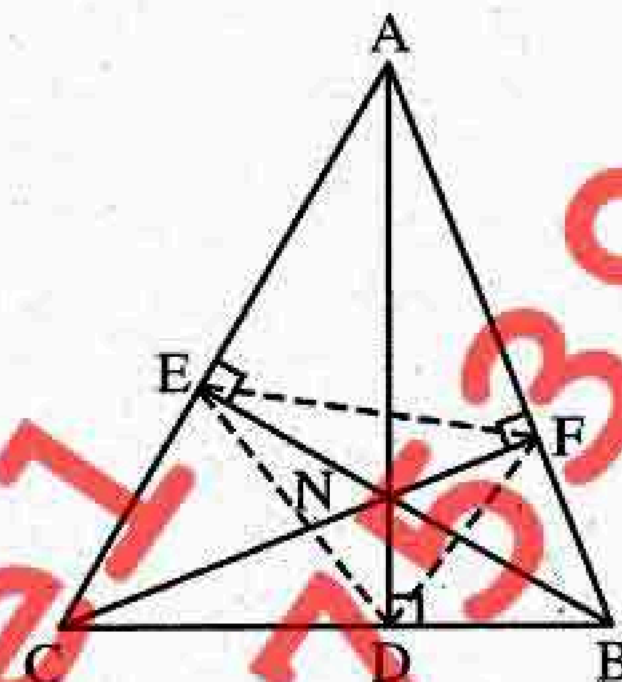
Remarks

In the opposite figure :

If $\overline{AD}$, $\overline{BE}$, $\overline{CF}$ are the altitudes of Δ ABC , then :

- $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are concurrent at one point (say N)
- From the figure we can get six cyclic quadrilaterals , they are :

NFBD , NECD , NFAE , FBCE , DCAF and EABD

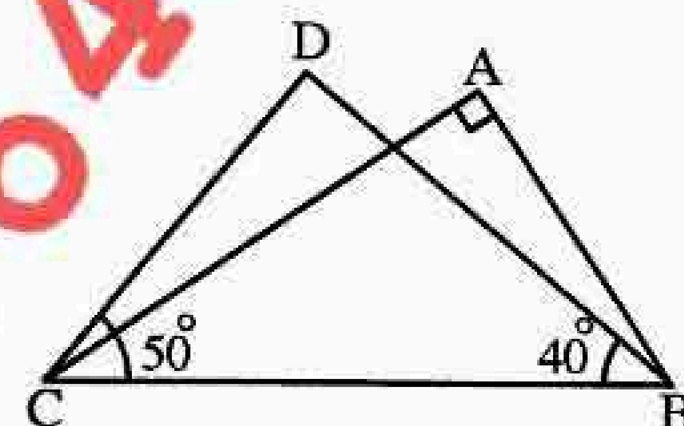


Examples :

1

In the opposite figure :

m (∠ A) = 90° , m (∠ DBC) = 40° , m (∠ DCB) = 50°



(1) **Prove that :** The figure ABCD is a cyclic quadrilateral

(2) Determine where is the center of the circle passes through the vertices of the figure ABCD

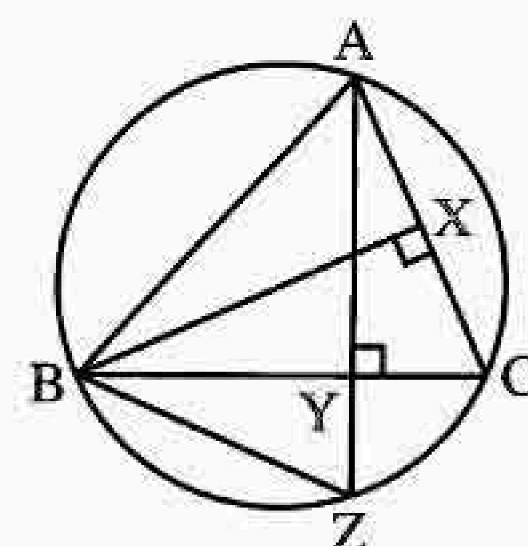
(Cairo 2014)

2

In the opposite figure :

ABC is a triangle drawn in a circle , $\overline{BX} \perp \overline{AC}$, $\overrightarrow{AY} \perp \overline{BC}$

cuts it at Y and cuts the circle at Z



Prove that :

(1) ABYX is a cyclic quadrilateral.

(2) $\overrightarrow{BC}$ bisects ∠ XBZ

(El-Gharbia 17 , El-Beheira 17)

3

ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and intersects $\overrightarrow{BD}$ at X , $\overrightarrow{DY}$ bisects ∠ CDB and intersects $\overline{AC}$ at Y

Prove that :

(1) AXYD is a cyclic quadrilateral.

(2) m (∠ AYX) = 45°

(Alexandria 2016 , Sharkia 2012)

4

ABC is a triangle inscribed in a circle , X ∈ $\widehat{AB}$, Y ∈ $\widehat{AC}$, where m $(\widehat{AX})$ = m $(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$

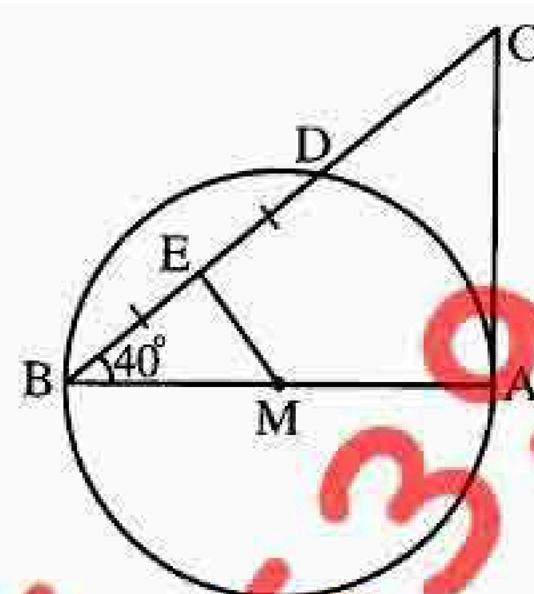
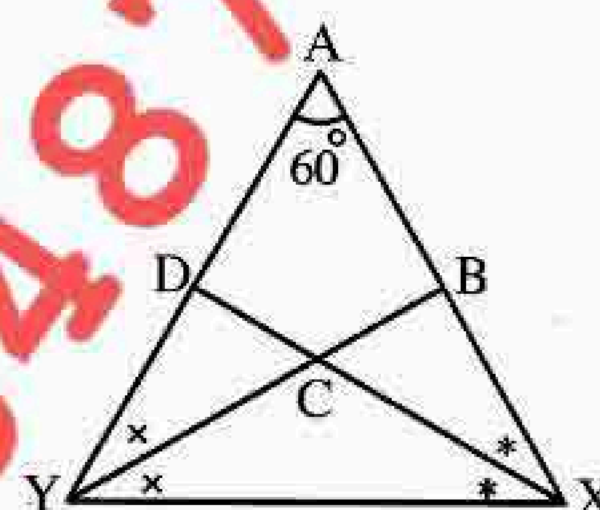
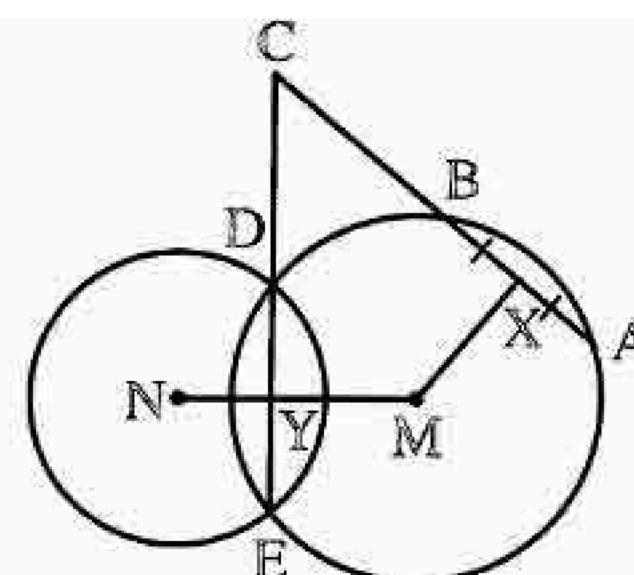
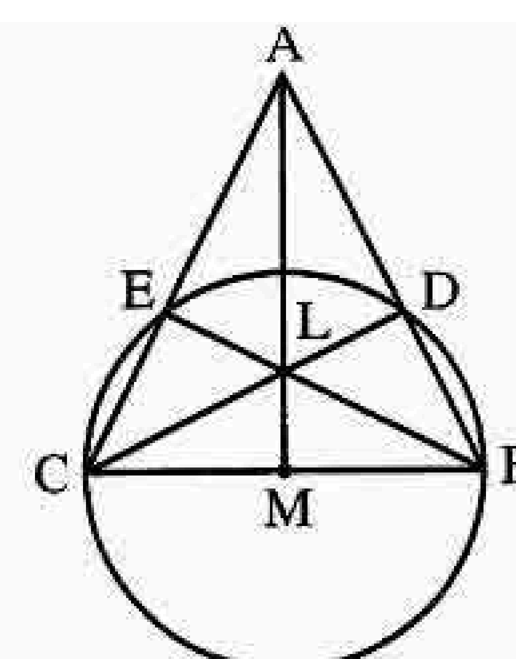
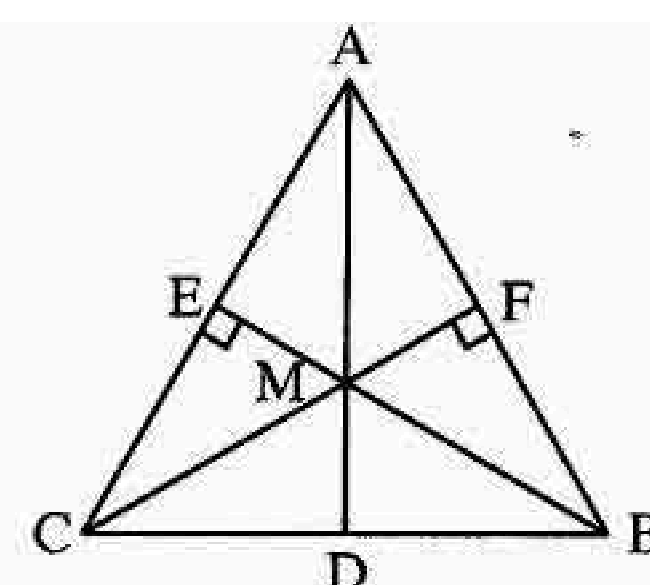
Prove that :

(1) BCED is a cyclic quadrilateral.

(2) m (∠ DEB) = m (∠ XAB)

(Alexandria 2015)

5	**In the opposite figure :** ABCD is a parallelogram, $E \in \overrightarrow{CD}$ where BE = AD **Prove that :** ABDE is a cyclic quadrilateral. *(S. Sinai 2013)*
6	**In the opposite figure :** $E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, m $(\widehat{AB})$ = 110° and m (∠ CBE) = 85° **Find :** m (∠ BDC) *(El-Beheira 14, Port Said 13)* « 30° »
7	**In the opposite figure :** m (∠ ABE) = 100° and m (∠ CAD) = 40° **Prove that :** m $(\widehat{CD})$ = m $(\widehat{AD})$ *(El-Gharbia 17, Souhag 15, Alexandria 14)*
8	**In the opposite figure :** ABCD is a quadrilateral inscribed in a circle M where $M \in \overline{AB}$, CB = CD and m (∠ BCD) = 140° **Find :** (1) m (∠ A) (2) m (∠ D) *(Matrouh 17, Kafr El-Sheikh 14)* « 40°, 110° »
9	ABC is an acute-angled triangle inscribed in a circle. Draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and cut the circle at E. Draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N **Prove that :** (1) The figure ANDC is a cyclic quadrilateral. (2) m (∠ BND) = m (∠ BED) *(Kafr El-Sheikh 2016, Beheira 2015)*
10	**In the opposite figure :** M and N are two intersecting circles at A and B, $\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D, $\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C and m (∠ C) = 70° (1) **Find :** m (∠ F) (2) **Prove that :** $\overleftrightarrow{CD} // \overleftrightarrow{EF}$ *(El-Monofia 17)* « 110° »

11	**In the opposite figure :** $\overline{AB}$ is a diameter in a circle of centre M , $\overrightarrow{AC}$ is a tangent to the circle at A , E is the midpoint of $\overline{DB}$, m (∠ B) = 40° (1) **Prove that :** The figure AMEC is a cyclic quadrilateral. (2) **Find :** m (∠ C)  *(El-Wadi El-Gedied 2014)* « 50° »
12	**In the opposite figure :** Δ AXY in which m (∠ A) = 60° , $\overrightarrow{XD}$ bisects ∠ AXY , $\overrightarrow{YB}$ bisects ∠ AYX **Prove that :** ABCD is a cyclic quadrilateral.  *(El-Beheira 2016)*
13	**In the opposite figure :** X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$ (1) **Prove that :** CXMY is a cyclic quadrilateral (2) Find the centre of the circle which passes through the vertices of the figure CXMY *(El-Ismailia 17)* 
14	**In the opposite figure :** $\overline{BC}$ is a diameter in the circle M **Prove that :** The figure LMBD is a cyclic quadrilateral.  *(Ismailia 2012)*
15	**In the opposite figure :** Δ ABC , $\overline{BE} \perp \overline{AC}$, $\overline{CF} \perp \overline{AB}$, $\overline{CF} \cap \overline{BE} = \{M\}$, $\overrightarrow{AM} \cap \overleftrightarrow{BC} = \{D\}$ **Prove that :** MDCE is a cyclic quadrilateral. *(South Sinai 17)* 

Solutions

1

In Δ BDC : m ($\angle$ BDC) = 180° – (40° + 50°) = 90°

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BDC) = 90°

but they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ The figure ABCD is a cyclic quadrilateral (Q.E.D. 1)

, in Δ ABC : $\because$ m ($\angle$ A) = 90°

$\therefore$ $\overline{BC}$ is a diameter in the circle

$\therefore$ The midpoint of $\overline{BC}$ is the centre of the circumcircle of the figure ABCD (Q.E.D. 2)

2

$\because$ m ($\angle$ AXB) = m ($\angle$ AYB) = 90° and they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ The figure ABYX is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ XAY) = m ($\angle$ XBY)

(they are drawn on $\overline{XY}$ and on one side of it)

, $\because$ m ($\angle$ CAZ) = m ($\angle$ CBZ)

(two inscribed angles of the same arc $\overset{\frown}{CZ}$)

$\therefore$ m ($\angle$ XBC) = m ($\angle$ CBZ)

$\therefore$ $\overrightarrow{BC}$ bisects $\angle$ XBZ (Q.E.D.2)

3

$\because$ ABCD is a square , $\overline{AC}$ and $\overline{BD}$ are two diagonals of the square

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BDC)

$\therefore$ $\frac{1}{2}$ m ($\angle$ BAC) = $\frac{1}{2}$ m ($\angle$ BDC)

$\therefore$ m ($\angle$ XAY) = m ($\angle$ XDY) but they are drawn on $\overline{XY}$ and on one side of it

$\therefore$ The figure AXYD is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ADX) = 45°

(they are drawn on $\overline{AX}$ and on one side of it) (Q.E.D.2)

4

$\because$ m ($\overset{\frown}{AX}$) = m ($\overset{\frown}{AY}$)

$\therefore$ m ($\angle$ ACX) = m ($\angle$ ABY)

and they are drawn on $\overline{DE}$ and on the same side of it

$\therefore$ The figure BCED is a cyclic quadrilateral. (Q.E.D.1)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ DCB) (they are drawn on $\overline{DB}$ and on one side of it)

$\because$ m ($\angle$ XAB) = m ($\angle$ XCB)

(two inscribed angles of the same arc $\overset{\frown}{XB}$)

$\therefore$ m ($\angle$ DEB) = m ($\angle$ XAB) (Q.E.D.2)

5

$\because$ ABCD is a parallelogram $\therefore$ AD = BC

$\because$ BE = AD $\therefore$ BC = BE

In Δ BCE : $\therefore$ m ($\angle$ C) = m ($\angle$ BEC) ,

$\because$ m ($\angle$ BAD) = m ($\angle$ C)

$\therefore$ m ($\angle$ BAD) = m ($\angle$ BED) and they are drawn on $\overline{BD}$ and on one side of it

$\therefore$ The figure ABDE is a cyclic quadrilateral. (Q.E.D.)

6

$\because$ m ($\overset{\frown}{AB}$) = 110°

$\therefore$ m ($\angle$ BDA) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$) = $\frac{110°}{2}$ = 55°

$\because$ $\angle$ CBE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ CBE) = m ($\angle$ ADC) = 85°

$\therefore$ m ($\angle$ BDC) = m ($\angle$ ADC) – m ($\angle$ BDA)

= 85° – 55° = 30° (The req.)

7

$\because$ $\angle$ ABE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD : m ($\angle$ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD)

$\therefore$ CD = AD

$\therefore$ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$) (Q.E.D.)

8

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) = 180° – 140° = 40° (First req.)

$\because$ CB = CD

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) = $\frac{180° - 140°}{2}$ = 20°

$\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ m ($\angle$ D) = 90° + 20° = 110° (Second req.)

9

$\because \overline{CN} \perp \overline{AB}$, $\overline{AD} \perp \overline{BC}$

$\therefore m(\angle ANC) = m(\angle ADC) = 90°$

and they are drawn on $\overline{AC}$

and in one side of it

$\therefore$ The figure ANDC is a cyclic quadrilateral (Q.E.D. 1)

$\because \angle BND$ is an exterior angle of the cyclic quadrilateral ANDC

$\therefore m(\angle BND) = m(\angle ACB)$

$\because m(\angle ACB) = m(\angle E)$

(two inscribed angles of the same arc $\overset{\frown}{AB}$)

$\therefore m(\angle BND) = m(\angle BED)$ (Q.E.D. 2)

10

Construction : Draw $\overline{AB}$

Proof : $\because$ the figure ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) = 180° - 70° = 110°$

$\because$ The figure ABFE is a cyclic quadrilateral and $\angle BAD$ is an exterior angle of it

$\therefore m(\angle F) = m(\angle BAD) = 110°$ (First req.)

$\therefore m(\angle F) + m(\angle C) = 110° + 70° = 180°$

but they are two interior angles on the same side of the transversal $\overleftrightarrow{FC}$

$\therefore \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$ (Second req.)

11

$\because \overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AC}$ is a tangent to the circle M at A

$\therefore \overline{AC} \perp \overline{AB}$ $\therefore m(\angle CAM) = 90°$

, $\because$ E is the midpoint of $\overline{BD}$ $\therefore \overline{ME} \perp \overline{DB}$

$\therefore m(\angle MEC) = 90°$

$\therefore m(\angle CAM) + m(\angle CEM) = 90° + 90° = 180°$

$\therefore$ The figure AMEC is a cyclic quadrilateral. (First req.)

In ΔABC : $m(\angle C) = 180° - (40° + 90°) = 50°$ (Second req.)

12

In ΔAXY : $\because m(\angle A) = 60°$

$\therefore m(\angle X) + m(\angle Y) = 180° - 60° = 120°$

$\therefore \frac{1}{2} m(\angle X) + \frac{1}{2} m(\angle Y) = \frac{1}{2} \times 120° = 60°$

$\therefore m(\angle CXY) + m(\angle CYX) = 60°$

$\therefore m(\angle XCY) = 180° - 60° = 120°$

, $\because \overrightarrow{YB} \cap \overrightarrow{XD} = \{C\}$

$\therefore m(\angle BCD) = m(\angle XCY) = 120°$ (V.O.A.)

$\therefore m(\angle BCD) + m(\angle A) = 120° + 60° = 180°$

$\therefore$ The figure ABCD is a cyclic quadrilateral. (Q.E.D.)

13

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ (1)

, $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{ED}$ is the common chord

$\therefore \overleftrightarrow{MN} \perp \overline{ED}$ (2)

From (1) and (2) : $\therefore m(\angle MXC) + m(\angle MYC) = 180°$

$\therefore$ The figure CXMY is a cyclic quadrilateral (Q.E.D.1)

, $\because m(\angle MXC) = 90°$

$\therefore$ The centre of the circle which passes through the vertices of the figure CXMY is the midpoint of $\overline{MC}$ (Q.E.D.2)

14

$\because \overline{BC}$ is a diameter in the circle

$\therefore m(\angle BDC) = 90°$, $m(\angle CEB) = 90°$

$\therefore \overline{BE}$, $\overline{CD}$ are altitudes in ΔABC

$\because \overline{BE} \cap \overline{CD} = \{L\}$

$\therefore$ L is the point of intersection of the altitudes of ΔABC

$\therefore \overline{AM}$ is an altitude of ΔABC

$\therefore m(\angle AMB) = 90°$

$\therefore m(\angle LMB) + m(\angle BDL) = 90° + 90° = 180°$

$\therefore$ The figure LMBD is a cyclic quadrilateral (Q.E.D.)

15

$\because \overline{BE} \perp \overline{AC}$

, $\overline{CF} \perp \overline{AB}$, $\because \overline{BE} \cap \overline{CF} = \{M\}$

$\therefore$ M is the point of intersection of the altitudes of ΔABC

, $\because \overline{AD}$ passes through the point M

$\therefore \overline{AD} \perp \overline{BC}$

$\therefore m(\angle MDC) + m(\angle MEC) = 180°$

$\therefore$ The figure MDCE is a cyclic quadrilateral (Q.E.D.)

Exercises

[A] : Choose The Correct Answer : -

1 It is possible to draw a circle passing through the vertices of a ....

(a) rhombus. (b) square. (c) trapezium. (d) parallelogram.

2 If m (∠ A) = $\frac{1}{2}$ m (∠ C) in a cyclic quadrilateral ABCD , then m (∠ A) = ...°

(a) 20 (b) 30 (c) 60 (d) 120

3 **In the opposite figure :**

m (∠ A) = 48° , then

the measure of major arc $\widehat{BD}$ = ...

(a) 260° (b) 265° (c) 264° (d) 262°

4 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ ABD) = 25°

, then m (∠ C) = ...

(a) 50° (b) 100° (c) 115° (d) 125°

5 If two chords intersect at a point inside a circle then the measure of the included angle equals .... of the two opposite arcs.

(a) half of the difference (b) half of the sum

(c) twice the sum (d) twice the difference

6 The measure of the arc that is opposite the inscribed angle of measure 60° = ...°

(a) 60 (b) 30 (c) 120 (d) 90

7 The inscribed angle drawn in a semicircle is ....

(a) an acute. (b) an obtuse. (c) a straight. (d) a right.

8 ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 60° (b) 30° (c) 90° (d) 120°

9 **In the opposite figure :**

m (∠ B) = 140°

, then m (∠ D) = ...

(a) 40° (b) 60° (c) 30° (d) 50°

10

In the opposite figure :

m (∠ ABH) = 100° , m ($\widehat{AD}$) = m ($\widehat{DC}$)

, then m (∠ ACD) = ...°

(a) 100 (b) 80 (c) 40 (d) 30

11

In the opposite figure :

ABCD is a cyclic quadrilateral , m (∠ D) = x° , (∠ B) = 2 x°

, then x = ...

(a) 120° (b) 100°

(c) 60° (d) 50°

12

The measure of the arc which represents $\frac{1}{3}$ the measure of the circle equals ....

(a) 60° (b) 120° (c) 90° (d) 240°

13

The inscribed angle which opposite to the minor arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

14

The sum of measures of the two opposite angles in the cyclic quadrilateral equal ....°

(a) 180 (b) 120 (c) 100 (d) 30

15

In the opposite figure :

m (∠ A) = x° , m (∠ C) = 2 x°

, then x = ...

(a) 60° (b) 50° (c) 80° (d) 20°

16

In the opposite figure :

m (∠ ADC) = 80° , then m (∠ ABE) = ...°

(a) 10 (b) 80

(c) 60 (d) 100

17

In the opposite figure :

ABCD is a cyclic quadrilateral and AD = AO , m (∠ D) = 65° , then :

First : m (∠ B) = ...°

(a) 65 (b) 115 (c) 90 (d) 45

Second : m (∠ AOD) = ...°

(a) 65 (b) 90 (c) 25 (d) 60

18

The ratio between the measure of the inscribed angle to the measure of the central angle subtended by the same arc equals ....

(a) 2 : 1 (b) 1 : 2 (c) 2 : 2 (d) 2 : 3

19 The measure of the arc which equals half the measure of the circle equals ……………

(a) 360° (b) 180° (c) 120° (d) 90°

20 Which of the following shapes is a cyclic quadrilateral ?

(a) rhombus (b) rectangle (c) parallelogram (d) trapezium

21 **In the opposite figure :**

M is a circle , m (∠ MBC) = 32°

, then m $(\overset{\frown}{BC}$ the minor) = ……………

(a) 116° (b) 23° (c) 58° (d) 64°

22 In the cyclic quadrilateral each two opposite angles ……………

(a) equal. (b) complementary. (c) alternate. (d) supplementary.

23 **In the opposite figure :**

If M is a circle

, m (∠ BCD) = 130°

, then m (∠ BAD) = ……………°

(a) 50 (b) 130 (c) 65 (d) 260

24 **In the opposite figure :**

If m (∠ BAD) = 60°

, then m (∠ BCF) = ……………°

(a) 30 (b) 60

(c) 80 (d) 120

25 **In the opposite figure :**

If M is the centre of the circle

, m (∠ BMD) = 110°

, then m (∠ C) = ……………

(a) 70° (b) 110° (c) 125° (d) 55°

26 **In the opposite figure :**

If $\overline{AB}$ is a diameter in circle

, m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{DE})$ = m $(\overset{\frown}{EF})$ = m $(\overset{\frown}{FB})$

, then m (∠ DXE) = ……………

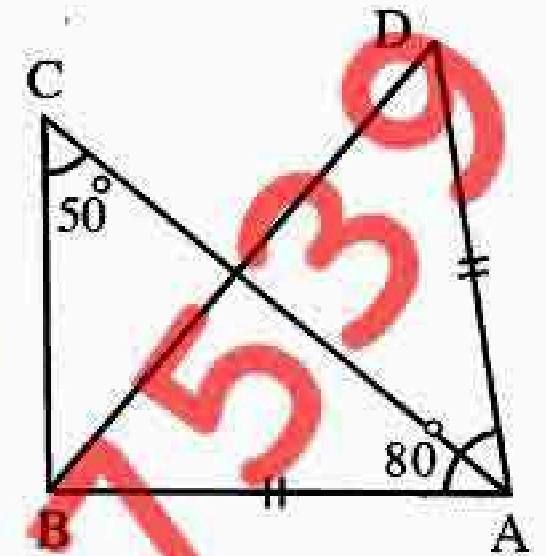
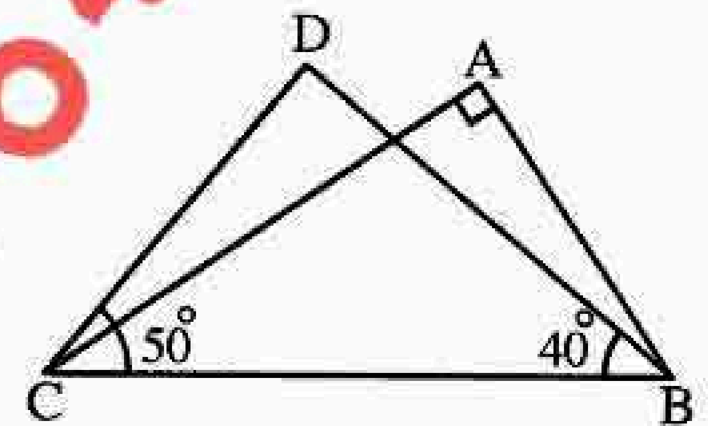
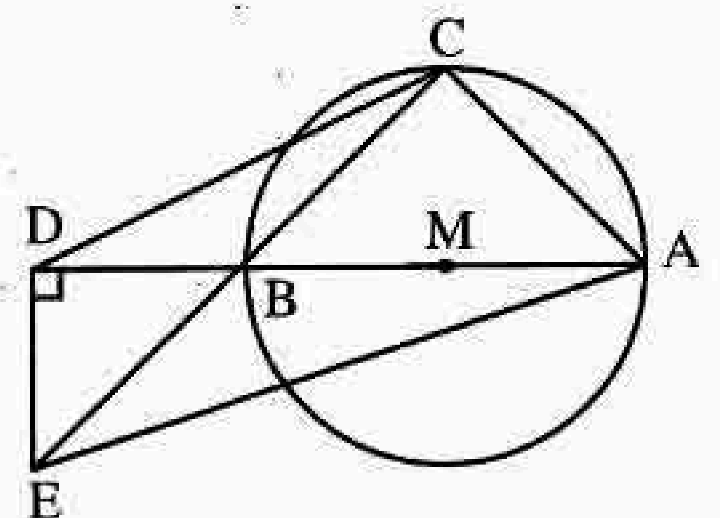
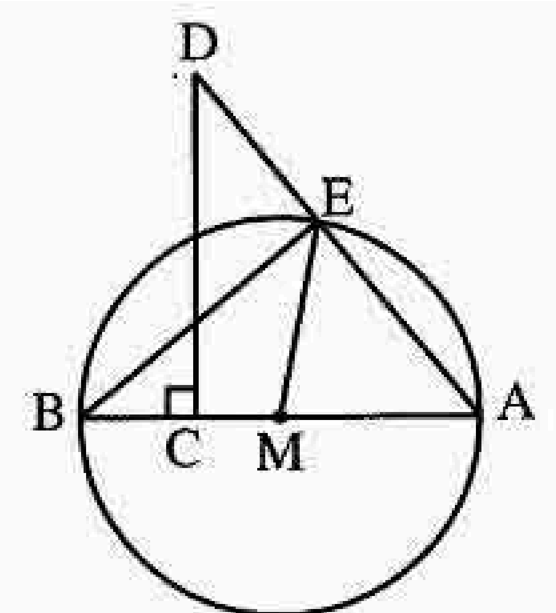
(a) 72° (b) 54° (c) 36° (d) 18°

27 The number of circles that pass through three collinear points equals ……………

(a) zero. (b) one. (c) three. (d) infinite number.

28	Number of the circles that pass through three non-collinear points equals .... (a) zero (b) one (c) three (d) an infinite number
29	**In the opposite figure :** If C is the midpoint of $\widehat{AB}$, then AB .... 2 AC (a) < (b) > (c) ≥ (d) =
30	**In the opposite figure :** m (∠ BCD) = 115° , m (∠ A) = x + 15° , then the value of x = ...° (a) 130 (b) 100 (c) 50 (d) 40
31	**In the opposite figure :** If M is a circle , E ∈ $\overrightarrow{DC}$, m (∠ BAD) = 70° , then m (∠ BCE) = ...° (a) 35 (b) 70 (c) 100 (d) 110
32	**In the opposite figure :** In the circle M , if m (∠ AMC) = 140° , then m (∠ ADC) = ... (a) 40° (b) 70° (c) 110° (d) 140°
33	**In the opposite figure :** $\overline{AB} \cap \overline{CD}$ = {E} , m ($\widehat{AC}$) = 60° , m ($\widehat{BD}$) = 100° , then m (∠ DEB) = ...° (a) 160 (b) 60 (c) 80 (d) 100
34	The measure of the inscribed angle is .... the measure of the central angle , subtended by the same arc. (a) half (b) third (c) quarter (d) double
35	The measure of the central angle is .... the measure of the arc which is opposite to it. (a) twice (b) half (c) equals (d) more than
36	If ABCD is a cyclic quadrilateral and m (∠ B) = $\frac{1}{2}$ m (∠ D) , then m (∠ B) = ... (a) 90° (b) 60° (c) 120° (d) 180°

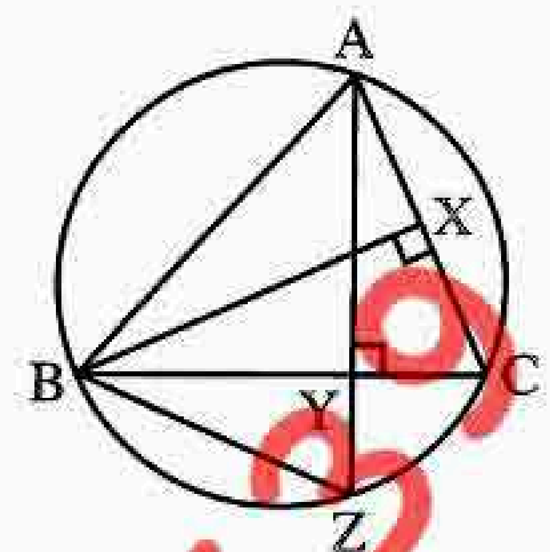
[B] : Essay Problems : -

1	🕮 **In the opposite figure :** AB = AD , m (∠ A) = 80° and m (∠ C) = 50° **Prove that :** The points A , B , C and D have one circle passing through them. *(Suez 2016 , South Sinai 2015 , Port Said 2014)* 
2	**In the opposite figure :** m (∠ A) = 90° , m (∠ DBC) = 40° , m (∠ DCB) = 50° (1) **Prove that :** The figure ABCD is a cyclic quadrilateral (2) Determine where is the center of the circle passes through the vertices of the figure ABCD *(Cairo 2014)* 
3	**In the opposite figure :** $\overline{AB}$ is a diameter in the circle M Draw $\overrightarrow{DE} \perp \overrightarrow{AB}$ and $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$ **Prove that :** ACDE is a cyclic quadrilateral. *(El-Fayoum 2011)* 
4	**In the opposite figure :** $\overline{AB}$ is a diameter in circle M in which $\overline{AE}$ is a chord and $\overrightarrow{CD} \perp \overline{AB}$, $\overrightarrow{CD}$ intersects $\overrightarrow{AE}$ at D **Prove that :** (1) The points D , E , C and B have one circle passing through them. (2) m (∠ AME) = 2 m (∠ D) *(Kafr El-Sheikh 2011)* 
5	🕮 ABC is a triangle inscribed in a circle , $X \in \widehat{AB}$, $Y \in \widehat{AC}$, where m $(\widehat{AX})$ = m $(\widehat{AY})$, $\overline{CX} \cap \overline{AB} = \{D\}$ and $\overline{BY} \cap \overline{AC} = \{E\}$ **Prove that :** (1) BCED is a cyclic quadrilateral. (2) m (∠ DEB) = m (∠ XAB) *(Alexandria 2015)*

6

In the opposite figure :

ABC is a triangle drawn in a circle , $\overline{BX} \perp \overline{AC}$, $\overrightarrow{AY} \perp \overline{BC}$ cuts it at Y and cuts the circle at Z



Prove that :

(1) ABYX is a cyclic quadrilateral.

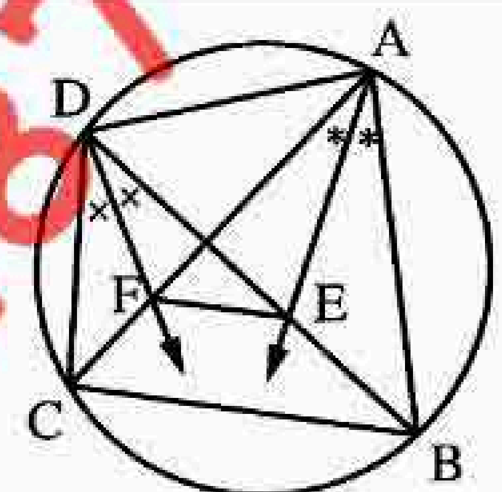
(2) $\overrightarrow{BC}$ bisects ∠ XBZ

(El-Gharbia 17 , El-Beheira 17)

7

📖 **In the opposite figure :**

ABCD is a cyclic quadrilateral which has $\overrightarrow{AE}$ bisects ∠ BAC and $\overrightarrow{DF}$ bisects ∠ BDC



Prove that :

(1) AEFD is a cyclic quadrilateral.

(2) $\overline{EF}$ // $\overline{BC}$

(Luxor 2016 , El-Menia , El-Dakahlia 2013)

8

📖 ABCD is a square , $\overrightarrow{AX}$ bisects ∠ BAC and intersects $\overline{BD}$ at X , $\overrightarrow{DY}$ bisects ∠ CDB and intersects $\overline{AC}$ at Y

Prove that :

(1) AXYD is a cyclic quadrilateral.

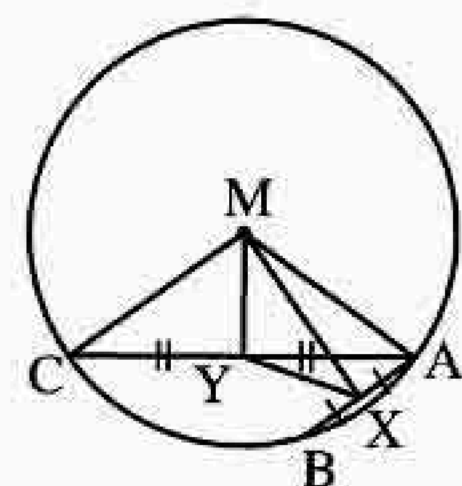
(2) m (∠ AYX) = 45°

(Alexandria 2016 , Sharkia 2012)

9

📖 **In the opposite figure :**

A circle with centre M , X and Y are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.



Prove that :

(1) AXYM is a cyclic quadrilateral.

(2) m (∠ MXY) = m (∠ MCY)

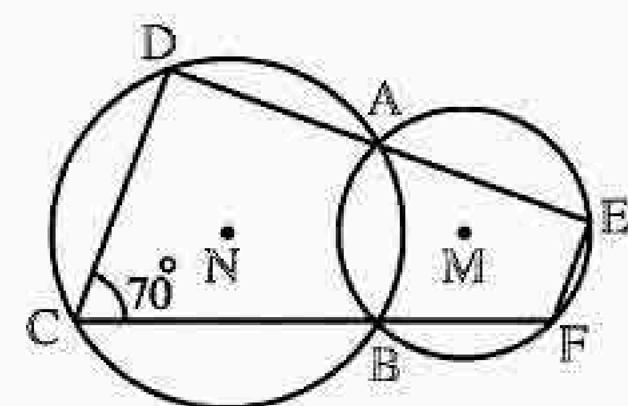
(3) $\overline{AM}$ is a diameter in the circle passing through the points A , X , Y and M

(Red Sea 2012)

10

📖 **In the opposite figure :**

M and N are two intersecting circles at A and B , $\overleftrightarrow{AD}$ is drawn to intersect circle M at E and circle N at D , $\overleftrightarrow{BC}$ is drawn to intersect circle M at F and circle N at C and m (∠ C) = 70°



(1) **Find :** m (∠ F) (2) **Prove that :** $\overleftrightarrow{CD}$ // $\overleftrightarrow{EF}$ *(El-Monofia 17)* « 110° »

Homework

[A] : Choose The Correct Answer : -

1 We can draw a circle passes through the vertices of ....

(a) rectangle. (b) rhombus. (c) trapezium. (d) parallelogram.

2 The figure is said to be cyclic quadrilateral if the measure of any exterior angle at any vertex equal to .... of the interior angle at the opposite vertex.

(a) the measure. (b) half the measure.
(c) twice the measure. (d) third the measure.

3 **In the opposite figure :**

m (∠ B) = 120°

, $\overline{BC}$ // $\overline{AD}$, then m (∠ BCF) = ...°

(a) 30 (b) 60 (c) 80 (d) 120

4 **In the opposite figure :**

H ∈ $\overrightarrow{DA}$, $\overrightarrow{AO}$ bisects ∠ HAB

, m (∠ HAO) = 55°

, then m (∠ C) = ...

(a) 55° (b) 75° (c) 110° (d) 125°

5 **In the opposite figure :**

M is a circle in which

$\overline{AB} \perp \overline{CD}$, then m ($\overset{\frown}{AC}$) + m ($\overset{\frown}{BD}$) = ...°

(a) 45 (b) 90
(c) 180 (d) 270

6 A circle can be drawn passing the vertices of ....

(a) a trapezium. (b) a rhombus. (c) a parallelogram. (d) a rectangle.

7 The number of common tangents of two touching circles externally equals ....

(a) 0 (b) 1 (c) 2 (d) 3

8 The number of common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

9 The measure of the inscribed angle drawn in a semicircle equals ....

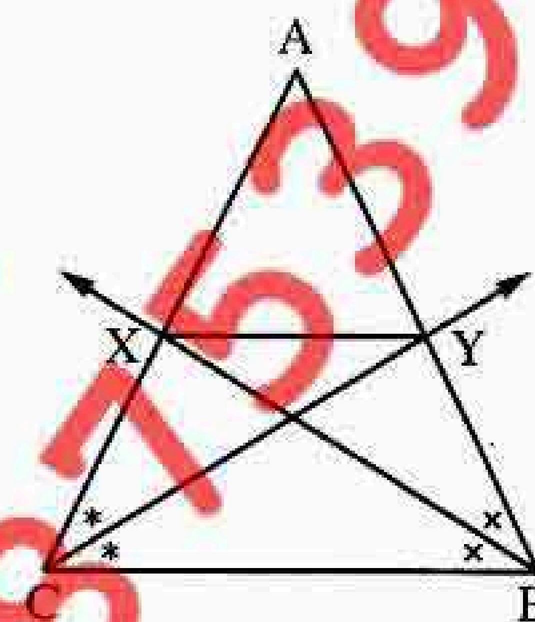
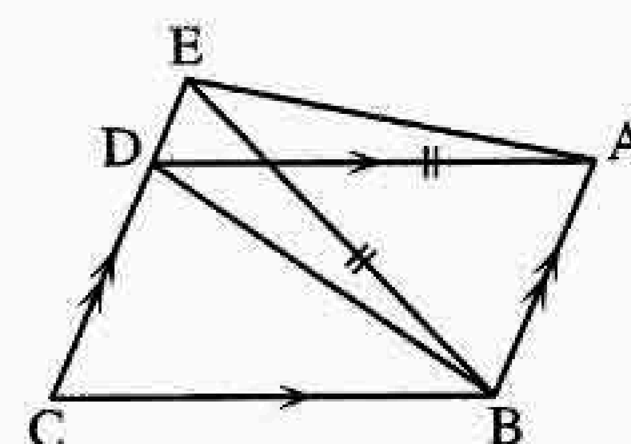
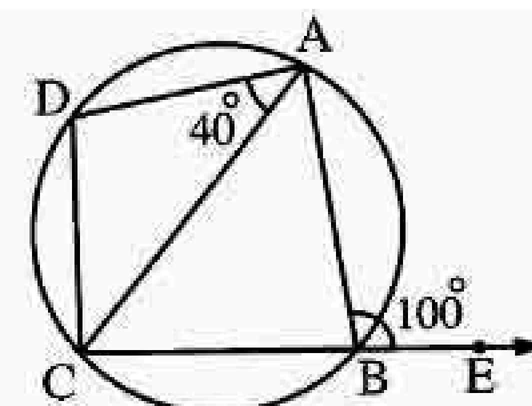
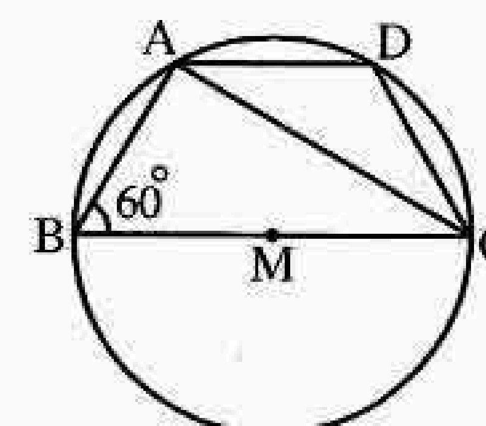
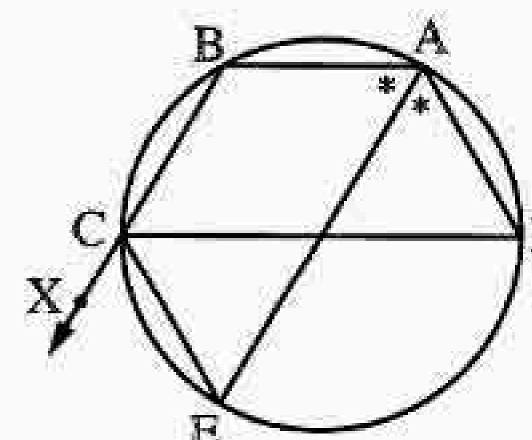
(a) 45° (b) 90° (c) 120° (d) 80°

10	If DHWQ is a cyclic quadrilateral with a right-angle at vertices Q , then .... is a diameter in its circle. (a) $\overline{DQ}$ (b) $\overline{HW}$ (c) $\overline{WD}$ (d) $\overline{DH}$
11	M and N are two intersecting circles their radii are 5 cm. , 2 cm. , then MN ∈ .... (a)]3 , 7[(b) [3 , 7] (c) [3 , 7[(d)]3 , 7]
12	**In the opposite figure :** M is a circle , $\overline{MA} \perp \overline{MB}$, then m (∠ ACB) = ...° (a) 45 (b) 90 (c) 135 (d) 145
13	**In the opposite figure :** If E ∈ $\overrightarrow{BC}$, $\overrightarrow{CX}$ bisects ∠ DCE , m (∠ XCE) = 62° , then m (∠ A) = ... (a) 62° (b) 118° (c) 56° (d) 124°
14	**In the opposite figure :** m (∠ AHC) = ... (a) 25° (b) 50° (c) 75° (d) 100°
15	**In the opposite figure :** If m (∠ A) = 120° , then m (∠ C) = ... (a) 60° (b) 90° (c) 120° (d) 180°
16	**In the opposite figure :** If m (∠ BAD) = 120° , then m (∠ CBD) = ... (a) 15° (b) 30° (c) 45° (d) 60°
17	**In the opposite figure :** If m ($\overset{\frown}{AC}$) = 30° , $\overline{AB}$ // $\overline{CD}$, then m (∠ BED) = ... (a) 10° (b) 15° (c) 30° (d) 60°

18	The length of the arc which represents $\frac{1}{4}$ of the perimeter of the circle = ... (a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$
19	Number of circles passing through a given point .... (a) one circle. (b) two circles. (c) three circles. (d) infinite number of circles.
20	Two circles M and N with radii lengths 8 cm. and 5 cm. respectively , are touching when MN $\in$ .... (a) $]13 , 3[$ (b) $]3 , 13[$ (c) $\mathbb{R} - [3 , 13]$ (d) $\{13 , 3\}$
21	If the surface of the circle M $\cap$ the surface of the circle N = $\{A\}$, then the two circles M and N are .... (a) distant. (b) concentric. (c) touching externally. (d) intersecting.
22	M and N are two circles their two radii lengths are 5 cm. and 3 cm. respectively. If MN = 8 cm. , then the two circles are .... (a) touching internally. (b) intersecting. (c) touching externally. (d) distant.
23	The number of circles which can be drawn passes through the endpoints of a line segment $\overline{AB}$ equals .... (a) 1 (b) 2 (c) 3 (d) an infinite number.
24	$\overline{AB}$ is a line segment , then the number of the circles passing through the two points A , B is .... (a) 1 (b) 2 (c) 3 (d) infinite number.
25	If A and B are two points in the plane , if AB = 4 cm. , then the smallest radius length of circle passing through by A and B is .... cm. (a) 2 (b) 3 (c) 4 (d) 5
26	The centres of all circles passing through the points A and B lie on .... (a) $\overline{AB}$ (b) midpoint of $\overline{AB}$ (c) the symmetry axis of $\overline{AB}$ (d) the perpendicular to $\overline{AB}$ from B
27	One of the following statments identify one and only one circle , if we have .... (a) radius length and one of its points. (b) two of its points. (c) only one of its points. (d) its centre and one of its points.
28	The centre of the inscribed circle of any triangle is the intersection point .... (a) its medians. (b) its heights. (c) the symmetric axes of its sides. (d) bisectors of its interior angles.

29	The centre of the circumcircle of any triangle is the point of intersection of .... (a) the interior bisectors of its angles. (b) the exterior bisectors of its angles. (c) its heights. (d) the symmetric axes of its sides.
30	M and N are two intersecting circles the lengths of their radii are 3 cm. and 5 cm. , then MN $\in$ .... (a) [2 , 8] (b) [2 , 8[(c)]2 , 8] (d)]2 , 8[
31	M and N are two intersecting circles , $r_1 = 3$ cm. , $r_2 = 5$ cm. respectively , then MN $\in$ .... (a)]0 , 5[(b)]2 , 8[(c)]8 , ∞[(d)]2 , ∞[
32	If the circle M $\cap$ the circle N = {A , B} , then the two circles M and N are .... (a) intersecting. (b) concentric. (c) touching externally. (d) distant.
33	M and N are two circles of radii lengths 9 cm. , 4 cm. , MN = 5 cm. , then the two circles are .... (a) intersecting. (b) touching internally. (c) touching externally. (d) distant.
34	The surface of the circle M $\cap$ the surface of the circle N = {A} and the radius length of one of them is 3 cm. and MN = 8 cm. , then the radius length of the other circle = ... cm. (a) 5 (b) 6 (c) 11 (d) 16
35	The centre of the inscribed circle of any triangle is the point of intersection of its .... (a) altitudes. (b) medians. (c) axes of symmetry of its sides. (d) bisectors of its interior angles.
36	The centre of the circumcircle of the triangle is the intersection point of its .... (a) altitudes of triangle. (b) medians of a triangle. (c) perpendicular bisectors of the sides of a triangle. (d) bisectors of its angles.
37	If the two circles M , N are touching externally , the radius length of the circle M is 4 cm. , if MN = 7 cm. then the circumference of the circle N is .... cm. (a) 4 π (b) 6 π (c) 7 π (d) π
38	If M , N are two touching circles internally , their radii lengths are 5 cm. , 9 cm. , then MN = ... cm. (a) 14 (b) 4 (c) 5 (d) 9

[B] : Essay Problems : -

1	🕮 **In the opposite figure :** ABC is a triangle in which : AB = AC , $\overrightarrow{BX}$ bisects ∠ B and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects ∠ C and intersects $\overline{AB}$ at Y **Prove that :** (1) BCXY is a cyclic quadrilateral. (2) $\overleftrightarrow{XY}$ // $\overleftrightarrow{BC}$  *(Assiut 2011)*
2	🕮 ABC is an isosceles triangle which has AB = AC , D is the midpoint of $\overline{BC}$, draw $\overrightarrow{BE} \perp \overline{AC}$, where $\overrightarrow{BE} \cap \overline{AC} = \{E\}$ **Prove that :** The points A , B , D and E have one circle passing through them. *(Dakahlia 2012)*
3	🕮 **In the opposite figure :** ABCD is a parallelogram , E ∈ $\overrightarrow{CD}$ where BE = AD **Prove that :** ABDE is a cyclic quadrilateral.  *(S. Sinai 2013)*
4	🕮 **In the opposite figure :** m (∠ ABE) = 100° and m (∠ CAD) = 40° **Prove that :** m $(\overset{\frown}{CD})$ = m $(\overset{\frown}{AD})$  *(El-Gharbia 17 , Souhag 15 , Alexandria 14)*
5	**In the opposite figure :** ABCD is a cyclic quadrilateral , $\overline{CB}$ is a diameter in the circle M , m (∠ ABC) = 60° , the length of $\overset{\frown}{AD}$ = the length of $\overset{\frown}{CD}$ **Prove that :** $\overrightarrow{CA}$ bisects ∠ DCB  *(Monofia 2008)*
6	**In the opposite figure :** ABCD is a quadrilateral inscribed in a circle M where $\overrightarrow{AE}$ bisects ∠ A and cuts the circle at E **Prove that :** $\overrightarrow{CE}$ bisects ∠ XCD  *(Assiut 12)*

7

In the opposite figure :

ABCD is a quadrilateral inscribed in a circle M where

$M \in \overline{AB}$, CB = CD and m (∠ BCD) = 140°

Find : (1) m (∠ A)

(2) m (∠ D)

(Matrouh 17 , Kafr El-Sheikh 14) « 40° , 110° »

8

ABC is an acute-angled triangle inscribed in a circle. Draw $\overrightarrow{AD} \perp \overline{BC}$ to cut $\overline{BC}$ at D and cut the circle at E. Draw $\overrightarrow{CN} \perp \overline{AB}$ to cut $\overline{AB}$ at N

Prove that :

(1) The figure ANDC is a cyclic quadrilateral.

(2) m (∠ BND) = m (∠ BED) *(Kafr El-Sheikh 2016 , Beheira 2015)*

9

In the opposite figure :

ABCD is a parallelogram ,

the circle which passes through

the points B , C and D intersects $\overline{AB}$ at E

Prove that : AD = ED *(El-Fayoum 2011)*

10

In the opposite figure :

$\overline{BC}$ is a diameter in the circle M and $\overline{ED} \perp \overline{BC}$

Prove that :

(1) The figure ABDE is a cyclic quadrilateral.

(2) m (∠ CED) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$ *(Giza 2009)*

11

In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ touch the circle M at B and C respectively

, m (∠ A) = 45°

Prove that :

(1) The figure ABMC is a cyclic quadrilateral.

(2) Δ MCD is an isosceles triangle. *(South Sinai 2012)*

12

In the opposite figure :

$\overline{AB}$ is a diameter in a circle of centre M

, $\overrightarrow{AC}$ is a tangent to the circle at A

, E is the midpoint of $\overline{DB}$, m (∠ B) = 40°

(1) **Prove that :** The figure AMEC is a cyclic quadrilateral.

(2) **Find :** m (∠ C) *(El-Wadi El-Gedied 2014)* « 50° »

13

In the opposite figure :

M is a circle , D is the midpoint of the chord $\overline{EC}$

, $\overleftrightarrow{BC}$ is a tangent to the circle M at C and $\overline{AB}$ // $\overline{MC}$

Prove that :

The figure ABCD is a cyclic quadrilateral.

(Cairo 2012)

14

In the opposite figure :

Δ AXY in which m ($\angle$ A) = 60°

, $\overrightarrow{XD}$ bisects $\angle$ AXY , $\overrightarrow{YB}$ bisects $\angle$ AYX

Prove that :

ABCD is a cyclic quadrilateral.

(El-Beheira 2016)

15

In the opposite figure :

$\overline{AB}$ // $\overline{DE}$, $\overline{BC}$ // $\overline{DF}$

and m ($\angle$ ADE) + m ($\angle$ CDF) = 180°

Prove that :

The figure ABCD is cyclic quadrilateral.

(Monofia 2006)

16

In the opposite figure :

X is the midpoint of $\overline{AB}$, $\overline{MN} \cap \overline{EC} = \{Y\}$

(1) **Prove that :** CXMY is a cyclic quadrilateral

(2) Find the centre of the circle which passes through the vertices of the figure CXMY

(El-Ismailia 17)

17

In the opposite figure :

$\overline{BC}$ is a diameter in the circle M

Prove that :

The figure LMBD is a cyclic quadrilateral.

(Ismailia 2012)

18

$\overline{AB}$ is a diameter in the circle M , $\overline{AC}$ is a chord in it , D is the midpoint of $\overline{AC}$,

$\overrightarrow{DM}$ is drawn to cut the circle at E. Draw $\overrightarrow{BF} \perp \overline{AB}$ to cut $\overrightarrow{AC}$ at F

Prove that :

(1) The figure MBFD is a cyclic quadrilateral.

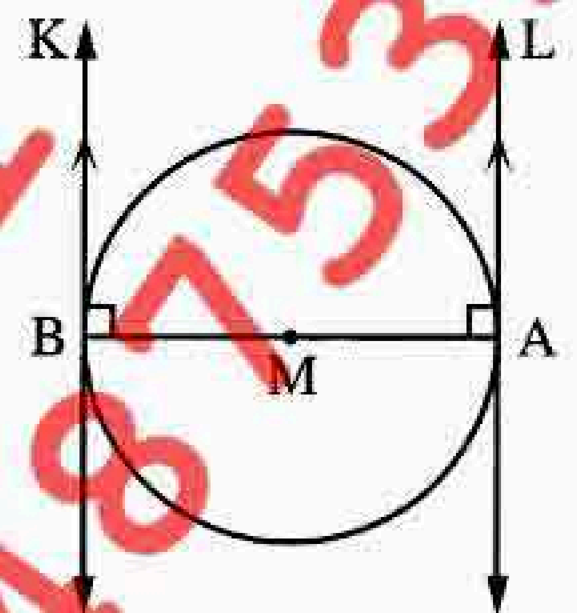
(2) m ($\angle$ F) = 2 m ($\angle$ BAE)

(Port Said 2008)

Lesson [6] : The Relation Between The Tangents Of A Circle

First : The two tangents drawn at the two ends of a diameter in a circle are parallel.

i.e. **In the opposite figure :**



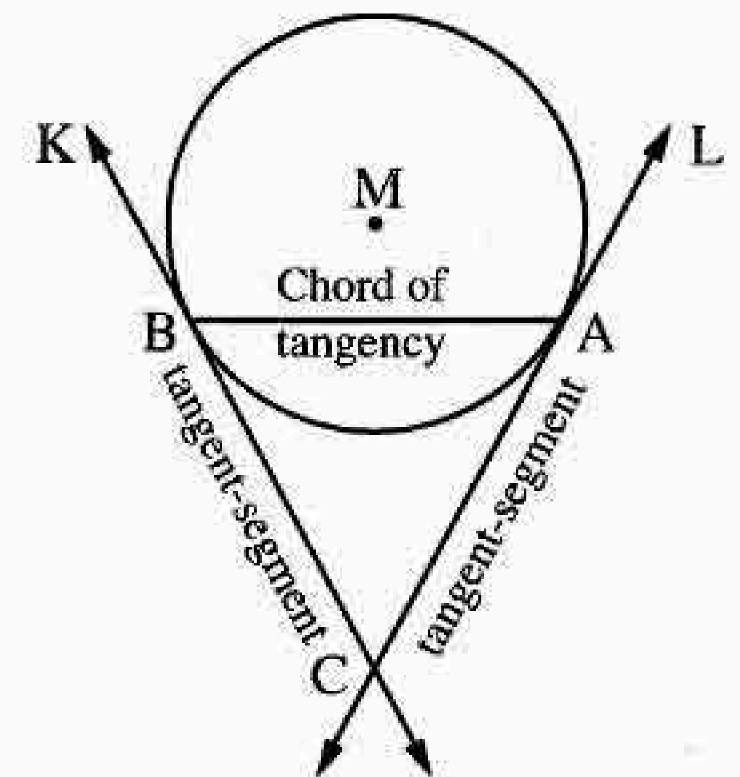
If $\overline{AB}$ is a diameter in the circle M and the two straight lines L and K are two tangents to the circle at A and B respectively,

then the straight line L // the straight line K

(because the straight line L $\perp$ $\overline{AB}$ and the straight line K $\perp$ $\overline{AB}$)

Second : The two tangents drawn at the two ends of a chord of a circle are intersecting.

i.e. **In the opposite figure :**



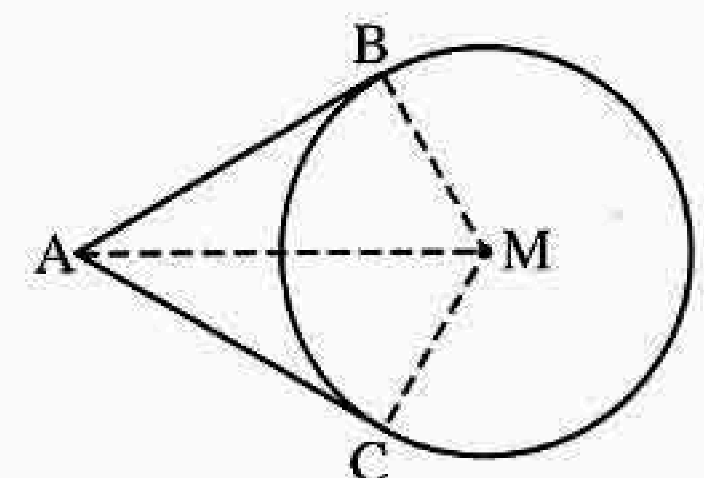
If $\overline{AB}$ is a chord in the circle M and the two straight lines L and K ar`e two tangents to the circle at A and B respectively, then the two straight lines L and K are intersecting at a point outside the circle M (Say C) and $\overline{AC}$, $\overline{BC}$ are called tangent - segments and $\overline{AB}$ is called a chord of tangency.

Theorem 4

The two tangent-segments drawn to a circle from a point outside it are equal in length.

A is a point outside the circle M ,
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C respectively.

AB = AC

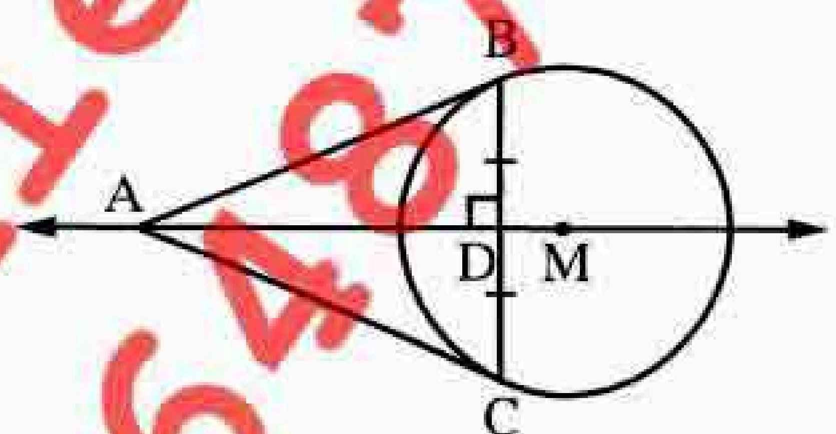


Corollaries of theorem (4) :

Corollary 1

The straight line passing through the centre of the circle and the intersection point of the two tangents is an axis of symmetry to the chord of tangency of those two tangents.

In the opposite figure :



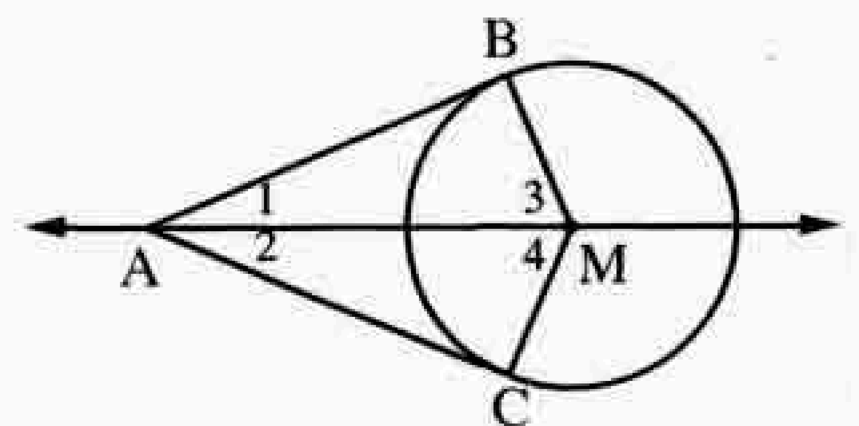
If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively, then $\overleftrightarrow{AM}$ is the axis of symmetry to $\overline{BC}$

i.e. $\overleftrightarrow{AM} \perp \overline{BC}$, BD = CD

Corollary 2

The straight line passing through the centre of the circle and the intersection point of its two tangents bisects the angle between these two tangents. It also bisects the angle between the two radii passing through the two points of tangency.

In the opposite figure :

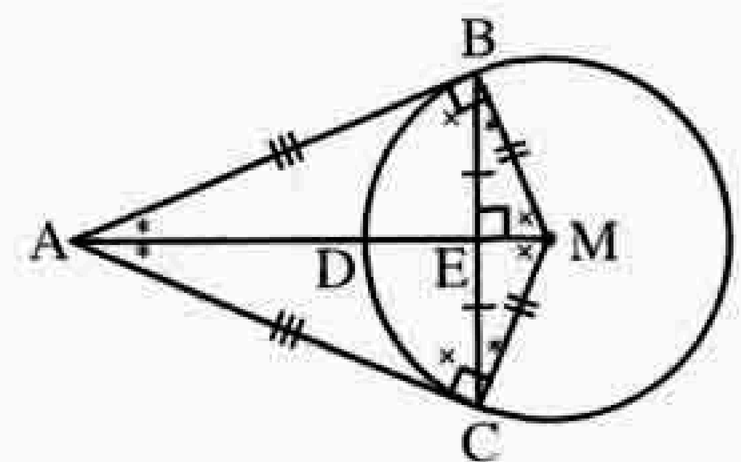


If $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are two tangents to the circle M at B and C respectively then :

- $\overrightarrow{AM}$ bisects ∠ BAC ∴ m (∠ 1) = m (∠ 2)
- $\overrightarrow{MA}$ bisects ∠ BMC ∴ m (∠ 3) = m (∠ 4)

Remarks on theorem (4) and its corollaries

In the opposite figure :



1 AB = AC **2** MB = MC = r

3 BE = CE , $\overleftrightarrow{AM} \perp \overline{BC}$

4 m (∠ ABM) = m (∠ ACM) = 90°

i.e. The figure ABMC is a cyclic quadrilateral.

5 m (∠ BAM) = m (∠ BCM) = m (∠ CAM) = m (∠ CBM)

6 m (∠ AMB) = m (∠ ACB) = m (∠ AMC) = m (∠ ABC)

Definition

The inscribed circle of a polygon is the circle which touches all of its sides internally.

Remark

The centre of the inscribed circle of any triangle is the point of intersection of the bisectors of its interior angles.

The common tangents to two circles

- It is said that the tangent $\overleftrightarrow{AB}$ is an internal common tangent to the two circles M and N if the two circles M and N are on two different sides of the tangent.
- It is said that the tangent $\overleftrightarrow{AB}$ is an external common tangent of the two circles M and N if the two circles M and N are on the same side of the tangent.

The following table shows the number of the common tangents to two circles in their different situations (locations) :

Two distant circles	Two circles touching externally
L_4, L_3, L_1, L_2, N, M	L_3, L_1, L_2, N, M
4 common tangents • L_1 and L_2 (external) • L_3 and L_4 (internal)	**3 common tangents** • L_1 and L_2 (external) • L_3 (internal)
Two intersecting circles	**Two circles touching internally**
L_1, L_2, N, M	L, M N
2 common tangents • L_1 and L_2 (external) • There are no internal tangents	**One common tangent** • L is the common tangent (external) • There are no internal tangents
One circle inside the other	
M N M	
There are no common tangents	

Lesson [7] : Angle Of Tangency

Definition

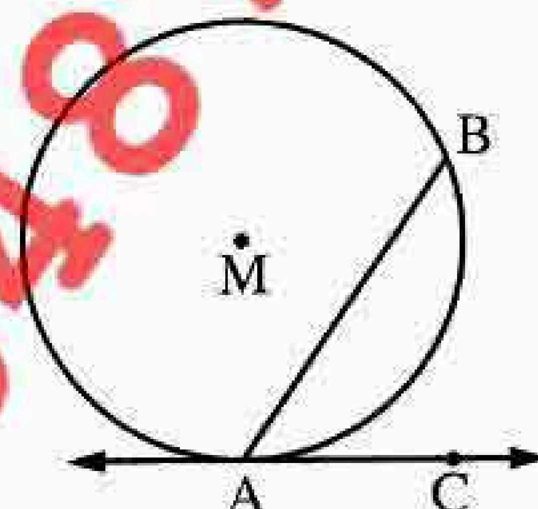
The angle of tangency is the angle which is composed of the union of two rays , one of them is a tangent to the circle and the other contains a chord of the circle passing through the point of tangency.

In the opposite figure :

If $\overleftrightarrow{AC}$ is a tangent to the circle at A and $\overrightarrow{AB}$ contains the chord $\overline{AB}$

, then ∠ BAC is an angle of tangency in the circle M , its chord is $\overline{AB}$

$\overline{AB}$ is called the chord of tangency of the angle of tangency ∠ BAC



i.e. **The measure of the angle of tangency = $\frac{1}{2}$ the measure of the arc intercepted by its sides.**

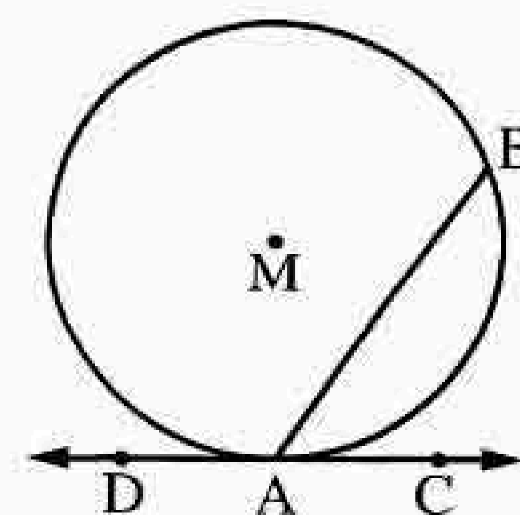
In the opposite figure :

- ∠ BAC is an angle of tangency that intercepts $\overset{\frown}{AB}$ between its sides.

  ∴ m (∠ BAC) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$)

- ∠ BAD is an angle of tangency that intercepts the major $\overset{\frown}{AB}$ between its sides.

  ∴ m (∠ BAD) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$ the major)



Theorem 5

The measure of the angle of tangency is equal to the measure of the inscribed angle subtended by the same arc.

Corollary

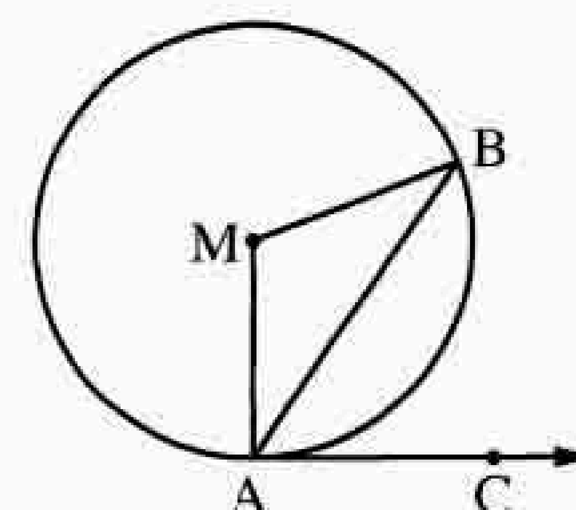
The measure of the angle of tangency is half the measure of the central angle subtended by the same arc.

In the opposite figure :

m (∠ BAC) (tangency angle) = $\frac{1}{2}$ m ($\overset{\frown}{AB}$)

, ∵ m (∠ AMB) (central angle) = m ($\overset{\frown}{AB}$)

∴ m (∠ BAC) (tangency angle) = $\frac{1}{2}$ m (∠ AMB) (central angle)



Remark

The angle of tangency is supplementary to the drawn inscribed angle on the chord of the angle of tangency and in one side of it.

Very Important Notes :

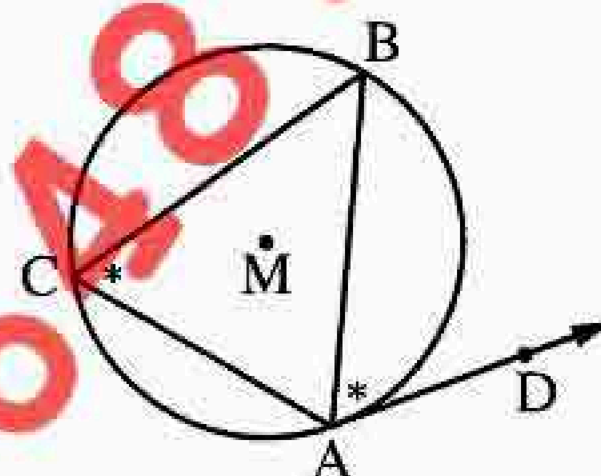
If a ray is drawn from one end of a chord of a circle so that the angle between this ray and the chord is equal in measure to the inscribed angle subtended by the chord in the alternate side , then this ray is a tangent to the circle.

Thus in the opposite figure :

If $\overline{AB}$ is a chord in the circle M ,

$\overrightarrow{AD}$ is drawn such that m (∠ BAD) = m (∠ C) ,

then $\overrightarrow{AD}$ is a tangent to the circle M



Examples :

1

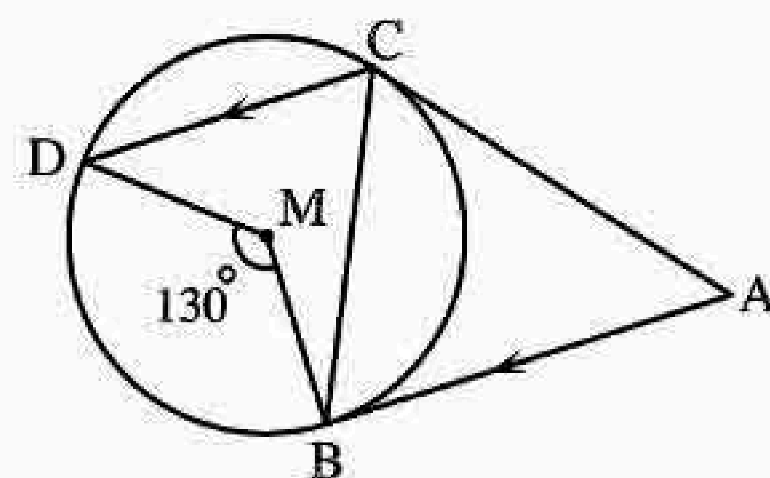
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AB}$ // $\overline{CD}$ and m (∠ BMD) = 130°

(1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

(2) **Find :** m (∠ A) *(El-Fayoum 17 , El-Gharbia 16 , El-Kalyoubia 16 , El-Menia 15 , Cairo 14)* « 50° »



2

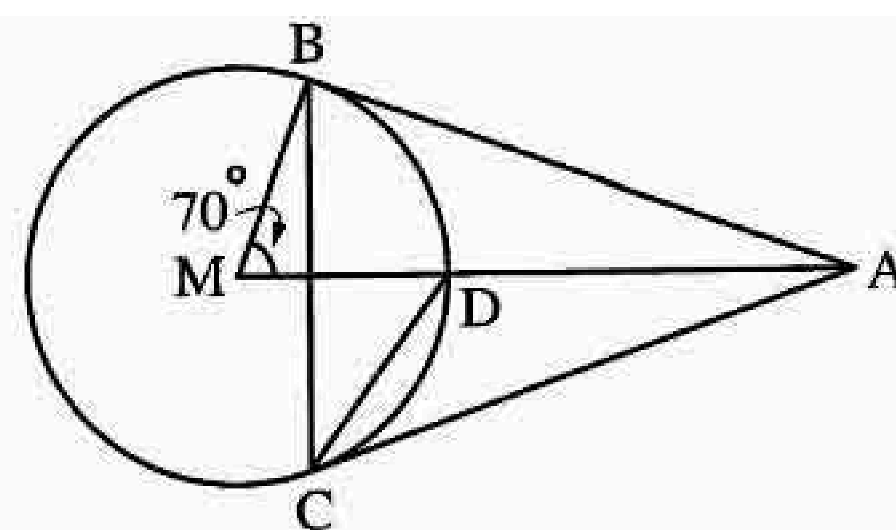
In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A

, m (∠ AMB) = 70°

Find : (1) m (∠ ABC)

(2) m (∠ ACD)



(El-Ismailia 17) « 70° , 35° »

3

In the opposite figure :

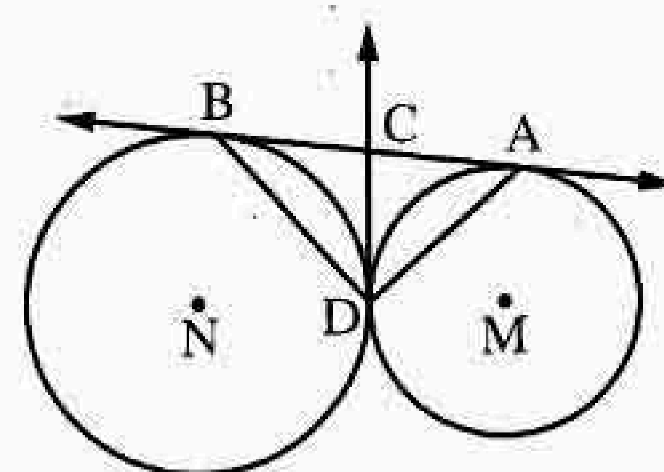
M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to them at A and B

$\overrightarrow{DC}$ is a common tangent to the two circles at D ,

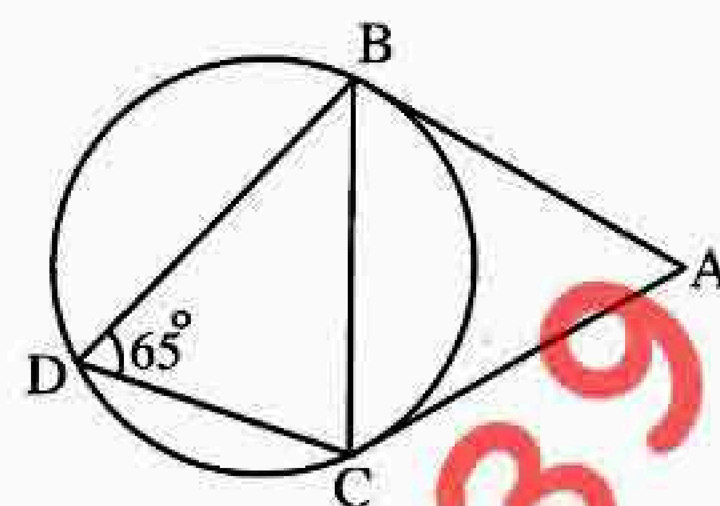
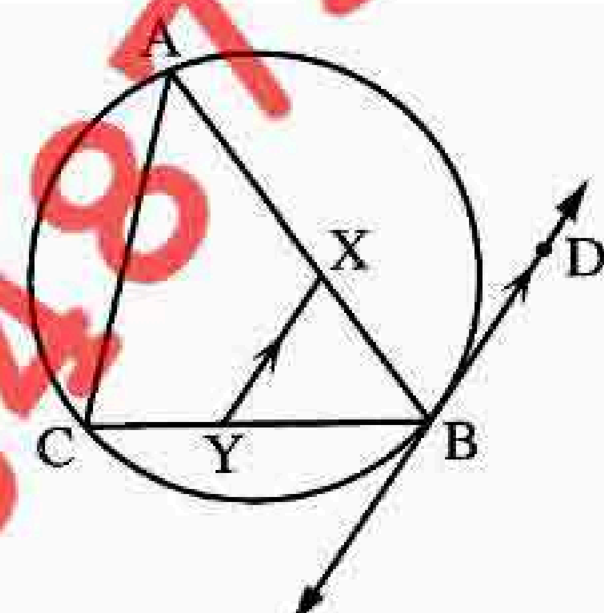
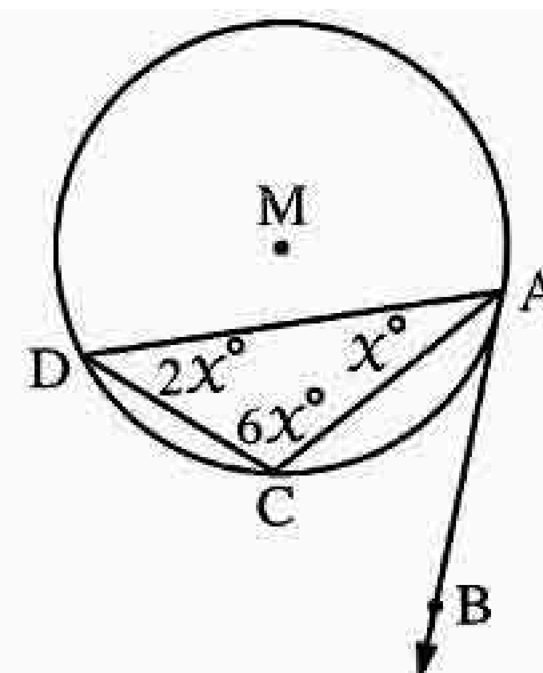
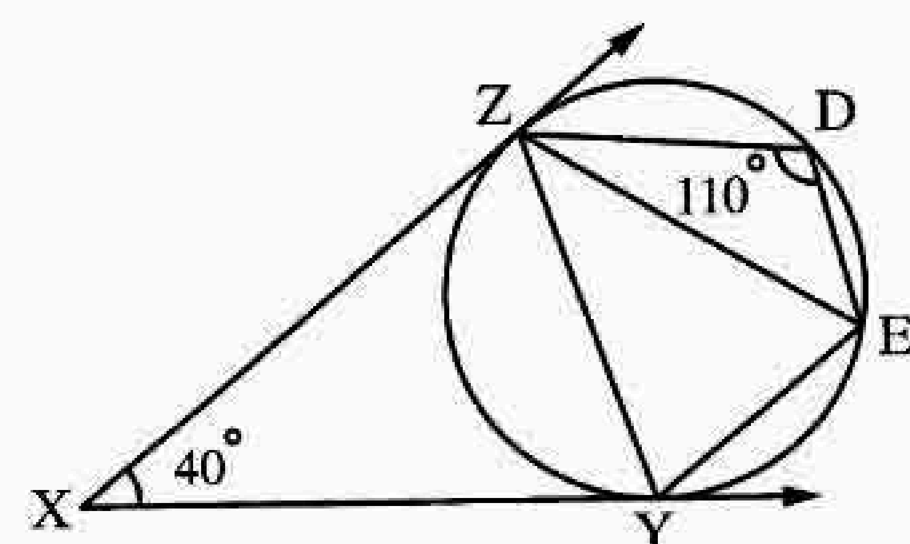
where $\overrightarrow{DC} \cap \overleftrightarrow{AB} = \{C\}$

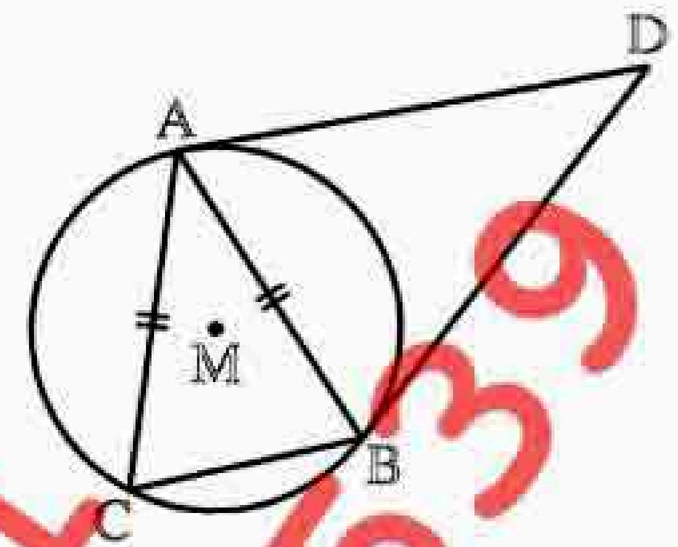
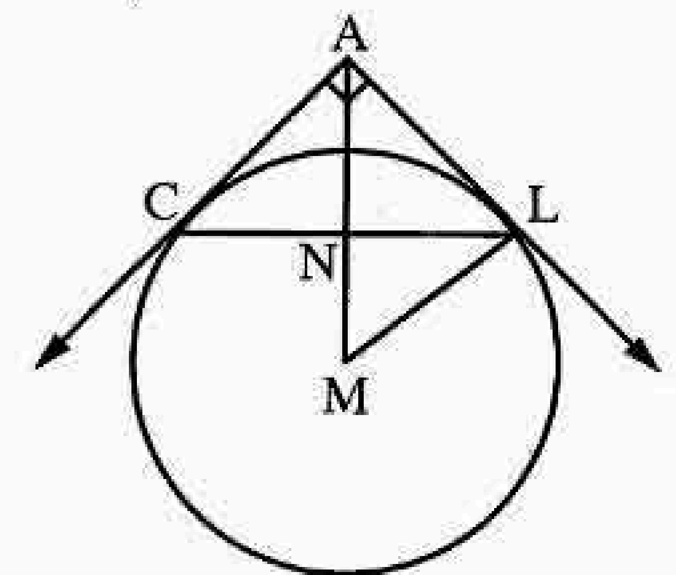
Prove that : (1) C is the midpoint of $\overline{AB}$

(2) $\overline{AD} \perp \overline{BD}$



(Alex. 2014 , 2016 , South Sinai 2012)

4	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , m (∠ BDC) = 65° **Find with proof :** m (∠ BAC)  *(South Sinai 17 , El-Menia 16 , Beni Suef 14)* « 50° »
5	📖 **In the opposite figure :** ABC is a triangle inscribed in a circle , $\overleftrightarrow{BD}$ is a tangent to the circle at B , X ∈ $\overline{AB}$ and Y ∈ $\overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$ **Prove that :** AXYC is a cyclic quadrilateral.  *(Cairo 17 , El-Kalyoubia 14 , Port Said 13)*
6	**In the opposite figure :** $\overrightarrow{AB}$ is a tangent to the circle M **Find :** m (∠ BAC)  *(El-Wadi El-Gedied 17)* « 40° »
7	**In the opposite figure :** $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X , m (∠ D) = 110° , m (∠ X) = 40° **Prove that :** m $(\overset{\frown}{ZDE})$ = m $(\overset{\frown}{ZY})$  *(Assiut 17 , El-Gharbia 17)*
8	📖 ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle and $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , If m (∠ AEB) = 70° and m (∠ ADC) = 125° , **prove that :** (1) AB = AC (2) $\overrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E *(Alex. 17)*

9	**In the opposite figure :** $\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B C $\in$ the circle M such that AB = AC **Prove that :** $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD  *(Cairo 17 , Damietta 16 , N. Sinai 14)*
10	ABCD is a parallelogram in which AC = BC **Prove that :** $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC *(Port Said 17 , Ismailia 16 , El-Menia 13)*
11	ABC is a triangle inscribed in a circle. $\overleftrightarrow{AD}$ is a tangent to the circle at A , X $\in \overline{AB}$ and Y $\in \overline{AC}$ where $\overline{XY}$ // $\overline{BC}$ **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y *(El-Fayoum 17 , Alexandria 15 , El-Beheira 14)*
12	**In the opposite figure :** $\overline{AL}$ and $\overline{AC}$ are two tangent - segments to the circle M at L and C , $\overline{AL} \perp \overline{AC}$, AC = 7 cm.  (1) **Find with proof :** the length of $\overline{AL}$ (2) **Prove that :** $\overleftrightarrow{AL}$ is a tangent to the circle passing through the vertices of the triangle ANC *(El-Sharkia 2016)* « 7 cm. »

Solutions

1

$\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ M)
(inscribed and central angles of the same arc $\overset{\frown}{BD}$)
$\therefore$ m ($\angle$ BCD) = $\frac{1}{2} \times 130° = 65°$ (1)
$\because$ $\overline{AB}$ // $\overline{CD}$
$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65° (alternate angles)
$\because$ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M
$\therefore$ AB = AC
$\therefore$ m ($\angle$ ACB) = m ($\angle$ ABC) = 65° (2)
From (1) , (2) : $\therefore$ m ($\angle$ ACB) = m ($\angle$ BCD) = 65°
$\therefore$ $\overrightarrow{CB}$ bisects $\angle$ ACD (First req.)
, m ($\angle$ A) = 180° – 2 × 65° = 50° (Second req.)

2

$\because$ $\overline{AB}$ is a tangent-segment to the circle M at B
, $\overline{MB}$ is a radius $\therefore$ m ($\angle$ AMB) = 90°
From Δ ABM : m ($\angle$ MAB) = 180° – (90° + 70°)
= 20°
, $\because$ $\overrightarrow{AM}$ bisects $\angle$ BAC
$\therefore$ m ($\angle$ BAC) = 2 × 20° = 40°
, $\because$ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M
$\therefore$ AB = AC
$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 40°}{2} = 70°$
(First req.)
, $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ BMD)
(inscribed and central angles of the same arc $\overset{\frown}{BD}$)
$\therefore$ m ($\angle$ BCD) = $\frac{1}{2} \times 70° = 35°$
$\therefore$ m ($\angle$ ACD) = 70° – 35° = 35° (Second req.)

3

$\because$ $\overline{CA}$, $\overline{CD}$ are two tangent-segments to the circle M
$\therefore$ CA = CD (1)
$\because$ $\overline{CD}$ and $\overline{CB}$ are two tangent-segments to the circle N
$\therefore$ CD = CB (2)
From (1) and (2) : $\therefore$ CA = CD = CB
$\therefore$ C is the midpoint of $\overline{AB}$ (Q.E.D. 1)
$\therefore$ Δ ABD in which $\overline{DC}$ is a median , DC = $\frac{1}{2}$ AB
$\therefore$ $\overline{AD} \perp \overline{BD}$ (Q.E.D. 2)

4

$\because$ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C
$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)
, $\because$ m ($\angle$ ABC) (the tangency angle)
= m ($\angle$ BDC) (the inscribed angle) = 65°
$\therefore$ m ($\angle$ BAC) = 180° – (65° + 65°) = 50° (The req.)

5

$\because$ m ($\angle$ ABD) (the tangency angle)
= m ($\angle$ C) (the inscribed angle) (1)
$\because$ $\overleftrightarrow{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{XB}$ is a transversal to them
$\therefore$ m ($\angle$ YXB) = m ($\angle$ XBD) (alternate angles) (2)
From (1) and (2) : $\therefore$ m ($\angle$ C) = m ($\angle$ YXB)
$\therefore$ The figure AXYC is a cyclic quadrilateral.
(Q.E.D.)

6

In Δ ACD : $\because$ $x° + 6x° + 2x° = 180°$
$\therefore$ $9x° = 180°$ $\therefore$ $x° = 20°$
$\therefore$ m ($\angle$ ADC) = 2 × 20° = 40°
$\therefore$ m ($\angle$ BAC) (the tangency angle)
= m ($\angle$ ADC) (the inscribed angle)
$\therefore$ m ($\angle$ BAC) = 40° (The req.)

7

$\because$ $\overrightarrow{XZ}$ and $\overrightarrow{XY}$ are two tangents to the circle
$\therefore$ XZ = XY
$\therefore$ m ($\angle$ XZY) = $\frac{180° - 40°}{2} = 70°$
, $\because$ m ($\angle$ ZEY) (the inscribed angle)
= m ($\angle$ XZY) (the tangency angle)
$\therefore$ m ($\angle$ ZEY) = 70° (1)
, $\because$ the figure YZDE is a cyclic quadrilateral
$\therefore$ m ($\angle$ ZYE) + m ($\angle$ D) = 180°
$\therefore$ m ($\angle$ ZYE) = 180° – 110° = 70° (2)
From (1) and (2) :
m ($\angle$ ZEY) = m ($\angle$ ZYE) = 70°
$\therefore$ ZE = ZY
$\therefore$ m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$) (Q.E.D.)

8

$\because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle ABC) = 180° - 125°$

$= 55°$ (1)

$\because \overrightarrow{EA}$, $\overrightarrow{EB}$ are two tangents to the circle at A and B

$\therefore EA = EB$ $\because m(\angle E) = 70°$

$\therefore m(\angle EAB) = \frac{180° - 70°}{2} = 55°$

$\because \overrightarrow{EA}$ is a tangent to the circle at A

$\therefore m(\angle EAB)$ (tangency) $= m(\angle ACB)$ (inscribed)

$\therefore m(\angle ACB) = 55°$ (2)

From (1) and (2) : $\therefore m(\angle ACB) = m(\angle ABC) = 55°$

$\therefore AB = AC$ (Q.E.D. 1)

$\therefore m(\angle BAC) = 180° - 2 \times 55° = 70°$

$\therefore m(\angle BAC) = m(\angle E) = 70°$

$\therefore \overrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E (Q.E.D. 2)

9

$\because \overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

$\therefore DA = DB$

$\therefore m(\angle 1) = m(\angle 2)$

$\therefore m(\angle D) = 180° - 2\, m(\angle 1)$ (1)

In ΔABC : $\because AB = AC$

$\therefore m(\angle 3) = m(\angle 4)$

$\therefore m(\angle BAC) = 180° - 2\, m(\angle 4)$ (2)

$\because \overline{AD}$ is a tangent-segment to the circle

$\therefore m(\angle 4)$ (inscribed) $= m(\angle 1)$ (tangency) (3)

From (1) , (2) and (3) : $\therefore m(\angle D) = m(\angle BAC)$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of ΔABD (Q.E.D.)

10

In ΔABC :

$\because AC = BC$

$\therefore m(\angle B) = m(\angle BAC)$ (1)

$\because \overline{AB} \,//\, \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal to them

$\therefore m(\angle DCA) = m(\angle BAC)$ (alternate angles) (2)

From (1) and (2) : $\therefore m(\angle B) = m(\angle DCA)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circumcircle of ΔABC

(Q.E.D.)

11

$\because \overleftrightarrow{AD}$ is a tangent to the circle at A

$\therefore m(\angle DAC)$ (tangency)

$= m(\angle B)$ (inscribed)

$\because \overleftrightarrow{XY} \,//\, \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal to them

$\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles)

$\therefore m(\angle DAC) = m(\angle AXY)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

12

$\because \overrightarrow{AL}$, $\overrightarrow{AC}$ are two tangents to the circle M

$\therefore AL = AC = 7$ cm. (First req.)

$\therefore$ In ΔALC : $m(\angle ALC) = m(\angle ACL)$

, $\because m(\angle LAC) = 90°$

$\therefore m(\angle ACL) = \frac{180° - 90°}{2} = 45°$

, $\because \overrightarrow{AM}$ bisects $\angle A$ $\therefore m(\angle LAM) = 45°$

$\therefore m(\angle LAN) = m(\angle ACL) = 45°$

$\therefore \overrightarrow{AL}$ is a tangent to the circle passing through the vertices of ΔANC (Second req.)

Exercises

[A] : Choose The Correct Answer : -

1	The angle of tangency is included between …………… (a) two chords. (b) two tangents. (c) a chord and a tangent. (d) a chord and a diameter.
2	**In the opposite figure :** If AC = 8 cm. , AZ = 3 cm. , BZ = 2 cm. , then BC = …………… (a) 5 cm. (b) 7 cm. (c) 10 cm. (d) 13 cm.
3	If the figure ABCD ~ the figure XYZL , then m (∠ B) = m (∠ ……………) (a) X (b) Y (c) Z (d) L
4	The point of concurrence of the medians of the triangle divides each median in the ratio …………… from its base. (a) 2 : 1 (b) 1 : 2 (c) 2 : 3 (d) 1 : 3
5	ABC is a triangle in which AB = AC , m (∠ C) = 40° , then m (∠ A) = …………… (a) 40° (b) 80° (c) 100° (d) 120°
6	**In the opposite figure :** ABC is right-angled triangle at B , m (∠ C) = 30° , AB = 3 cm. , then AC = …………… cm. (a) 2 (b) 3 (c) $3\sqrt{3}$ (d) 6
7	The rhombus in which the lengths of its diagonals are L_1 and L_2 , its area = …………… (a) $L_1 L_2$ (b) $L_1 + L_2$ (c) $2 L_1 L_2$ (d) $\frac{1}{2} L_1 L_2$
8	**In the opposite figure :** If the side length of the square ABCD = 7 cm. and the side length of the square XYZL = 3 cm. , then the area of the shaded part = …………… cm². (a) (7 – 3) (b) 4 (7 – 3) (c) $(7 - 3)^2$ (d) $(7^2 - 3^2)$
9	A circle its radius length is 5 cm. , then its circumference = …………… cm. (a) 5 π (b) 7 π (c) 10 π (d) 25 π

10	**In the opposite figure :** The length of $\overline{BC}$ = ... cm. (a) 3 (b) 4 (c) 5 (d) 6
11	The corresponding angles of the two similar polygons are .... in measure. (a) equal (b) different (c) proportional (d) alternate
12	The image of the point (2 , 3) by rotation R (O , 180°) is the point .... (a) (2 , 3) (b) (– 2 , 3) (c) (2 , – 3) (d) (– 2 , – 3)
13	ABC is a triangle having one symmetric line and its side lengths are 10 , 5 and X cm. , then X = ... cm. (a) 5 (b) 8 (c) 10 (d) 12
14	**In the opposite figure :** AB = AC , AB = 2 X – 1 and AC = X + 2 , then X = ... (a) 3 (b) 5 (c) 11 (d) 14
15	A rectangular picture its length is 60 cm. and its width is 40 cm. We need to make a wooden frame its width is 5 cm. , then its total area is .... cm^2. (a) 3050 (b) 3500 (c) 2925 (d) 3250
16	The perimeter of the square whose area is 81 cm^2. is .... (a) 24 cm. (b) 8 cm. (c) 9 cm. (d) 36 cm.
17	The radius length of the circle whose centre is (7 , 4) and passes through the point (3 , 1) equals .... length unit. (a) 3 (b) 4 (c) 5 (d) 6
18	The image of the point (A , B) by rotation R (0 , 180°) the point .... (a) (– A , B) (b) (– A , – B) (c) (A , – B) (d) (A , B)
19	ABC is a triangle where $(AB)^2 = (AC)^2 + (BC)^2$, m (∠ B) = 40° , then m (∠ A) = ... (a) 40° (b) 50° (c) 90° (d) 130°
20	The number of the axes of symmetry in the equilateral triangle = ... (a) 1 (b) 2 (c) 3 (d) an infinite number.
21	If m_1 and m_2 are the slopes of two perpendicular straight lines , then .... (a) $m_1 + m_2 = 0$ (b) $m_1 - m_2 = -1$ (c) $m_1 = m_2$ (d) $m_1 \times m_2 = -1$

22 The sum of measures of the interior angles of the quadrilateral = ...

(a) 90° (b) 180° (c) 270° (d) 360°

23 A square of perimeter 20 cm. , then its area = ... cm^2.

(a) 20 (b) 25 (c) 50 (d) 100

24 If the radius length of the circle M equals 2 cm. , then its circumference equals ....

(a) 4π cm. (b) 5π cm. (c) 6π cm. (d) 7π cm.

25 The opposite figure represents a semicircle its centre is M and its radius length is r length unit, then the area of the opposite figure = ... square units.

(a) $2\pi r$ (b) πr (c) πr^2 (d) $\frac{\pi r^2}{2}$

26 **In the opposite figure :**

$\overrightarrow{CA}$, $\overrightarrow{CB}$ are two tangents to the circle M , m ($\angle$ C) = 60°

, then m ($\angle$ M) = ...°

(a) 90 (b) 100

(c) 110 (d) 120

27 **In the opposite figure :**

$\overline{CB}$ and $\overline{CD}$ are two tangent-segments at B and D

, m ($\angle$ C) = 70°

, then m ($\widehat{DB}$ the minor) = ...

(a) 180° (b) 90° (c) 100° (d) 110°

28 If m_1 , m_2 are two slopes of two parallel straight lines , then ....

(a) $m_1 + m_2 = 0$ (b) $m_1 = m_2$ (c) $m_1 \times m_2 = -1$ (d) $m_1 - m_2 = -1$

29 If the projection of a line segment on a straight line is a point , then the line segment .... the straight line.

(a) // (b) $\perp$ (c) $\in$ (d) $\subset$

30 Δ XYZ is right-angled triangle at Y , then XZ .... YZ

(a) < (b) > (c) = (d) twice

31 The sum of lengths of any two sides of a triangle .... the length of the third side.

(a) < (b) > (c) = (d) $\leq$

32 The diagonals are equal in length and not perpendicular in ....

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

33 The area of a square whose diagonal length is 6 cm. equals .... cm^2.

(a) 36 (b) 18 (c) 24 (d) 9

34 The longest chord in the circle is called ....

(a) diameter. (b) tangent. (c) secant. (d) radius.

35 If the area of the circle M = 16π cm^2, A is a point on its plane where MA = 8 cm.
, then A is ....

(a) outside the circle. (b) inside the circle.
(c) on the circle. (d) on the centre of the circle.

36 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents of the circle M

, m (∠ MBC) = 30°

, if AB = 4 cm. then the length of $\overline{BC}$ = ... cm.

(a) 3 (b) 4 (c) 5 (d) 8

37 **In the opposite figure :**

If m (∠ CBD) = 50°

, then m (∠ AMD) = ...

(a) 40° (b) 50°
(c) 80° (d) 100°

38 If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (4 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (8 , – 2)

39 ABC is a right-angled triangle at B where AB = 6 cm. , BC = 8 cm.
, then its area = ... cm^2.

(a) 48 (b) 14 (c) 24 (d) 7

40 The numbers 5 , 4 , ... can be side lengths of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

41 **In the opposite figure :**

ABCD is quadrilateral , m (∠ ABD) = 30°

, m (∠ DCE) = 120°

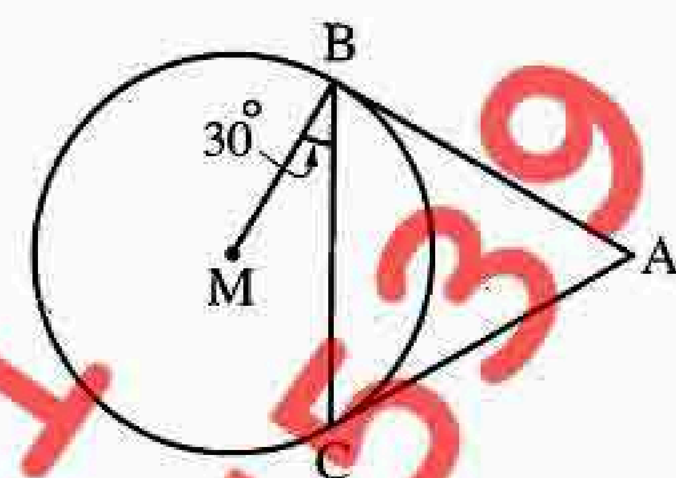
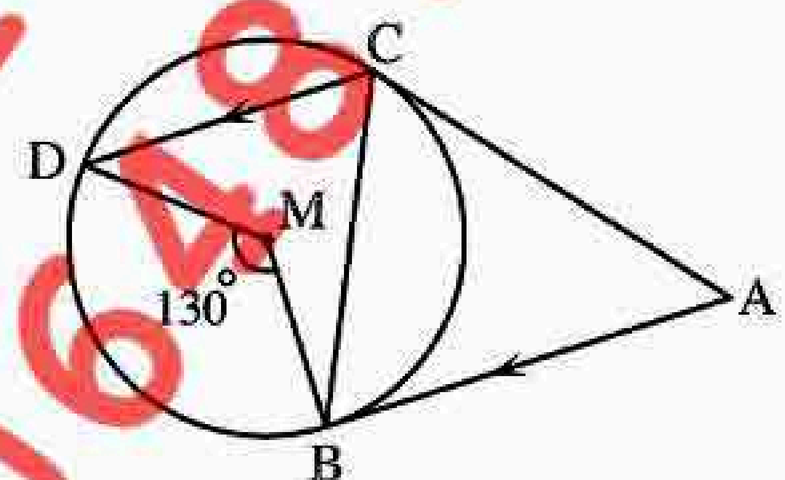
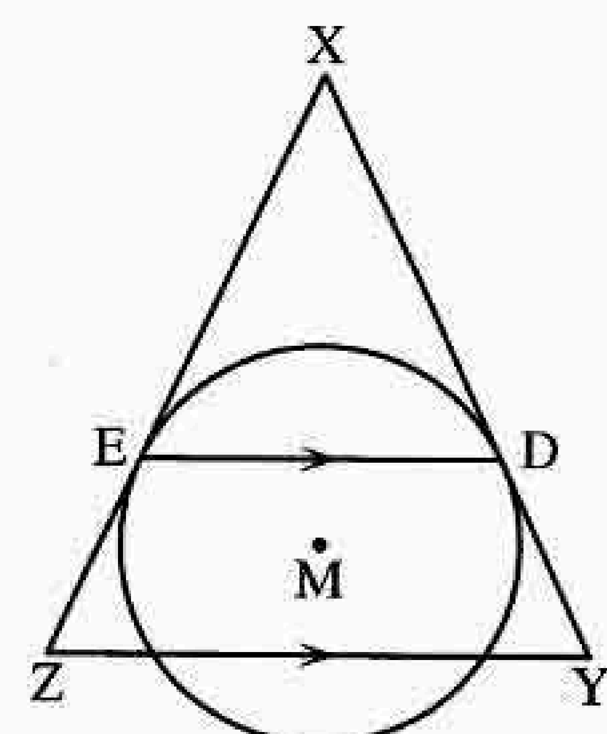
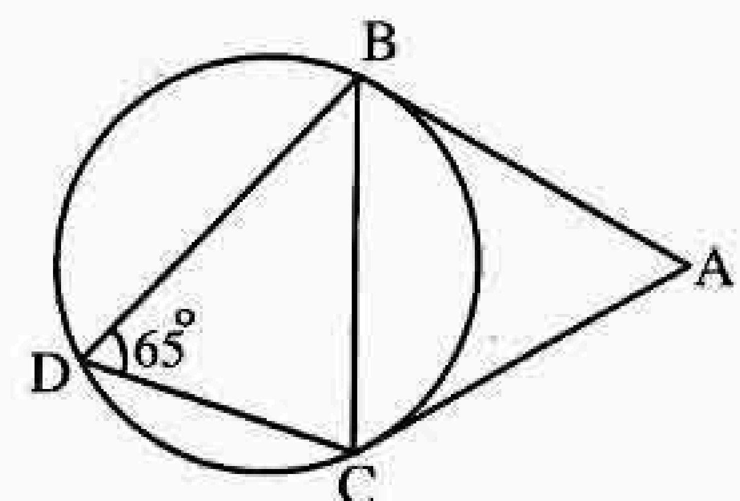
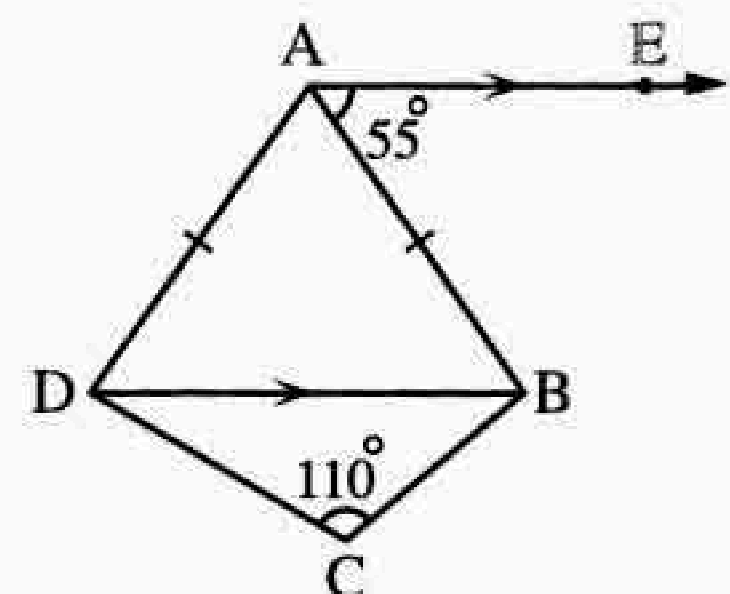
, then ABCD is ....

(a) a rectangle. (b) a rhombus.
(c) a cyclic quadrilateral. (d) a parallelogram.

42 The number of the symmetry axes of square is ....

(a) 1 (b) 2 (c) 3 (d) 4

[B] : Essay Problems : -

1	**In the opposite figure :** If $\overline{AB}$ and $\overline{AC}$ are two tangent - segments to the circle M and m (∠ MBC) = 30° **Prove that :** Δ ABC is equilateral.  *(Kafr El-Sheikh 2011)*
2	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M , $\overline{AB}$ // $\overline{CD}$ and m (∠ BMD) = 130° (1) **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD (2) **Find :** m (∠ A) *(El-Fayoum 17 , El-Gharbia 16 , El-Kalyoubia 16 , El-Menia 15 , Cairo 14)* « 50° » 
3	**In the opposite figure :** XYZ is a triangle , $\overline{XY}$ and $\overline{XZ}$ touch the circle M at D and E If $\overline{DE}$ // $\overline{YZ}$, **prove that :** The figure DYZE is a cyclic quadrilateral.  *(Alex. 2004)*
4	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C , m (∠ BDC) = 65° **Find with proof :** m (∠ BAC)  *(South Sinai 17 , El-Menia 16 , Beni Suef 14)* « 50° »
5	**In the opposite figure :** $\overrightarrow{AE}$ // $\overline{DB}$, m (∠ BAE) = 55° , m (∠ C) = 110° and AB = AD **Prove that :** (1) The figure ABCD is a cyclic quadrilateral. (2) $\overrightarrow{AE}$ is a tangent to the circumcircle of the quadrilateral ABCD  *(Beheira 05)*

6

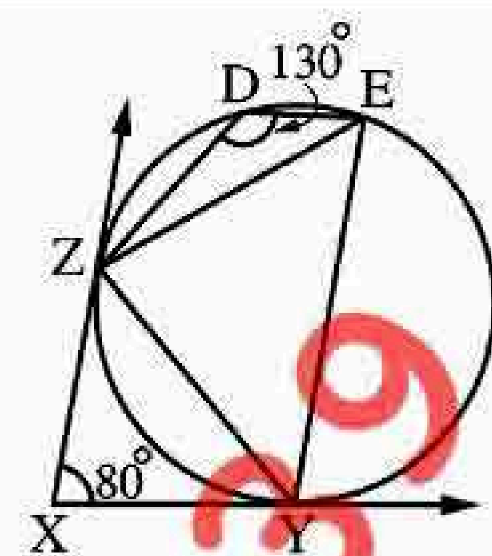
In the opposite figure :

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y and Z

, m (∠ YXZ) = 80°

and m (∠ EDZ) = 130°

Prove that : (1) ZE = ZY (2) $\overline{XZ}$ // $\overline{YE}$



(Giza 2009)

7

📖 **In the opposite figure :**

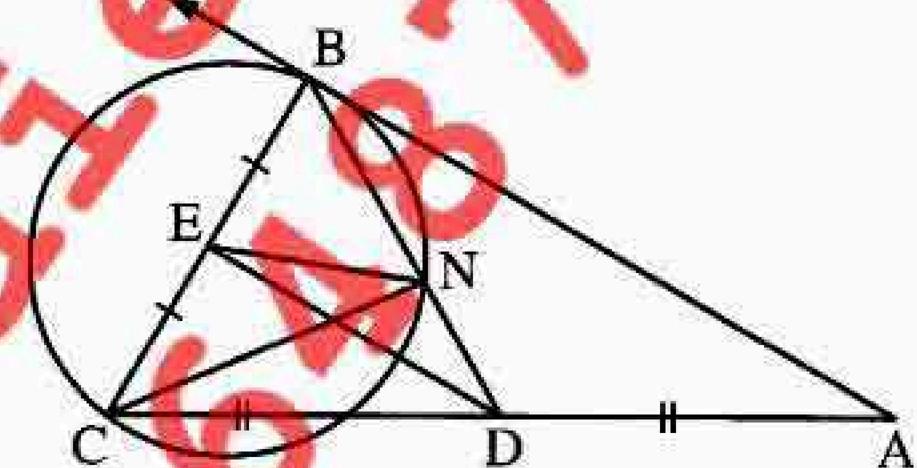
$\overrightarrow{AB}$ is a tangent to the circle M , $\overline{AC}$ is a secant to it

, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

and $\overline{BD}$ ∩ the circle M = {N} **Prove that :**

(1) $\overline{AB}$ // $\overline{DE}$

(2) The points N , D , C , E have one circle passing through them.



(Port Said 15)

8

In the opposite figure :

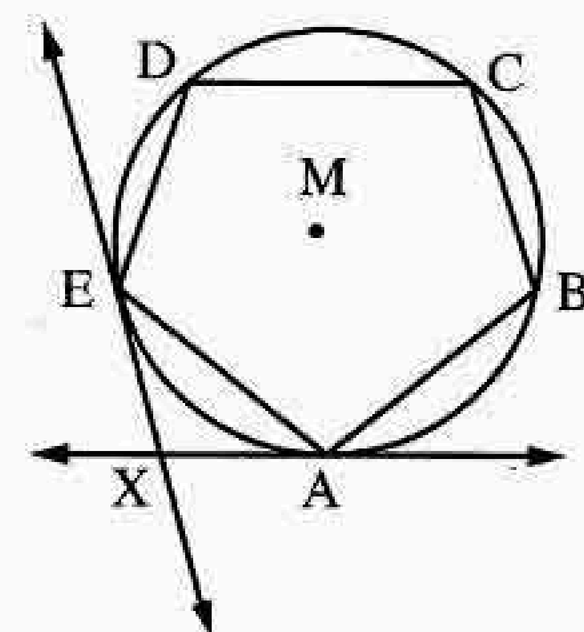
ABCDE is a regular pentagon inscribed in the circle M ,

$\overleftrightarrow{AX}$ is a tangent to the circle at A , $\overleftrightarrow{EX}$ is a tangent to the circle at E where $\overleftrightarrow{AX}$ ∩ $\overleftrightarrow{EX}$ = {X}

Find :

(1) m ($\overset{\frown}{AE}$)

(2) m (∠ AXE)



(Matrouh 2011) « 72° , 108° »

9

📖 ABCD is a parallelogram in which AC = BC

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle circumscribed about the triangle ABC

(Port Said 17 , Ismailia 16 , El-Menia 13)

10

📖 ABC is a triangle inscribed in a circle. $\overleftrightarrow{AD}$ is a tangent to the circle at A ,

X ∈ $\overline{AB}$ and Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

(El-Fayoum 17 , Alexandria 15 , El-Beheira 14)

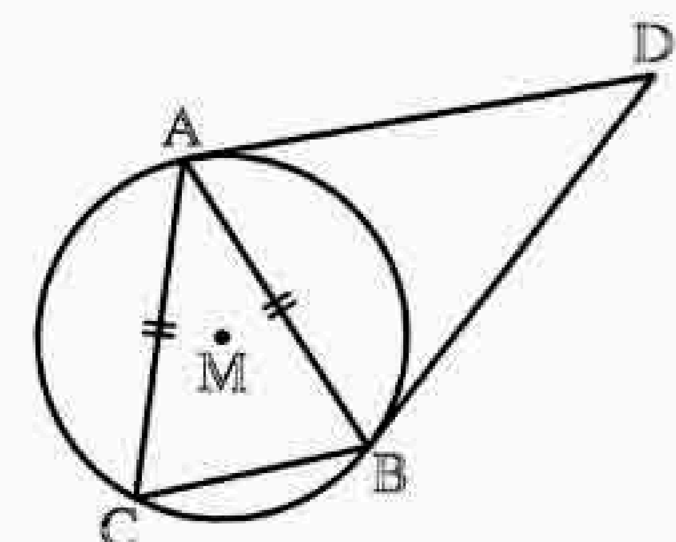
11

In the opposite figure :

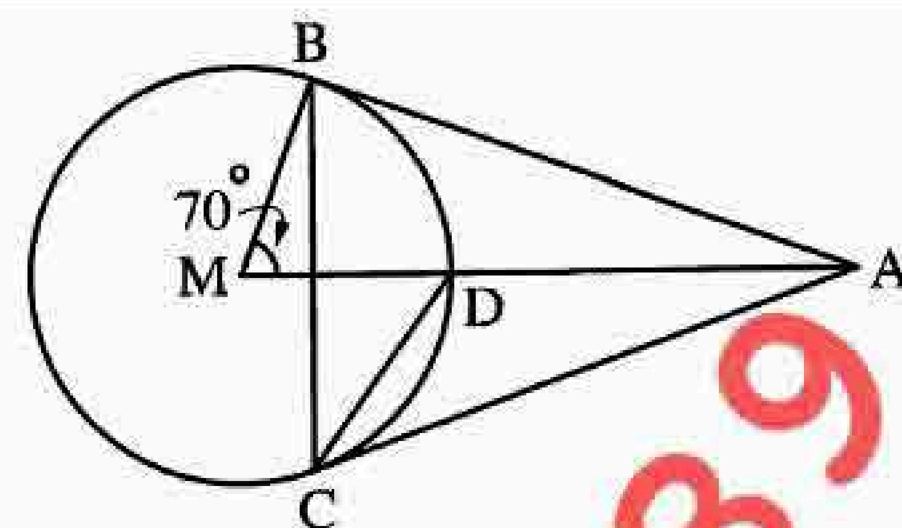
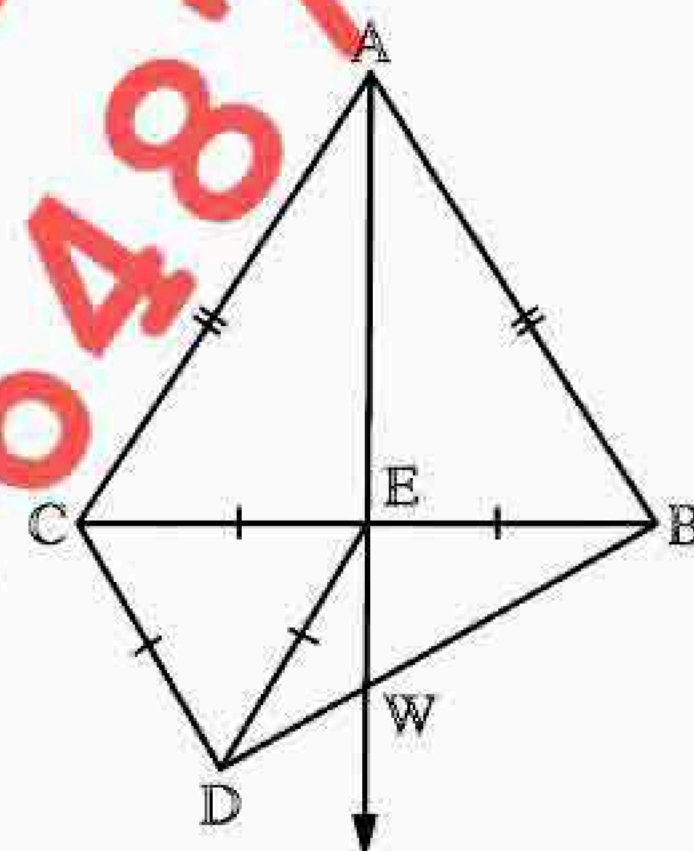
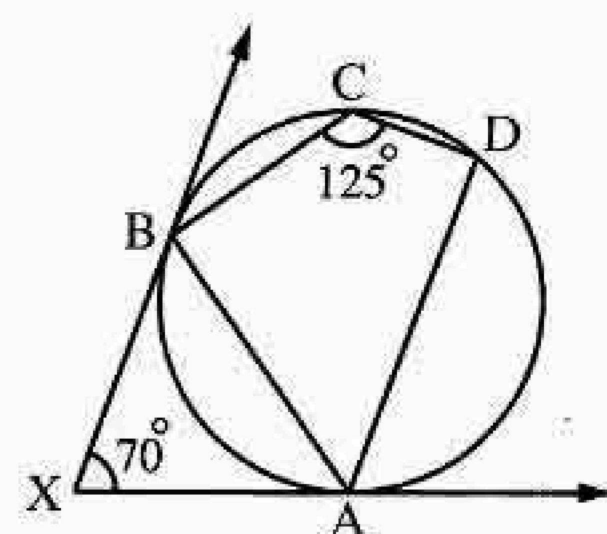
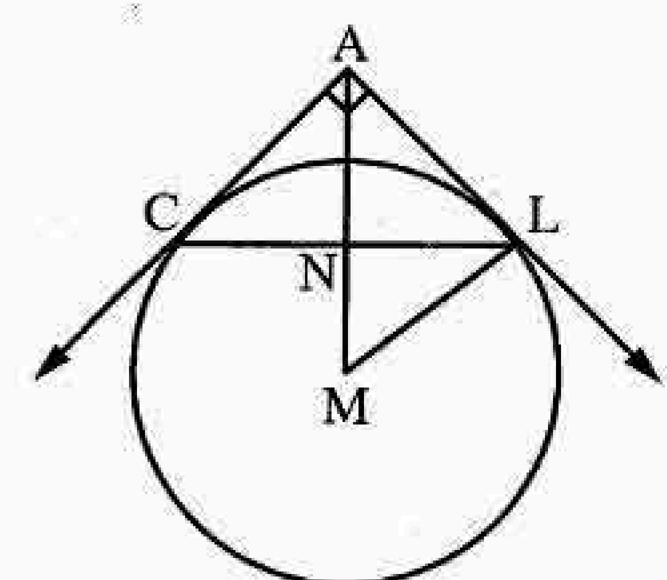
$\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

C ∈ the circle M such that AB = AC

Prove that : $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD



(Cairo 17 , Damietta 16 , N. Sinai 14)

12	**In the opposite figure :** $\overline{AB}$ and $\overline{AC}$ are two tangent-segments drawn from A , m (∠ AMB) = 70° **Find :** (1) m (∠ ABC) (2) m (∠ ACD)  *(El-Ismailia 17)* « 70° , 35° »
13	**In the opposite figure :** ABC and DCE are two equilateral triangles , E is the midpoint of $\overline{BC}$, $\overrightarrow{AE} \cap \overline{BD} = \{W\}$ (1) **Prove that :** $\overline{AC}$ is a tangent-segment to the circle which passes through the vertices of Δ CED (2) **Prove that :** CDWE is a cyclic quadrilateral. (3) **Find :** The centre of the circle which passes through the vertices of the quadrilateral CDWE  *(El-Sharkia 17)*
14	**In the opposite figure :** $\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B , m (∠ AXB) = 70° and m (∠ DCB) = 125° **Prove that :** (1) $\overrightarrow{AB}$ bisects ∠ DAX (2) $\overline{AD}$ // $\overrightarrow{XB}$  *(Luxor 2016 , Qena 2016 , El-Beheira 2011)*
15	**In the opposite figure :** $\overline{AL}$ and $\overline{AC}$ are two tangent - segments to the circle M at L and C , $\overline{AL} \perp \overline{AC}$, AC = 7 cm. (1) **Find with proof :** the length of $\overline{AL}$ (2) **Prove that :** $\overrightarrow{AL}$ is a tangent to the circle passing through the vertices of the triangle ANC  *(El-Sharkia 2016)* « 7 cm. »

Homework

[A] : Choose The Correct Answer : -

1	If the area of the circle is 9π cm^2, then its radius length = ... cm. (a) 9 (b) 2 (c) (– 3) (d) 3
2	The two tangent-segments drawn from a point outside a circle are .... (a) equal in length. (b) not equal in length. (c) perpendicular. (d) parallel.
3	**In the opposite figure :** $\overleftrightarrow{ED}$ is a tangent of the circle at B , m (∠ ABC) = 65° , m (∠ CBE) = 75° , then m (∠ C) = ...° (a) 20 (b) 40 (c) 50 (d) 80
4	The distance between the two points (6 , 0) , (– 4 , 0) equals .... length units. (a) – 10 (b) 10 (c) 2 (d) 24
5	The two angles A and C in the right-angled triangle at B are .... (a) complementary. (b) supplementary. (c) adjacent. (d) vertically opposite angles.
6	The number of symmetric axes of the square is .... (a) 1 (b) 2 (c) 3 (d) 4
7	The area of the triangle whose base length is 10 cm. and its height is 6 cm. equals .... cm^2 (a) 6 (b) 10 (c) 30 (d) 60
8	**In the opposite figure :** The length of $\overline{AB}$ = ... cm. (a) $10\sqrt{3}$ (b) 10 (c) 5 (d) $5\sqrt{3}$
9	The area of the rhombus whose diagonal lengths are 8 cm. and 10 cm. equals .... cm^2. (a) 2 (b) 18 (c) 40 (d) 80

10 **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD} // \overline{BC}$

, $\overline{AD}$ is a diameter of circle M

, then the area of the shaded region is ....

(a) 70 cm^2. (b) 147 cm^2. (c) 170 cm^2. (d) 224 cm^2.

11 Circumference of a circle is 6 π cm. , L is a straight line at a distance of 3 cm. from its centre , then L is ....

(a) a tangent to the circle. (b) a secant to the circle.

(c) outside the circle. (d) the diameter to the circle.

12 If the measure of the tangency angle = 70° , then the measure of the central angle subtended by the same arc = ...°

(a) 35 (b) 70 (c) 140 (d) 105

13 **In the opposite figure :**

If $\overrightarrow{BD}$ is a tangent to the circle M

, m (∠ BAM) = 25°

, then m (∠ ABD) = ...

(a) 25° (b) 50° (c) 65° (d) 120°

14 The sum of measures of the accumulative angles at a point = ...°

(a) 80 (b) 120 (c) 360 (d) 630

15 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

16 The medians of triangle intersect at a same point which divides each in the ratio .... from its base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 2

17 **In the opposite figure :**

ABC is a triangle , AB = AC

, m (∠ B) = 50°

, then m (∠ A) = ...

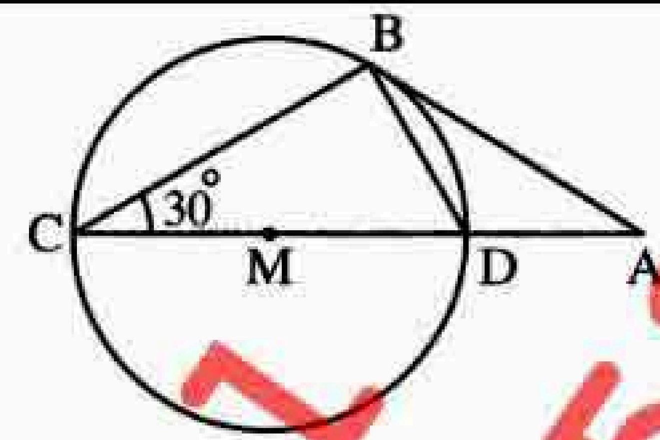
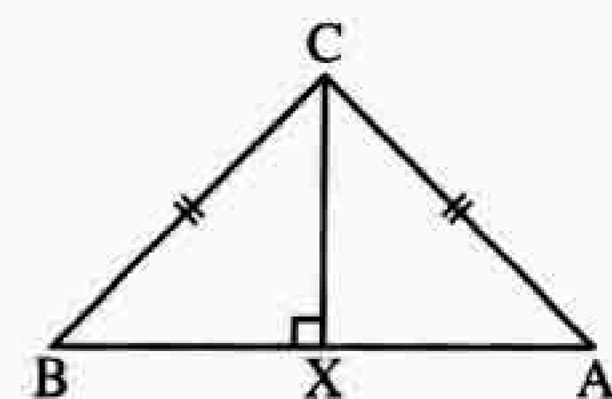
(a) 100° (b) 90° (c) 80° (d) 70°

18 The area of the rhombus whose diagonal lengths are 6 cm. , 8 cm. is .... cm^2.

(a) 2 (b) 14 (c) 24 (d) 48

19 In a regular hexagon , the measure of the angle of its vertex equals ....

(a) 60° (b) 108° (c) 120° (d) 135°

20	A circle whose circumference 20 π cm. its area = ... π cm^2. (a) 10 (b) 100 (c) 200 (d) 400
21	**In the opposite figure :** $\overline{AB}$ is a tangent of the circle M , then m (∠ ABC) = ...  (a) 120° (b) 110° (c) 90° (d) 30°
22	The angle whose measure is 50° complements an angle of measure .... (a) 90° (b) 130° (c) 50° (d) 40°
23	If cos 2 $x = \frac{1}{2}$ where x is an acute angle , then m (∠ x) = ...° (a) 15 (b) 30 (c) 45 (d) 60
24	If the ratio between the measures of the angles of a triangle is 2 : 3 : 4 , then the measure of the greatest angle is .... (a) 40° (b) 90° (c) 45° (d) 80°
25	**In the opposite figure :** CA = CB , $\overline{CX} \perp \overline{AB}$, AB = 2 CX , then m (∠ A) = ...  (a) 30° (b) 60° (c) 90° (d) 45°
26	If the side length of a rhombus is L cm. , then its perimeter = ... cm. (a) L^2 (b) $2L^2$ (c) 4 L (d) $2\sqrt{2}$ L
27	The number of sides of the regular polygon in which the measure one of its interior angles 135° = ... sides. (a) 4 (b) 6 (c) 8 (d) 10
28	If M is a circle of radius length r cm. , then the length of the semicircle = ... cm. (a) 2 π r (b) $\frac{1}{4}$ π r (c) $\frac{1}{2}$ π r (d) π r
29	If A ∈ the circle M of diameter length 6 cm. , then MA = ... cm. (a) 3 (b) 4 (c) 5 (d) 6
30	Which of the following points does not belong to the circle that its centre is the origin and its radius is 7 cm.? (a) (0 , 7) (b) (0 , – 7) (c) (7 , 0) (d) (7 , 7)

31 If the point $A \in$ the circle M and its diameter length equals 6 cm. , then MA = ... cm.

(a) 4 (b) 6 (c) 3 (d) 8

32 If M is a circle of diameter length 7 cm. , A is a point on its plane and MA = 4 cm. , then the position of A with respect to this circle is ....

(a) inside the circle. (b) outside the circle.

(c) on the circle. (d) coincide on the centre M

33 If M is a circle of a diameter length equals 14 cm. , MA = $(2x + 3)$ cm. where A lies on the circle , then x = ...

(a) 5 (b) 3 (c) 2 (d) 1

34 The number of symmetric axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

35 Number of the axes of symmetry of the semicircle is ....

(a) zero. (b) 1 (c) 2 (d) infinite.

36 The circle has .... number of axes of symmetry.

(a) 1 (b) 2 (c) 3 (d) an infinite

37 The number of the axes of symmetry of the semicircle .... the number of the axes of symmetry of the isosceles triangle.

(a) $>$ (b) $<$ (c) $=$ (d) $\geq$

38 If the straight line $L \cap$ the circle $M = \varnothing$, then L is .... of the circle.

(a) a secant (b) outside (c) a tangent (d) an axis of symmetry

39 If $\overleftrightarrow{AB} \cap$ the circle $M = \{A , B\}$, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\overleftrightarrow{AB}$ (b) $\overline{AB}$ (c) $\{A , B\}$ (d) $\overrightarrow{AB}$

40 The number of tangents can be drawn from a point lies on a circle equals ....

(a) one. (b) two. (c) four. (d) infinite number.

41 The tangent to a circle whose diameter length is 10 cm. , is at a distance of .... cm. from its centre.

(a) 4 (b) 5 (c) 6 (d) 10

42 A tangent to a circle of diameter length 8 cm. is at a distance of .... cm. from its centre.

(a) 4 (b) 3 (c) 8 (d) 6

43 A tangent to a circle of diameter length 6 cm. is at distance of .... cm. from its centre.

(a) 6 (b) 12 (c) 3 (d) 2

44 If the straight line L is outside a circle of radius length 3 cm. and its centre is the origin point M (0 , 0) , if L at distance x from its centre , then $x \in$ ....

(a) $[3 , \infty[$ (b) $]3 , \infty[$ (c) $[6 , \infty[$ (d) $]-\infty , -6[$

[B] : Essay Problems : -

1 📖 **In the opposite figure :**

M and N are two circles touching externally at D and $\overleftrightarrow{AB}$ is a common tangent to them at A and B

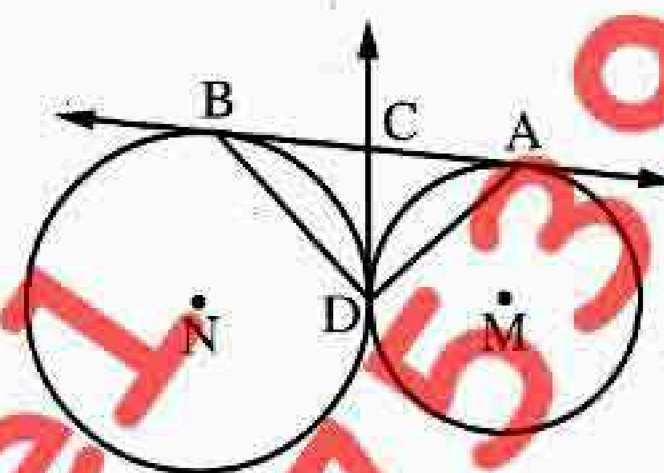
$\overrightarrow{DC}$ is a common tangent to the two circles at D ,

where $\overrightarrow{DC} \cap \overleftrightarrow{AB} = \{C\}$

Prove that : (1) C is the midpoint of $\overline{AB}$

(2) $\overline{AD} \perp \overline{BD}$

(Alex. 2014 , 2016 , South Sinai 2012)



2 **In the opposite figure :**

ABCD is a cyclic quadrilateral ,

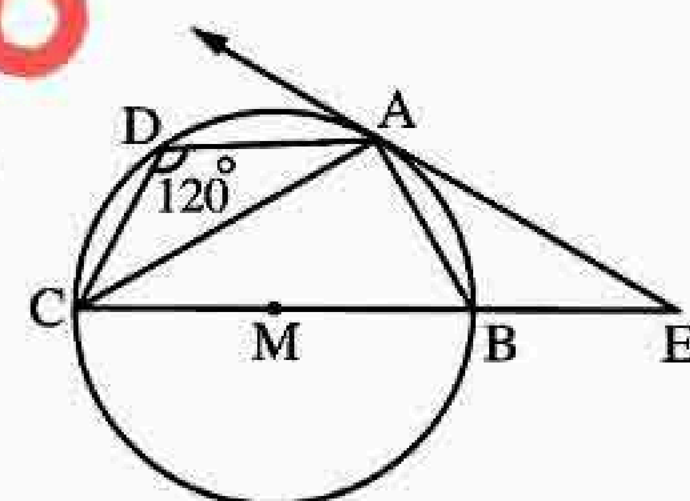
$\overline{BC}$ is a diameter ,

$\overrightarrow{EA}$ is a tangent for the circle at point A

and m (∠ ADC) = 120°

Prove that : (1) BA = BE (2) m (∠ ABE) = m (∠ EAC)

(Damietta 09)



3 **In the opposite figure :**

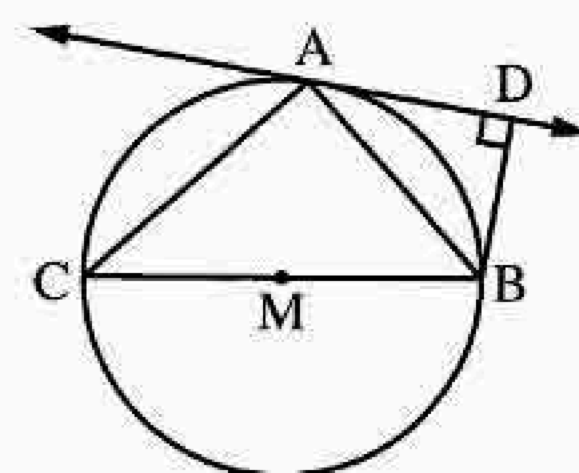
$\overleftrightarrow{AD}$ is a tangent to the circle M at A

, $\overline{BC}$ is a diameter in the circle M

and $\overline{BD} \perp \overleftrightarrow{AD}$

Prove that : m (∠ ABD) = m (∠ ABC)

(Port Said 2006)



4 **In the opposite figure :**

M and N are two circles intersecting at A and B

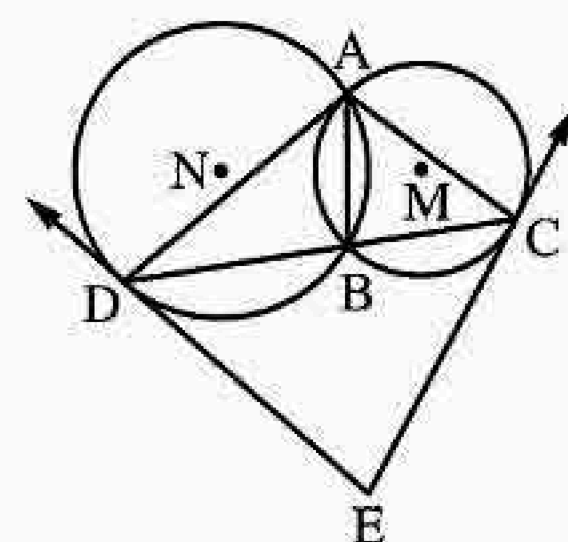
, B ∈ $\overline{CD}$, $\overrightarrow{EC}$ and $\overrightarrow{ED}$ are two tangents.

Prove that :

(1) m (∠ ECD) + m (∠ EDC) = m (∠ CAD)

(2) The figure ACED is a cyclic quadrilateral.

(Fayoum 2008)



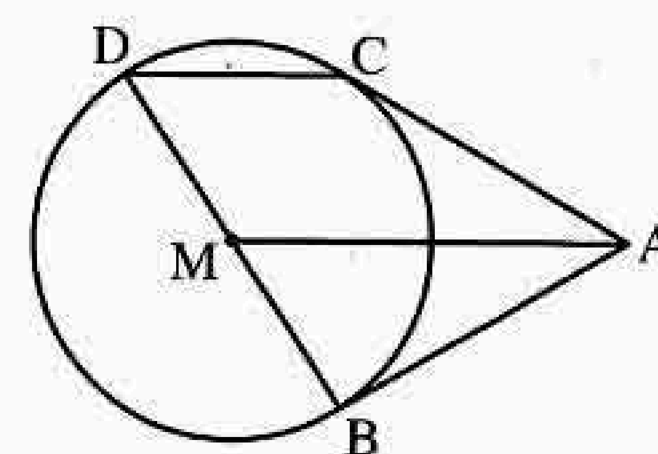
5 📖 **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

and $\overline{BD}$ is a diameter in the circle.

Prove that : $\overline{AM}$ // $\overline{CD}$

(EL-Monofia 2011)



6

In the opposite figure :

AB = AC and $\overleftrightarrow{EL}$ is a tangent to the circle at A

Prove that :

(1) m (∠ LAB) = m (∠ ABC)

(2) $\overrightarrow{AC}$ is a tangent to the circumcircle of Δ ADE



(South Sinai 2005)

7

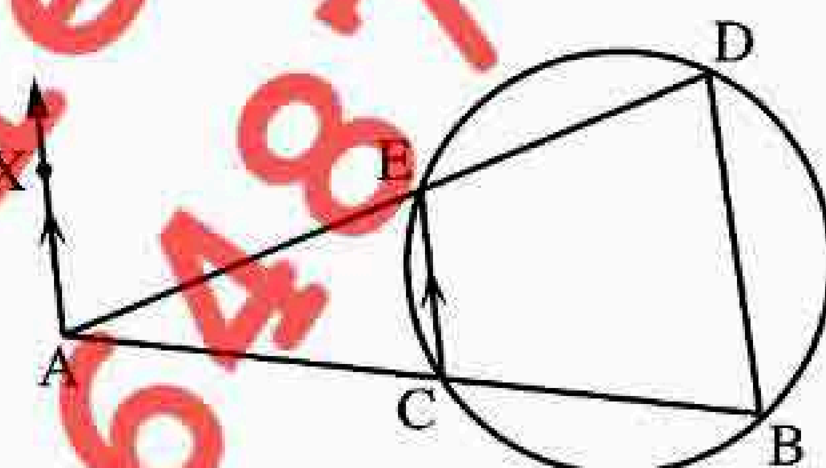
In the opposite figure :

The figure BCED is a cyclic quadrilateral

, $\overrightarrow{DE} \cap \overrightarrow{BC} = \{A\}$

and $\overrightarrow{AX}$ // $\overline{CE}$

Prove that : $\overleftrightarrow{AX}$ is a tangent to the circumcircle of Δ ABD



(Kafr El-Sheikh 2013)

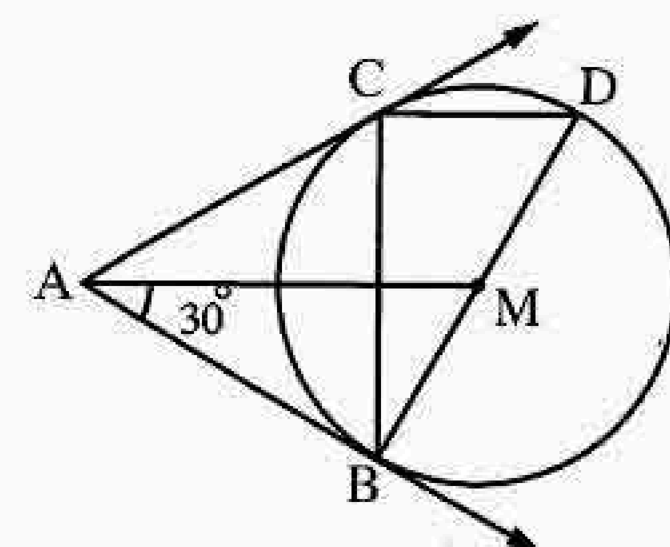
8

In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M

, $\overline{BD}$ is a diameter in it , m (∠ MAB) = 30°

Find : m (∠ ACD)



(El-Sharkia 2013) « 150° »

9

In each of the opposite figures :

$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are two tangents to the two circles M and N .

Prove that : AB = CD

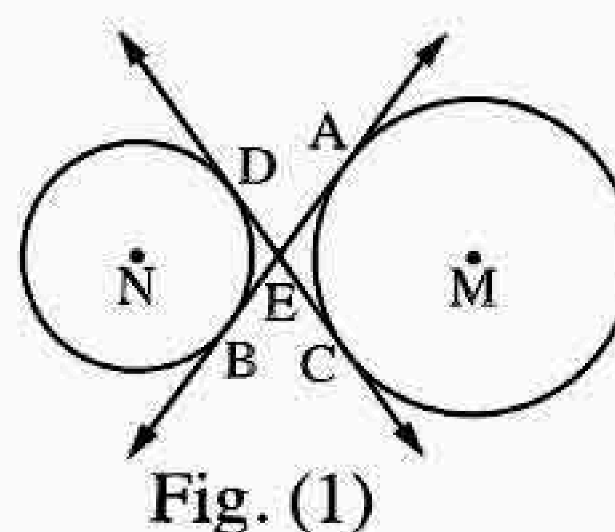


Fig. (1)

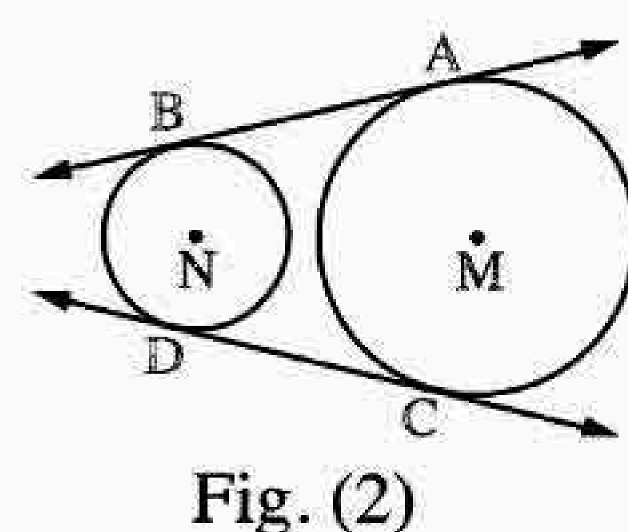


Fig. (2)

(Suez 11 , Port Said 13)

10

In the opposite figure :

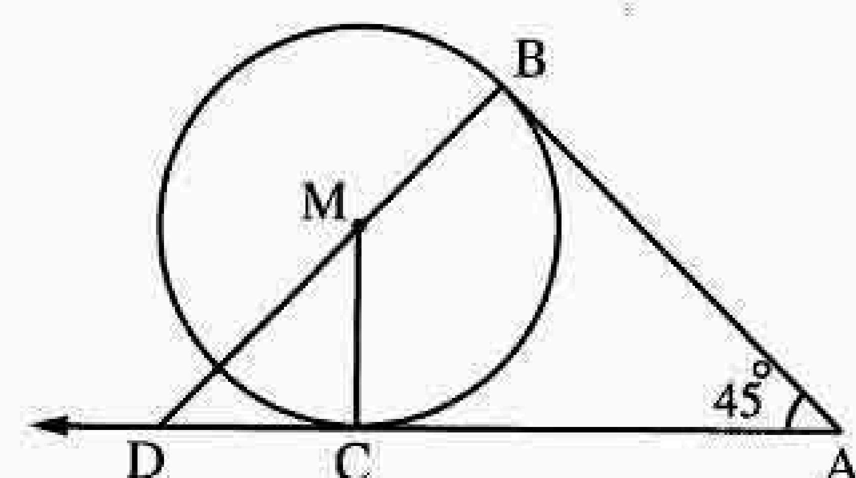
$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C respectively , m (∠ A) = 45°

, $\overrightarrow{BM} \cap \overrightarrow{AC} = \{D\}$

Prove that :

(1) The figure ABMC is cyclic quadrilateral.

(2) AD = AB + MB



(Helwan 2009)

11

In the opposite figure :

$\overleftrightarrow{AX}$ is a tangent

, m (∠ XAB) = 40°

and m (∠ ABC) = 110°

Find : m (∠ CDB)

(Kafr El-Sheikh 11) « 30° »

12

In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ touch the circle at B and C

, $\overline{AC}$ // $\overline{BD}$ and m (∠ A) = 40°

Find with proof :

(1) m (∠ ACB) (2) m (∠ ECD)

Then prove that : CB = CD

(Gharbia 04) « 70° , 70° »

13

In the opposite figure :

$\overline{AB}$ is a diameter of the semicircle ,

$\overrightarrow{CF}$ is a tangent to it at C and $\overrightarrow{DF}$ ⊥ $\overline{AB}$

(1) **Prove that :** The figure ADEC is a cyclic quadrilateral.

(2) **Prove that :** Δ FCE is isosceles.

(3) Determine the centre of the circle passing through the vertices of the quadrilateral ADEC

(Kafr El-Sheikh 2008)

14

In the opposite figure :

Two circles are intersecting at B and C

, A ∈ one of the two circles ,

$\overleftrightarrow{AF}$ is drawn as a tangent to it at A

, then $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are drawn to cut the other circle at D and E

Prove that : $\overleftrightarrow{AF}$ // $\overline{DE}$

(Kafr El-Sheikh 2009)

15

In the opposite figure :

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$ and Y ∈ $\overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

(Cairo 17 , El-Kalyoubia 14 , Port Said 13)

16 ABCD is a quadrilateral inscribed in a circle , E is a point outside the circle and $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are two tangents to the circle at A and B , If m (∠ AEB) = 70° and m (∠ ADC) = 125° , **prove that :**

(1) AB = AC

(2) $\overrightarrow{AC}$ is a tangent to the circle passing through the points A , B and E *(Alex. 17)*

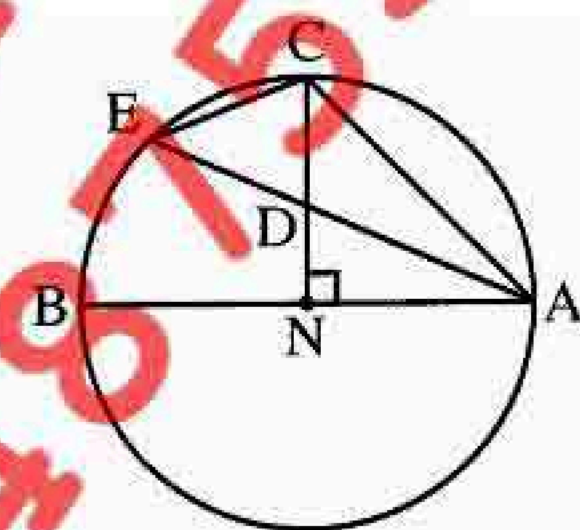
17 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle N

, $\overline{NC} \perp \overline{AB}$, D ∈ $\overline{NC}$

and $\overrightarrow{AD}$ is drawn to cut the circle at E

Prove that : $\overrightarrow{AC}$ is a tangent to the circle circumscribed about Δ CDE



(Kafr El-Sheikh 2008)

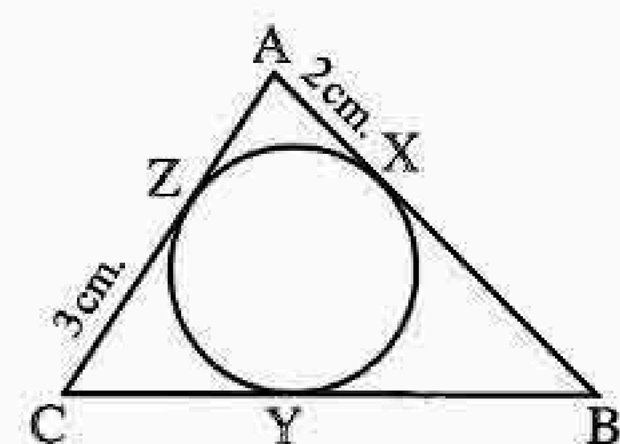
18 **In the opposite figure :**

Δ ABC touches the circle externally at X , Y and Z

If the perimeter of Δ ABC = 18 cm.

, AX = 2 m. and CZ = 3 cm.

Calculate : The length of $\overline{BY}$



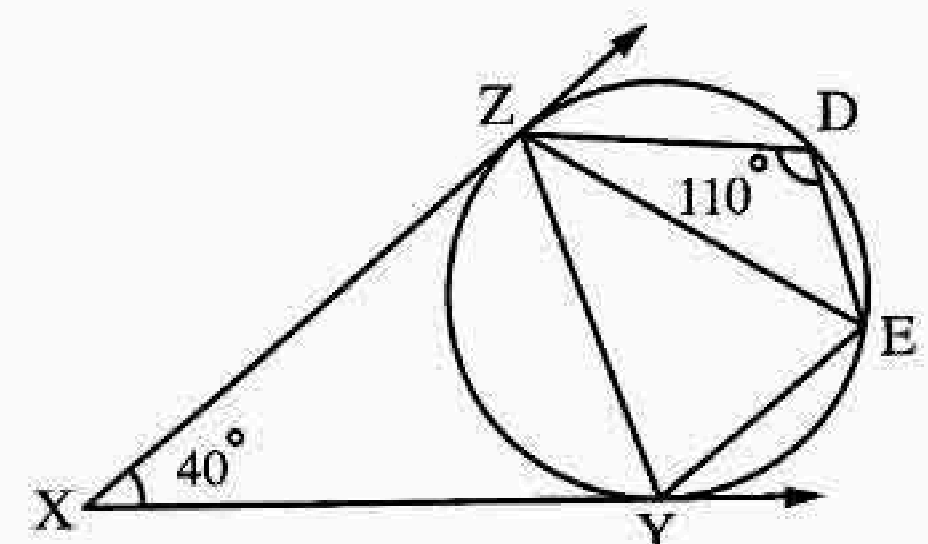
(Sharkia 03) « 4 cm. »

19 **In the opposite figure :**

$\overrightarrow{XY}$ and $\overrightarrow{XZ}$ are two tangents to the circle from the point X

, m (∠ D) = 110° , m (∠ X) = 40°

Prove that : m ($\overset{\frown}{ZDE}$) = m ($\overset{\frown}{ZY}$)



(Assiut 17 , El-Gharbia 17)

20 **In the opposite figure :**

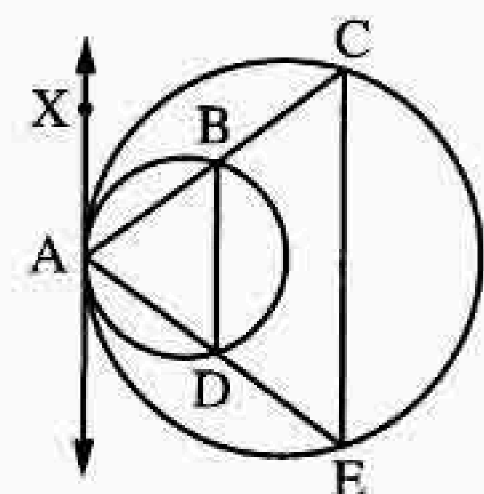
Two circles are touching internally at A

, $\overleftrightarrow{AX}$ is the common tangent to them at A

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ intersect the small circle at B , D

and the great circle at C , E

Prove that : $\overline{DB}$ // $\overline{EC}$



(El-Gharbia 15 , El-Monofia 14 , Souhag 13)

21 **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{CD}$ is a tangent to the circle at C

Draw $\overline{DE}$ // $\overline{AC}$ to cut $\overline{AB}$ at E

Prove that : BECD is a cyclic quadrilateral. *(Menia 2009)*

22 **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle in which

, AB = AD , m (∠ C) = 140° and m (∠ ADE) = 70°

Prove that :

$\overrightarrow{DE}$ is a tangent to the circle at D *(Menia 09)*

23 $\overline{AB}$ and $\overline{AC}$ are two chords in a circle such that AB = AC , D ∈ $\overline{BC}$ and $\overrightarrow{AD}$ is drawn to cut the circle at E

Prove that : $\overline{AC}$ is a tangent-segment to the circumcircle of Δ CDE *(Fayoum 09)*

24 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M where AB = 8 cm.

, $\overline{AC}$ is a chord in it. Draw $\overrightarrow{BD}$ to be a tangent to the circle to cut $\overrightarrow{AC}$ at D. If BD = 6 cm.

(1) **Prove that :** $\overleftrightarrow{AB}$ is a tangent to the circumcircle of Δ CBD

(2) **Find :** The length of $\overline{BC}$ *(Monofia 2009)* « 4.8 cm. »

25 **In the opposite figure :**

M , N are two circles touching externally at C ,

$\overline{AD}$ touching the circle M at D ,

$\overline{AB}$ touching the circle N at B

If m (∠ DAC) = 55° , m (∠ CNB) = 110° ,

MN = 6 cm. , AD = 5 cm.

Prove that :

(1) **Prove that :** AD = AC = AB

(2) **Find :** The perimeter of ABNMD

(3) **Prove that :** $\overrightarrow{NA}$ bisects ∠ CNB

(4) **Prove that :** $\overline{AD}$ is a tangent - segment to the circle passes through the points A , C and N

(El-Ismailia 2015)